CSI 2101 Discrete Structures — Winter 2011
Prof. Lucia Moura — University of Ottawa

Homework Assignment #3 (100 points, weight 6.25%)
Due: Tuesday, March 29, at 2:30pm (in lecture)

Induction

1. For which non-negative integers n is $n^2 \leq n!$? Prove your answer using induction.

2. Suppose a store offers gift certificates in denominations of \$25 and \$40. Determined the possible total amounts you can form using these gift certificates. Prove your answers using strong induction.

3. For this question you need a few definitions:

   Definition 1 (full binary trees)
   Full binary trees can be defined recursive as follows:
   BASIS STEP: A tree formed by a single vertex r is a full binary tree.
   RECURSIVE STEP: If T_1 and T_2 are disjoint full binary trees, then the tree $T_1 \cdot T_2$, consisting of a root r connected to the roots of the left subtree T_1 and the right subtree T_2, is a full binary tree.

   Definition 2 (leaves and internal vertices of full binary trees)
   BASIS STEP: A tree formed by a single vertex r has a leaf node, r, and no internal nodes.
   RECURSIVE STEP: The leaves of $T_1 \cdot T_2$ is the union of the set of leaves of T_1 and the set of leaves of T_2. The internal vertices of $T_1 \cdot T_2$ are the root r and the union of the set of internal vertices of T_1 and the set of internal vertices of T_2.

   Question: Use structural induction to show that $l(T)$, the number of leaves of a full binary tree T, is one more than $i(T)$, the number of internal vertices of T.

4. (Correctness of recursive programs) Consider the following recursive program

   procedure CALC(int a, int b)
   if $(b = 0)$ then return 0;
   else if (b mod $2 = 0$) then return CALC$(a + a, b/2)$;
   else return CALC$(a + a, \lfloor b/2 \rfloor)+a$;

   Use strong induction to show that CALC(a, b) computes $a*b$ for all nonnegative integers a and b.