CSI 2101 Discrete Structures — Winter 2012

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Quiz #5

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1. **S4.1, Exercise 31:** Use induction to show that $2|(n^2+n)$ for all positive integers n.

   Let $P(n)$ be the statement: $2|(n^2+n)$.

   Basis step: We show that $P(1)$ is true: $(1)^2+1=2$, and $2|2$.

   Inductive hypothesis: Let k be a positive integer and assume $P(k)$ is true, i.e. $2|(k^2+k)$. This means that there exists an integer j such that $k^2+k=2j$.

   Inductive step: Show that $P(k+1)$ is true, i.e. $2|((k+1)^2+(k+1))$. We have that:

$$\begin{aligned}(k+1)^2+(k+1) &= k^2+2k+1+k+1\\ &= (k^2+k)+2k+2\\ &= 2j+2k+2 && \text{by the inductive hypothesis}\\ &= 2(j+k+1)\end{aligned}$$

   Hence, $2|((k+1)^2+(k+1))$.

   Thus, for all positive integers n, $2|(n^2+n)$ as shown by induction.