CSI 2101 Discrete Structures | Winter 2012
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Quiz #4

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1. **S3.4, Exercise 6:** Show that if a, b, c, and d are integers such that $a|c$ and $b|d$, then $ab|cd$.

   If $a|c$, then there exists an integer m such that $am = c$, and if $b|d$, then there exists an integer n such that $bn = d$. Then $cd = (am)(bn) = (mn)(ab)$, and thus $ab|cd$.

2. **S3.4, Exercise 24:** Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$.

   If n is odd, we can write $n = 2k + 1$ for some integer k. Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$. To show that $n^2 \equiv 1 \pmod 8$, it is sufficient to show that $8|(n^2 - 1)$. We have that $n^2 - 1 = 4k^2 + 4k = 4k(k + 1)$. Now, we have two cases to consider: if k is even, there is some integer d such that $k = 2d$. Then $n^2 - 1 = 4(2d)(2d+1) = 8d(d+1)$, and clearly this is divisible by 8 since it is a multiple of 8. If k is odd, then there is some integer d such that $k = 2d + 1$. Then $n^2 = 4(2d + 1)(2d + 2) = 8(2d + 1)(d + 1)$, and again, this is divisible by 8. Thus, in both cases, $n^2 - 1$ is divisible by 8, so $n^2 \equiv 1 \pmod 8$.