CSI 2101 Discrete Structures — Winter 2011
Prof. Lucia Moura — University of Ottawa

Homework Assignment #4 (100 points, weight 6.25%)
Due: Friday, April 8, at 4:00pm (in lecture)

Recurrence relations

1. (30 points) Find the solution to:

$$a_n = 5a_{n-2} - 4a_{n-4}$$

with $a_0 = 3$, $a_1 = 2$, $a_2 = 6$, $a_3 = 8$.

2. (40 points) Find the solution of the recurrence relation

$$a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n,$$

with $a_0 = -2, a_1 = 0$ and $a_2 = 5$.

3. (30 points) Consider the following algorithm:

procedure mpower (a, m, n: integers with $m \geq 2$ $n \geq 0$)
if $n = 0$ then return 1
else if n is even then
    $x =$ mpower$(a, n/2, m)$
    return $x * x$ **mod** m
  else
    $x =$ mpower$(a, \lfloor n/2 \rfloor, m)$
    return $((x * x \text{ mod } m) * a)$ mod m

(a) Set up a divide-and-conquer recurrence relation for the number of modular multiplications required to compute a^n **mod** m, where a, m and n are positive integers, using this recursive algorithm above.

(b) Use the recurrence relation you found in part (a) to construct a big-O estimate for the number of modular multiplications used to compute a^n **mod** m using this recursive algorithm. (Hint: apply the master theorem).