CSI 2101 Discrete Structures — Winter 2010
Prof. Lucia Moura — University of Ottawa

Homework Assignment #3 (100 points, weight 6.25%)
Due: March 29 at 10:00a.m. (in tutorial)

Induction and Recursion: Your best 4 questions will be used to calculate your mark.

1.
   - (10 points) Exercise 32, page 280 (induction to prove divisibility facts).
   - (15 points) Prove that this is the following recursive algorithm correctly computes $2 - (\frac{1}{2})^n$, for all $n \geq 0$.

   ```
   procedure P(n:nonnegative integer)
      if n = 0 then return 1
               else return 1 + (1/2)P(n − 1)
   ```

2. (25 points) Exercise 6, page 291-292 (postage problem using math induction and strong induction).

3. (25 points) Exercise 64, page 282 (celebrity identification). Show the statement for $n \geq 1$. Note that finding the celebrity with x questions really means doing so with **at most** x questions.

4. (25 points) Exercise 32 page 309 (structural induction for strings).
   Hint: Use definition 2 (strings) and definition 3 (concatenation of strings). The structural induction can be done based on the definition of strings applied to string t.

5. (25 points) (Program Verification) Consider the following iterative program that computes the nth Fibonacci number.

   ```
   procedure iterativeFibonacci(n:nonnegative integer)
      if n = 0 then return 0
      else
      begin
          x ← 0
          y ← 1
          i ← 1
          while i ≤ n − 1 do
          begin
              z ← x + y
              x ← y
              y ← z
              i ← i + 1
          end
          return y
      end
   ```

   Use program verification techniques (Hoare triples, loop invariantes) to prove that the above algorithm correctly computes f_n, the nth Fibonacci number, for $n \geq 0$.