CSI2101-2009 - ASSIGNMENT#1;
STUDENT ID: .... NAME: ....

Hand in at the assignment drop box for this course at SITE 1st floor by the due dates:
Part 1. Propositional logic: Wednesday, January 28 at 12:30pm
Part 2. Predicate logic: Wednesday, February 4 at 12:30pm.

1. Propositional Logic (30/100 marks)

Instructions for Part 1. Answer these questions in a separate piece of paper, in order. Put your name and student id in all pages and staple them. (-3 points, if not followed)

(1) *(Ex 24 and 26, p.29; 2 marks) Use logical equivalences, to show that:*
- $(p \to q) \vee (p \to r)$ *and* $p \to (q \vee r)$ *are logically equivalent.*
- $\neg p \to (q \to r)$ *and* $q \to (p \vee r)$ *are logically equivalent.*

(2) *(Ex. 33, p 29, 2 marks) Show that* $(p \to q) \to (r \to s)$ *and* $(p \to r) \to (q \to s)$ *are not logically equivalent.*

(3) *(Ex. 56, p 30, 4 marks) Show that if* p*,* q*, and* r *are compound propositions such that* p *and* q *are logically equivalent and* q *and* r *are logically equivalent, then* p *and* r *are logically equivalent.*

(4) *(Ex. 52, p 29, 8 marks) A collection of logical operators is* functionally complete *if every compound proposition is logically equivalent to a compound proposition using only these logical operators. You have learned that* $\{\neg, \wedge\}$ *is a functionally complete collection of logical operators. The same is true for* $\{\neg, \vee\}$*. The logical operator NAND, denoted by* $|$*, is true when either* p *or* q*, or both, are false. Show that* $\{|\}$ *is a functionally complete collection of operators.*
Hint: check the steps used for Exercise 50. Note I suspect there is a typo in 50-c, where "Exercise 49" should read "Exercise 45").

(5) *(Ex 60, p 30, 6 marks) Which of these compound propositions are satisfiable/why?*
  (a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
  (b) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$
  (c) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$

(6) *(8 marks) Consider the boolean function* $Agreement : \{0,1\}^3 \to \{0,1\}$*, which is the function that has value 1 if all three inputs are identical (all are* 0 *or all are* 1*).*
  (a) *Write a truth table for a proposition that* represents *the boolean function* $Agreement(x, y, z)$*.*
  (b) *Find a compound proposition in DNF (disjunctive normal form) that represents the boolean function* $Agreement(x, y, z)$*.*
  (c) *Find a compound proposition in CNF (conjunctive normal form) that represents the boolean function* $Agreement(x, y, z)$*.*
  (d) *Draw a circuit that computes the boolean function* $Agreement(x, y, x)$*.*

2. Predicate Logic (75/100, due February 4, 12:30pm)

Ex.(7)-(14) are to be solved in the given space in these pages, while 15 and 16 can be handed in a separate page (staple all!) *(-7 if not followed)*

(7) *(Ex. 10, p. 47, 5 marks) Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement "x has a dog" and let $F(x)$ be the statement "x has a ferret". Express each of the following statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers and logical connectives. Let the domain consist of all students in your class.*

(a) *A student in your class has a cat, a dog and a ferret.*
A:

(b) *All students in your class have a cat, a dog, or a ferret.*
A:

(c) *Some student in your class has a cat and a ferret, but not a dog.*
A:

(d) *No student in your class has a cat, a dog and a ferret.*
A:

(e) *For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.*
A:

(8) *(Ex. 12, p. 47, 4*1+2+2+2=10 marks) Let $Q(x)$ be the statement "$x + 1 > 2x$". If the domain consists of all integers, what are these truth values?*

(a) $Q(0)$ *true* *false* *(circle one)*

(b) $Q(-1)$ *true* *false*

(c) $Q(1)$ *true* *false*

(d) $\exists x Q(x)$ *true* *false*
Justify:

(e) $\forall x Q(x)$ *true* *false*
Justify:

(f) $\exists x \neg Q(x)$ *true* *false*
Justify:

(g) $\forall x \neg Q(x)$ *true* *false*
Justify:

(9) *(Ex. 20, p. 47, 1+1+2+2+2=8 marks) Suppose the domain of the propositional function $P(x)$ consists of -5,-3,-1,1,3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions and conjunctions.*

(a) $\exists x P(x)$

(b) $\forall x P(x)$

(c) $\forall x((x \neq 1) \rightarrow P(x))$

(d) $\exists x((x \geq 0) \wedge P(x))$

(e) $\exists x(\neg P(x)) \wedge \forall x((x < 0) \rightarrow P(x))$

(10) *(Ex. 38, p. 49, 5 marks) Translate these system specifications into English where the predicate $S(x, y)$ is "x is in state y" and where the domain for x and y consists of all systems and all possible states, respectively.*

(a) $\exists S(x, \text{open})$

(b) $\forall x(S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$

(c) $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$

(d) $\exists x \neg S(x, \text{available})$

(e) $\forall x \neg S(x, \text{working})$

(11) *(Ex. 58, p.50, 3 marks) Suppose that Prolog facts are used to define predicates $mother(M, Y)$ and $father(F, X)$, which represent that "M is the mother of Y" and "F is the father of X", respectively. Give a Prolog rule to define the predicate $grandfather(X, Y)$, which represents that X is the grandfather of Y.* [Hint: You can write a disjunction in Prolog either by using a semicolon or by putting these predicates on separate lines.]

(12) *(Ex. 12, p. 59, 1+1+2+2+2+2= 10 marks) Let* $I(x)$ *be the statement "x has an internet connection" and* $C(x, y)$ *be the statement "x and y have chatted over the internet",where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements:*

c) *Jan and Sharon never chatted over the internet.*

d) *No one in the class has chatted with Bob.*

e) *Sanjay has chatted with everyone except Joseph.*

j) *Everyone in your class with an internet connection has chatted over the internet with at least one other student in your class.*

k) *Someone in your class has an internet connection but has not chatted with anyone else in your class.*

n) *There are at least two students in your class who have not chatted with the same person in your class.*

(13) *(Ex. 40, p. 62, 6 marks) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.*

(a) $\forall x \exists y (x = 1/y)$

(b) $\forall x \exists y (y^2 - x < 100)$

(c) $\forall x \forall y (x^2 \neq y^3)$

(14) *(Ex. 46, p. 62, 6 marks) Determine the truth value of the statement* $\exists x \forall y (x \leq y^2)$ *if the domain of the variables consists of*

(a) *the positive real numbers.*
Justification:

(b) *the integers.*
Justification:

(c) *the nonzero real numbers.*
Justification:

(15) *(Ex. 32, p.61, 3+3+3+3=12 marks) Express the negations of each of these statements so that all negation symbols immediately precede predicates (that is, no negation is outside a quantifier or an expression involving logical connectives). Show al the steps in your derivation.*

(a) $\exists z \forall y \forall x T(x, y, z)$
(b) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$
(c) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$
(d) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

(16) *(Ex. 50, p. 50, 5 marks) Show that* $\forall x P(x) \vee \forall x Q(x)$ *and* $\forall x (P(x) \vee Q(x))$ *are not logically equivalent.*
Hint: In order to do this, it is enough to show an "interpretation" (a choice for domain and meaning for $P(x)$ and $Q(x)$) for which these two statement have a different value (one is false and one is true).