CSI 4105 Design and Analysis of Algorithms II — Winter 2012
Computer Science — University of Ottawa

Homework Assignment #2 (100 points, weight 6.67%)
Due: Friday Mar 16, at 11:30 p.m. (in lecture)

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1. (25 points) Let NearbySet be the problem defined as follows. Given a graph G and a number k, is there a way to select a set $N \subseteq V(G)$ with $|N| = k$ such that every vertex in the graph is either in N or is connect by an edge to a vertex in N. Show that NearbySet is NP-complete.

2. (25 marks) Consider the treasure splitting problem: there are n objects $1, 2, \ldots, n$ each of value v_i, $1 \leq i \leq n$. Two pirates need to split the treasures evenly. The TreasureSplitting problem asks: given $v_1, v_2, \ldots, v_n$ is it possible to partition $\{1, 2, \ldots, n\}$ into two sets S_1, S_2 (partitioning means $S_1 \cup S_2 = \{1, 2, \ldots, n\}$ and $S_1 \cap S_2 = \emptyset$) such that

$$\sum_{i \in S_1} v_i = \sum_{j \in S_2} v_j \ ?$$

   Prove that TreasureSplitting is NP-complete.

3. (25 points) Consider a special case of QSAT (Quantified 3-SAT) in which the formula $\phi(x_1, \ldots, x_n)$ has no negated variables. We define the decision problem NNQSAT to be the problem of deciding the truth value of:

$$\exists x_1 \forall x_2 \ldots \exists x_{n-2} \forall x_{n-1} \exists x_n \ \phi(x_1, x_2, \ldots, x_n),$$

   where n is odd and $\phi(x_1, x_2, \ldots, x_n)$ is a 3-CNF formula with no negated variables. Give a polynomial time algorithm to solve NNQSAT; analyse the running time of the algorithm.

4. (25 points) Define the choice set and describe a backtracking algorithm for the problem: given G and k, find all k-vertex colourings of G.