CSI 2101 Discrete Structures — Winter 2010
Prof. Lucia Moura — University of Ottawa

Homework Assignment #1 (100 points, weight 6.25%)
Due: Monday Feb 1, at 10:00 p.m. (in tutorial)

Propositional Logic

1. (12 points) Use logical equivalences to show that $[\neg p \wedge (p \vee q)] \rightarrow q$ is a tautology.

2. (12 points) Recall that a collection of logical operators is *functionally complete* if every compound proposition is logically equivalent to a compound proposition using only these logical operators. The logical operator NOR, denoted by $\downarrow$, is true when both p or q are false, and is false otherwise. Show that $\{\downarrow\}$ is a functionally complete collection of operators, by proving the following steps.

   (a) Use truth tables to show that $p \downarrow p$ is logically equivalent to $\neg p$.

   (b) Use truth tables to show that $(p \downarrow q) \downarrow (p \downarrow q)$ is logically equivalent to $p \vee q$.

   (c) Complete the argument by using the fact that $\{\neg, \vee\}$ is functionally complete.

Predicate Logic

3. (12 points) Exercise 34, page 48. (English-predicates-negate-English)

4. (12 points) Exercise 36, page 49. (Counter examples to universally quantified statements)

5. (16 points) Exercise 32, page 61. Note: show your steps. (Negations of nested quantified statements)

Inference Rules

6. (14 points) Exercise 12, page 73. (Formal proofs)

7. (10 points) Exercise 20, page 74. Please justify a "yes" by referring to a rule of inference applied and a "no" by pointing out the fallacy. (Validity of arguments)

Proof Methods

8. (12 points) Use a proof by contraposition to prove the following:
   Let x, y and a be real numbers. If $x + y \geq a$ then $x \geq a/2$ or $y \geq a/2$.