ITI 1121. Introduction to Computing II *

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Version of February 2, 2013

Abstract

- Abstract data type: Stack
  - Stack-based algorithms

*These lecture notes are meant to be looked at on a computer screen. Do not print them unless it is necessary.

Evaluating arithmetic expressions

Stack-based algorithms are used for **syntactical analysis** (*parsing*).

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Compilers use similar algorithms to check the syntax of your programs and generate machine instructions (executable).

To verify that parentheses are balanced: '([])' is ok, but not '([)]' or ')((())('.

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During the **lexical analysis** (*scanning*) the source code is read from left to right and the characters are regrouped into **tokens**, which are successive characters that constitute numbers or identifiers. One of the tasks of the lexical analyser is to remove spaces from the input.

E.g.:

$$\cdot 10 \cdot + \cdot \cdot 2 + \cdot \cdot \cdot 300$$

where "·" represent blank spaces, is transformed into the following list of tokens:

[10,+,2,+,300]

The next step is the syntactical analysis (*parsing*) and consists in regrouping the tokens into grammatical units, for example the sub-expressions of RPN expressions (seen in class this week).

In the next slides, we look at simple examples of lexical and syntactical analysis.

```java
public class Test {

    public static void scan( String expression ) {

        Reader reader = new Reader( expression );

        while ( reader.hasMoreTokens() ) {
            System.out.println( reader.nextToken() );
        }
    }

    public static void main( String[] args  ) {
        scan( " 3     + 4 * 567   " );
    }
}
// > java Test
// INTEGER: 3
// SYMBOL: +
// INTEGER: 4
// SYMBOL: *
// INTEGER: 567
```

```
public class Token {
    private static final int INTEGER = 1;
    private static final int SYMBOL = 2;
    private int iValue;
    private String sValue;
    private int type;

    public Token( int iValue ) {
        this.iValue = iValue;
        type = INTEGER;
    }
    public Token( String sValue ) {
        this.sValue = sValue;
        type = SYMBOL;
    }

    public int iValue() { ... }
    public String sValue() { ... }

    public boolean isInteger() { return type == INTEGER; }
    public boolean isSymbol() { return type == SYMBOL; }
}
```

LR Scan

```
public static int execute( String expression ) {
    Token op = null; int l = 0, r = 0;

    Reader reader = new Reader( expression );
    l = reader.nextToken().iValue();

    while ( reader.hasMoreTokens() ) {
        op = reader.nextToken();
        r = reader.nextToken().iValue();
        l = eval( op, l, r );
    }
    return l;
}
```

eval(Token op, int l, int r)

```
public static int eval( Token op, int l, int r ) {

    int result = 0;

    if ( op.sValue().equals( "+" ) )
        result = l + r;
    else if ( op.sValue().equals( "-" ) )
        result = l - r;
    else if ( op.sValue().equals( "*" ) )
        result = l * r;
    else if ( op.sValue().equals( "/" ) )
        result = l / r;
    else
        System.err.println( "not a valid symbol" );

    return result;
}
```

Evaluating an arithmetic expression: LR Scan

Left-to-right algorithm:

```
Declare L, R and OP

Read L
While not end-of-expression
do:
  Read OP
  Read R
  Evaluate L OP R
  Store result in L
```

At the end of the loop the result can be found in L.

```
3 * 8 - 10
```

```
   3 + 4 - 5
   ^

   L  = 3
   OP =
   R  =

>  Read L
   While not end-of-expression
   do:
     Read OP
     Read R
     Evaluate L OP R
     Store result in L
```

```
3 + 4 - 5
  ^

L  = 3
OP = +
R  =

Read L
While not end-of-expression
do:
>    Read OP
     Read R
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```

```
   3 + 4 - 5
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   L  = 3
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   R  = 4

   Read L
   While not end-of-expression
   do:
     Read OP
>    Read R
     Evaluate L OP R
     Store result in L
```

```
3 + 4 - 5
    ^


L  = 3
OP = +
R  = 4

Read L
While not end-of-expression
do:
  Read OP
  Read R
>   Evaluate L OP R (7)
  Store result in L
```

```
3 + 4 - 5
    ^

L  = 7
OP = +
R  = 4

Read L
While not end-of-expression
do:
  Read OP
  Read R
  Evaluate L OP R
>   Store result in L
```

```
   3 + 4 - 5
         ^

   L  = 7
   OP = -
   R  = 4

   Read L
   While not end-of-expression
   do:
>    Read OP
     Read R
     Evaluate L OP R
     Store result in L
```

```
3 + 4 - 5
        ^


L  = 7
OP = -
R  = 5

Read L
While not end-of-expression
do:
  Read OP
>   Read R
  Evaluate L OP R
  Store result in L
```

```
   3 + 4 - 5
           ^

   L  = 7
   OP = -
   R  = 5

   Read L
   While not end-of-expression
   do:
     Read OP
     Read R
>    Evaluate L OP R (2)
     Store result in L
```

```
   3 + 4 - 5
           ^


   L  = 2
   OP = -
   R  = 5

   Read L
   While not end-of-expression
   do:
     Read OP
     Read R
     Evaluate L OP R
>    Store result in L
```

```
3 + 4 - 5
         ^

L  = 2
OP = -
R  = 5

Read L
While not end-of-expression
do:
  Read OP
  Read R
  Evaluate L OP R
  Store result in L
```

>

⇒ end of expression, exit the loop, L contains the result.

What do you think?

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Without **parentheses** the following expression cannot be evaluated correctly:

$\Rightarrow$ 7 - (3 - 2)

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Because the result of the left-to-right analysis corresponds to:

$\Rightarrow$ (7 - 3) - 2

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⇒ 7 - 3 * 2

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Since the left-to-right analysis corresponds to:

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Because the result of the left-to-right analysis corresponds to:

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Similarly the following expression cannot be evaluated by our simple algorithm:

$\Rightarrow$ 7 - 3 * 2

Since the left-to-right analysis corresponds to:

$\Rightarrow$ (7 - 3) * 2

But according to the **operator precedences**, the evaluation should have proceeded as follows:

$\Rightarrow$ 7 - (3 * 2)

Remarks

The left-to-right algorithm:

- Does not handle parentheses;
- Nor precedence.

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1. Use a different notation;
2. Develop more complex algorithms.

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- Does not handle parentheses;
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Solutions:

1. Use a different notation;
2. Develop more complex algorithms.

$\Rightarrow$ Both solutions involve stacks!

Notations

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```
 7  - (3  -  2)  =  7 3 2 - -
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prefix: the third notation consists in placing the operator before the operands, OP L R. The programming language Lisp uses a combination of parentheses and prefix notation: `(- 7 (* 3 2))`.

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[1] www.calculator.org/rpn.html

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Successively transform, one by one, all the sub-expressions following the same order of evaluation that you would normally follow to evaluate an infix expression.

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An infix expression $l \ \diamond \ r$ becomes $l \ r \ \diamond$, where l and r are sub-expressions and $\diamond$ is an operator.

$$9 \;/\; (2 \times 4 - 5)$$

$$9 \;/\; (\; 2 \;\times\; 4 \;-\; 5 \;)$$

$$9 \;/\; (\; \underbrace{2}_{l} \; \underbrace{\times}_{\diamond} \; \underbrace{4}_{r} \;-\; 5 \;)$$

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$$9 \;/\; (\; [\; 2 \; 4 \; \times \;] \;-\; 5 \;)$$

$$9 \;/\; (\; 2 \;\times\; 4 \;-\; 5\;)$$

$$9 \;/\; (\; \underbrace{2}_{l} \;\underbrace{\times}_{\diamond}\; \underbrace{4}_{r} \;-\; 5\;)$$

$$9 \;/\; (\; \overbrace{2}^{l} \;\overbrace{4}^{r}\; \overbrace{\times}^{\diamond} \;-\; 5\;)$$

$$9 \;/\; (\; [\,2\;4\;\times\,] \;-\; 5\;)$$

$$9 \;/\; (\; \underbrace{[\,2\;\;4\;\times\,]}_{l} \;\underbrace{-}_{\diamond}\; \underbrace{5}_{r}\;)$$

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$$\underbrace{9}_{l} \; \underbrace{/}_{\diamond} \; \underbrace{[\; 2 \; 4 \; \times \; 5 \; - \;]}_{r}$$

$$\overbrace{9}^{l} \; \overbrace{[\; 2 \; 4 \; \times \; 5 \; - \;]}^{r} \; \overbrace{/}^{\diamond}$$

$$9 \; 2 \; 4 \; \times \; 5 \; - \; /$$

Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace $l\ r \diamond$ by the result of the evaluation of $l \diamond r$.

$$9\ 2\ 4\ \times\ 5\ -\ /$$

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$$9\ 2\ 4\ \times\ 5\ -\ /$$

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$$9\ \overbrace{8}^{l \diamond r}\ 5\ -\ /$$

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$$9\ \overbrace{8}^{l \diamond r}\ 5\ -\ /$$

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$$9\ 2\ 4\ \times\ 5\ -\ /$$

$$9\ \underbrace{2}_{l}\ \underbrace{4}_{r}\ \underbrace{\times}_{\diamond}\ 5\ -\ /$$

$$9\ \overbrace{8}^{l\diamond r}\ 5\ -\ /$$

$$9\ \underbrace{8}_{l}\ \underbrace{5}_{r}\ \underbrace{-}_{\diamond}\ /$$

$$9\ \overbrace{3}^{l\ \diamond\ r}\ /$$

Evaluating a postfix expression (mentally)

Scan the expression from left to right. When the current element is an operator, apply the operator to its operands, i.e. replace $l\ r \diamond$ by the result of the evaluation of $l \diamond r$.

$$9\ 2\ 4\ \times\ 5\ -\ /$$

$$9\ \underbrace{2}_{l}\ \underbrace{4}_{r}\ \underbrace{\times}_{\diamond}\ 5\ -\ /$$

$$9\ \overbrace{8}^{l \diamond r}\ 5\ -\ /$$

$$9\ \underbrace{8}_{l}\ \underbrace{5}_{r}\ \underbrace{-}_{\diamond}\ /$$

$$9\ \overbrace{3}^{l\ \diamond\ r}\ /$$

$$\underbrace{9}_{l}\ \underbrace{3}_{r}\ \underbrace{/}_{\diamond}$$

Evaluating a postfix expression (mentally)

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$$9\ \overbrace{8}^{l \diamond r}\ 5\ -\ /$$

$$9\ \underbrace{8}_{l}\ \underbrace{5}_{r}\ \underbrace{-}_{\diamond}\ /$$

$$9\ \overbrace{3}^{l\ \diamond\ r}\ /$$

$$\underbrace{9}_{l}\ \underbrace{3}_{r}\ \underbrace{/}_{\diamond}$$

$$\overbrace{3}^{l\ \diamond\ r}$$

Evaluating a postfix expression

Until the end of the expression has been reached:

1. From left to right until the first operator;
2. Apply the operator to the two preceding operands;
3. Replace the operator and its operands by the result.

At the end we have result.

```
9   3   /   10   2   3 *   -   +
```

9 2 4 * 5 - /

Remarks: infix vs postfix

The order of the operands is the same for both notations, however operators are inserted at different places:

```
 2 + (3 * 4) = 2 3 4 * +

(2 + 3) * 4  = 2 3 + 4 *
```

Remarks: infix vs postfix

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```
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```

Evaluating an infix expression involves handling operators precedence and parenthesis — in the case of the postfix notation, those two concepts are embedded in the expression, i.e. the order of the operands and operators.

Algorithm: Eval Infix

What role will the stack be playing?

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```
operands = new stack;

while ( "has more tokens" ) {
  t = next token;
  if ( "t is an operand" ) {
    operands.push( "the integer value of t" );
  } else { // this is an operator
    op = "operator value of t";
    r = operands.pop();
    l = operands.pop();
    operands.push( "eval( l, op, r )" );
  }
}
return operands.pop();
```

Evaluating a postfix expression

The algorithm requires a stack (Numbers), a variable that contains the last element that was read (X) and two more variables, L and R, whose purpose is the same as before.

```
Numbers = [

While not end-of-expression
do:
  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers (right before left?!)
    L = POP Numbers
    Evaluate L X R; PUSH result onto Numbers
```

To obtain the final result: POP Numbers.

9 3 - 2 /

9 3 / 10 2 3 * - +

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                       ^


>  Numbers = [
   X =
   L =
   R =

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Create a new stack

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                      ^

   Numbers = [
   X = 9
   L =
   R =

   While not end-of-expression
   do:
>    Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Read X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                      ^

   Numbers = [9
   X = 9
   L =
   R =

   While not end-of-expression
   do:
     Read X
>    If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Push X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                        ^
```

```
   Numbers = [9
   X = 2
   L =
   R =

   While not end-of-expression
   do:
>    Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
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⇒ Read X

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⇒ Push X

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                          ^

   Numbers = [9 2
   X = 4
   L =
   R =

   While not end-of-expression
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       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ Read X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                          ^

   Numbers = [9 2 4
   X = 4
   L =
   R =

   While not end-of-expression
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```

⇒ Push X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                            ^

   Numbers = [9 2 4
   X = *
   L =
   R =

   While not end-of-expression
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   L =
   R = 4

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
>      R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Ah! X is an operator, pop the top element save into R

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                              ^


   Numbers = [9
   X = *
   L = 2
   R = 4

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
>      L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Top element is removed and saved into L

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                              ^
```

```
   Numbers = [9 8
   X = *
   L = 2
   R = 4

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
>      Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ Push the result of L X R, $2 \times 4 = 8$, onto the stack

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                ^

   Numbers = [9 8
   X = 5
   L = 2
   R = 4

   While not end-of-expression
   do:
>    Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Read X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                              ^

   Numbers = [9 8 5
   X = 5
   L = 2
   R = 4

   While not end-of-expression
   do:
     Read X
>    If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ Push X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                ^

   Numbers = [9 8 5
   X = -
   L = 2
   R = 4

   While not end-of-expression
   do:
>    Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Read X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                 ^

   Numbers = [9 8
   X = -
   L = 2
   R = 5

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
>      R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Remove the top element and save it into R

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                ^

   Numbers = [9
   X = -
   L = 8
   R = 5

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
>      L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Remove the top element and save it into L

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                ^

   Numbers = [9 3
   X = -
   L = 8
   R = 5

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
>      Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ Push the result of L X R, $8 - 5 = 3$, onto the stack

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                  ^

   Numbers = [9 3
   X = /
   L = 8
   R = 5

   While not end-of-expression
   do:
>    Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

⇒ Read X

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                   ^
```

```
   Numbers = [9
   X = /
   L = 8
   R = 3

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
>      R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ R = POP Numbers.

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                  ^

   Numbers = [
   X = /
   L = 9
   R = 3

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
>      L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ L = POP Numbers.

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                  ^

   Numbers = [3
   X = /
   L = 9
   R = 3

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
>      Evaluate L X R; PUSH result onto Numbers
```

$\Rightarrow$ Push L X R, $9 \div 3 = 3$, onto the stack sur la pile.

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                     ^

   Numbers = [3
   X = /
   L = 9
   R = 3

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
>      Evaluate L X R; PUSH result onto Numbers
```

⇒ End-of-expression

```
9 / ((2 * 4) - 5)  =  9 2 4 * 5 - /
                                    ^

   Numbers = [
   X = /
   L = 9
   R = 3

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Evaluate L X R; PUSH result onto Numbers
>
```

⇒ The result is "POP Numbers = 3"; the stack is now empty

Problem

Rather than evaluating an RPN expression, we would like to convert an RPN expression to infix (usual notation).

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Rather than evaluating an RPN expression, we would like to convert an RPN expression to infix (usual notation).

Hum?

Do we need a new algorithm?

No, a simple modification will do, replace "Evaluate L OP R" by "Concatenate (L OP R)".

Note: parentheses are essential (not all of them but some are).

This time the stack does not contain numbers but character strings that represent sub-expressions.

9 5 6 3 / - /

Postfix → infix

```
String rpnToInfix(String[] tokens)

   Numbers = [
   X =
   L =
   R =

   While not end-of-expression
   do:
     Read X
     If X isNumber, PUSH X onto Numbers
     If X isOperator,
       R = POP Numbers
       L = POP Numbers
       Concatenate ( L X R ); PUSH result onto Numbers
```

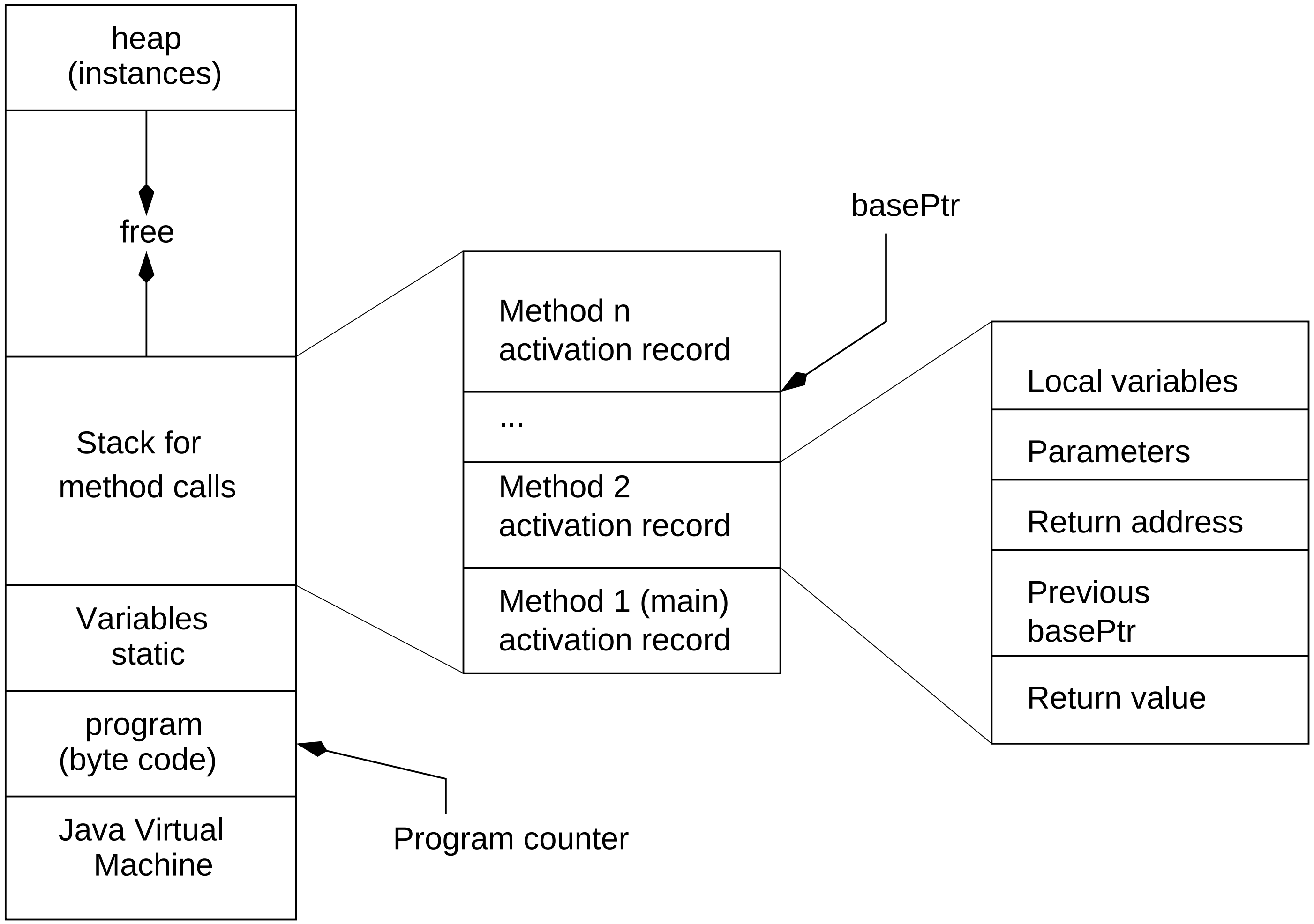
Postfix → ?

```
While not end-of-expression
do:
  Read X
  If X isNumber, PUSH X onto Numbers
  If X isOperator,
    R = POP Numbers
    L = POP Numbers
    Process L X R; PUSH result onto Numbers
```

We've seen an example where 'Process == Evaluate', then one where 'Process == Concatenate', but Process could also produce assembly code (i.e. machine instructions).

This shows how programs are compiled or translated.

Memory management



⇒ Schematic and simplified representation of the memory during the execution of a Java program.

Method call

The Java Virtual Machine (JVM) must:

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1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);

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Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;

Method call

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2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;

Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
5. Initial the local variables;

Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
5. Initial the local variables;
6. Start executing the instruction designated by the program counter.

Method call

The Java Virtual Machine (JVM) must:

1. Create a new activation record/block (which contains space for the local variables and the parameters among other things);
2. Save the current value of basePtr in the activation record and set the basePtr to the address of the current record;
3. Save the value of the program counter in the designated space of the activation record, set the program counter to the first instruction of the current method;
4. Copy the values of the effective parameters into the designated area of the current activation record;
5. Initial the local variables;
6. Start executing the instruction designated by the program counter.

$\Rightarrow$ activation block = *stack frame*, *call frame* or *activation record*.

When the method ends

The JVM must:

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1. Save the return value (at the designated space)

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2. Return the control to the calling method, i.e. set the program counter and basePtr back to their previous value;

When the method ends

The JVM must:

1. Save the return value (at the designated space)
2. Return the control to the calling method, i.e. set the program counter and basePtr back to their previous value;
3. Remove the current block;

When the method ends

The JVM must:

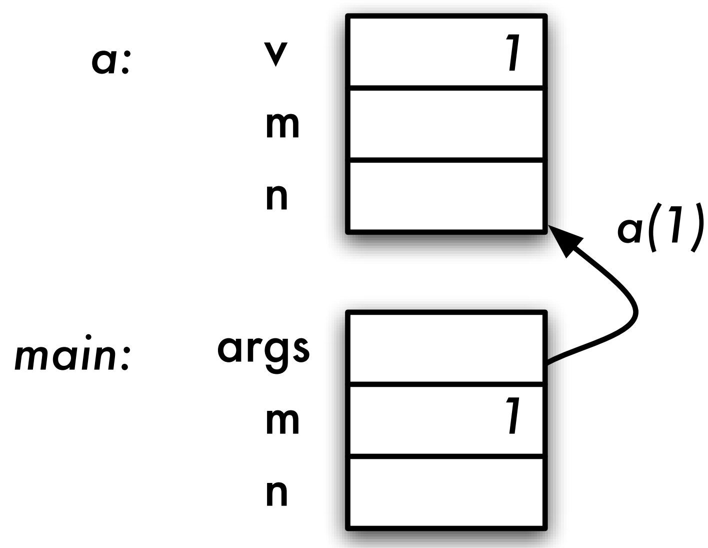
1. Save the return value (at the designated space)
2. Return the control to the calling method, i.e. set the program counter and basePtr back to their previous value;
3. Remove the current block;
4. Execute instruction designated by the current value of the program counter.

Example 1 (simplified)

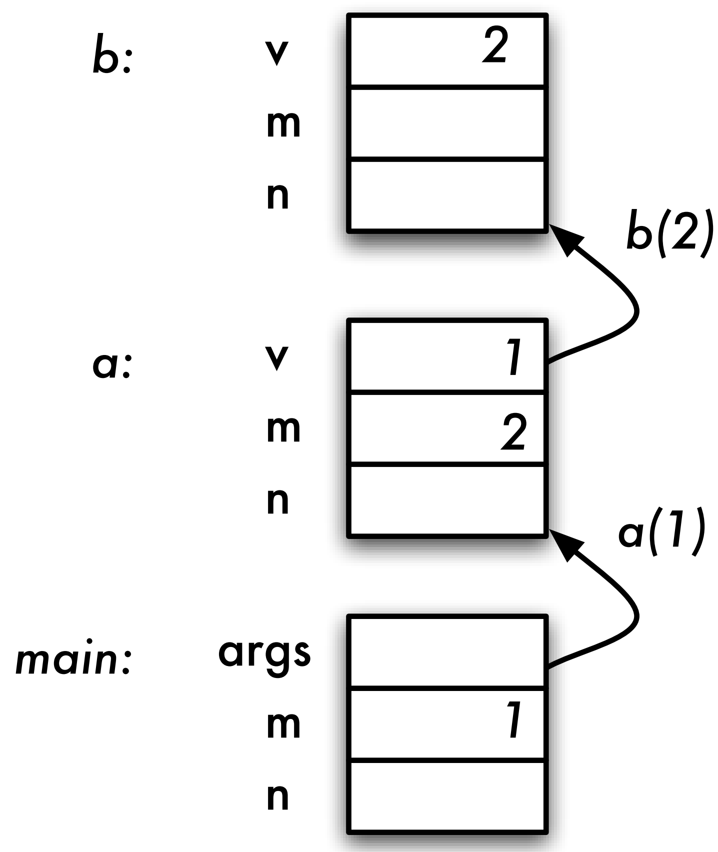
```
public class Calls {
    public static int c( int v ) {
        int n;
        n = v + 1;
        return n;
    }
    public static int b( int v ) {
        int m,n;
        m = v + 1;
        n = c( m );
        return n;
    }
    public static int a( int v ) {
        int m,n;
        m = v + 1;
        n = b( m );
        return n;
    }
```

```
    public static void main( String[] args ) {
        int m,n;
        m = 1;
        n = a( m );
        System.out.println( n );
    }
}
```

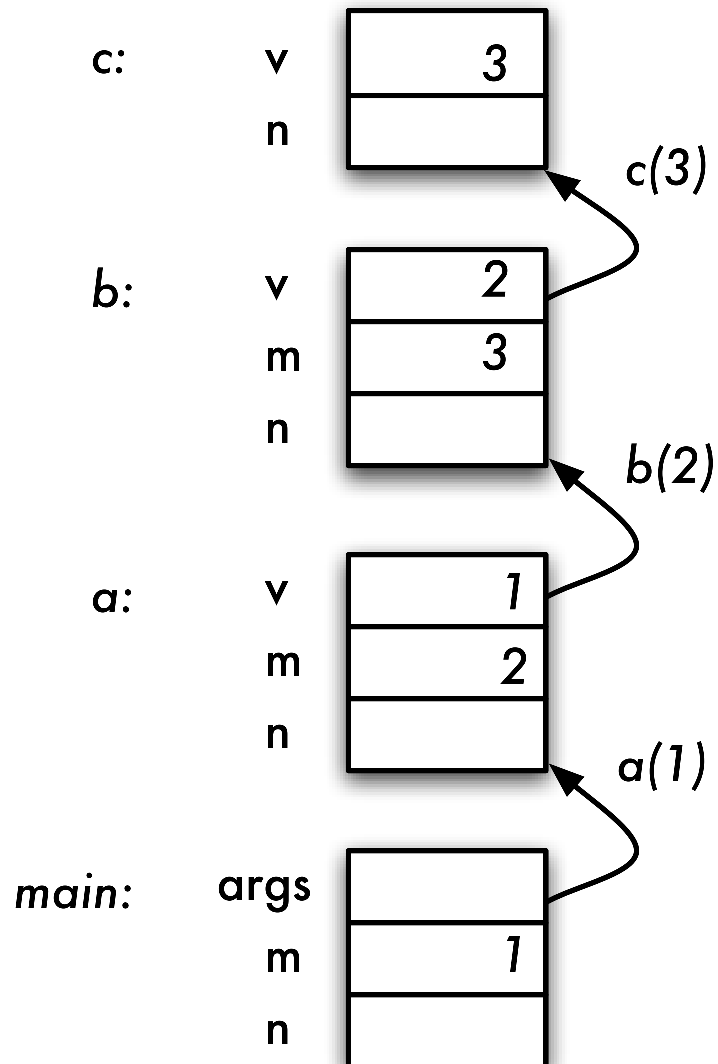
Example 1 (simplified)



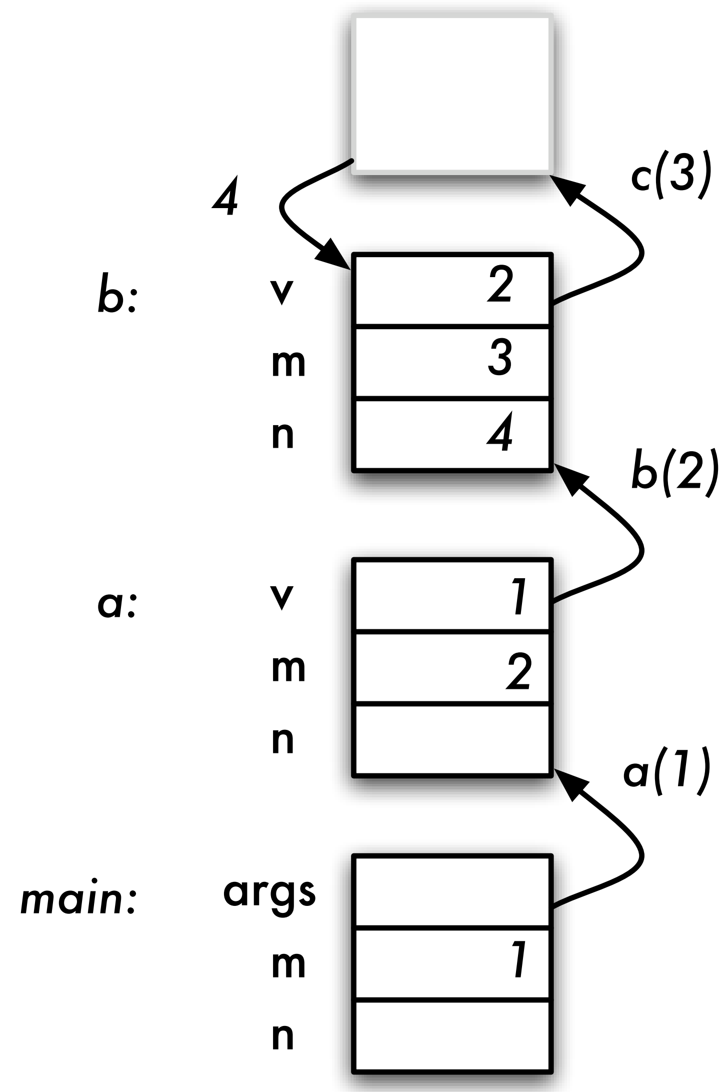
Example 1 (simplified)



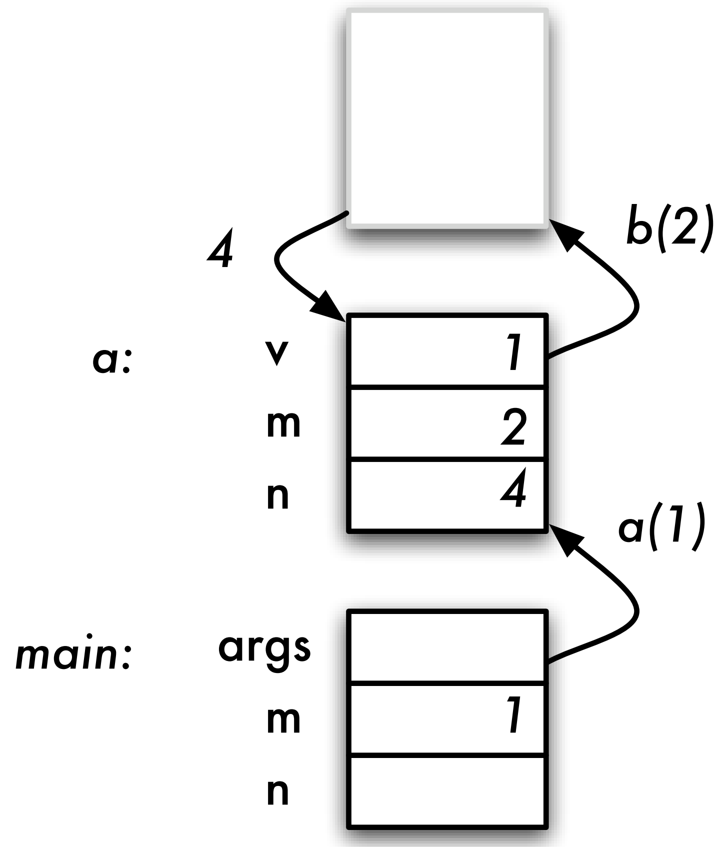
Example 1 (simplified)



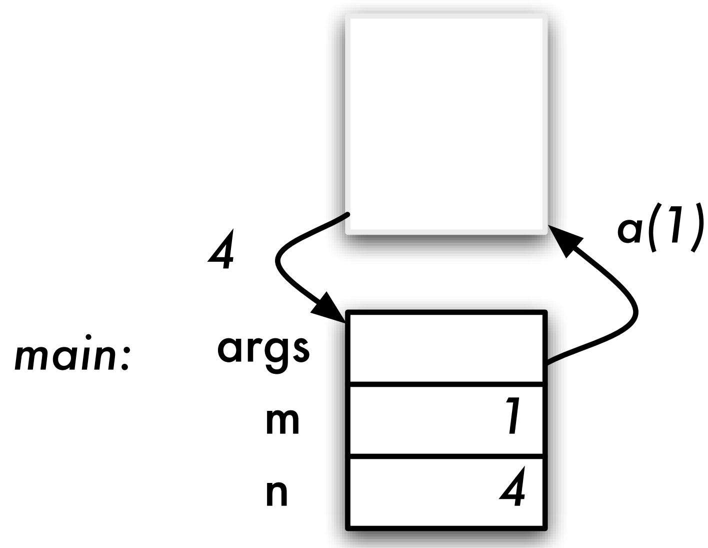
Example 1 (simplified)



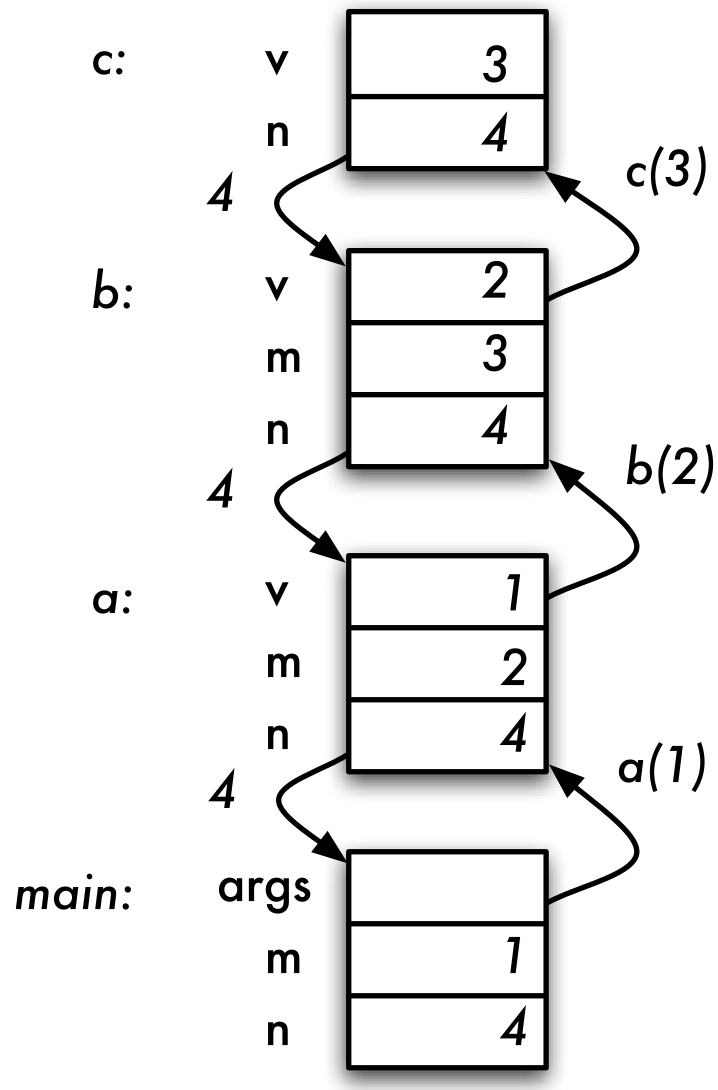
Example 1 (simplified)



Example 1 (simplified)



Example 1: summary



Example 2 (with a program counter)

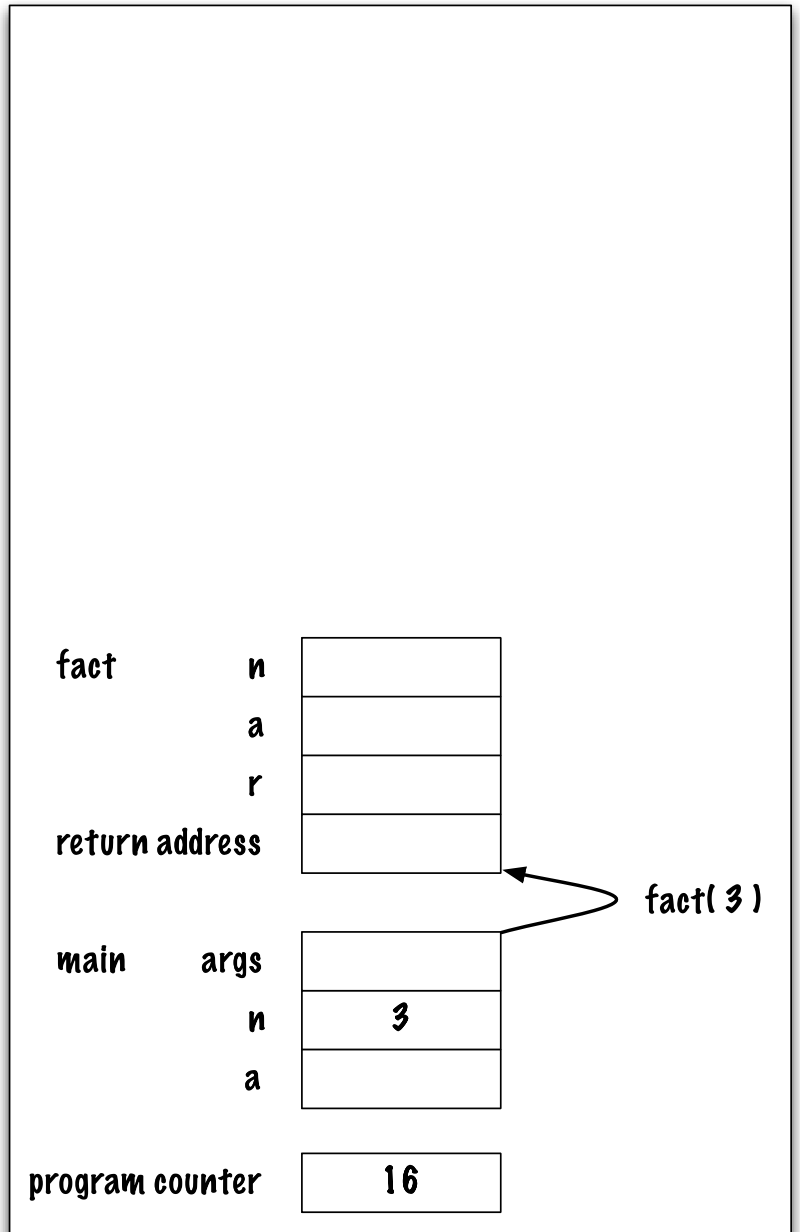
```
public class Fact {
    public static int fact( int n ) {
     // pre-condition: n >= 0
     int a, r;
     if ( n == 0 || n == 1 ) {
         a = 1;
     } else {
         r = fact( n-1 );
         a = n * r;
     }
     return a;
    }
    public static void main( String[] args ) {
     int a, n;
     n = 3;
     a = fact( n );
    }
}
```

main	args	
	n	
	a	

program counter	15

main	args	
	n	3
	a	

program counter	16


fact
n
a
r
return address
fact(3)
main
args
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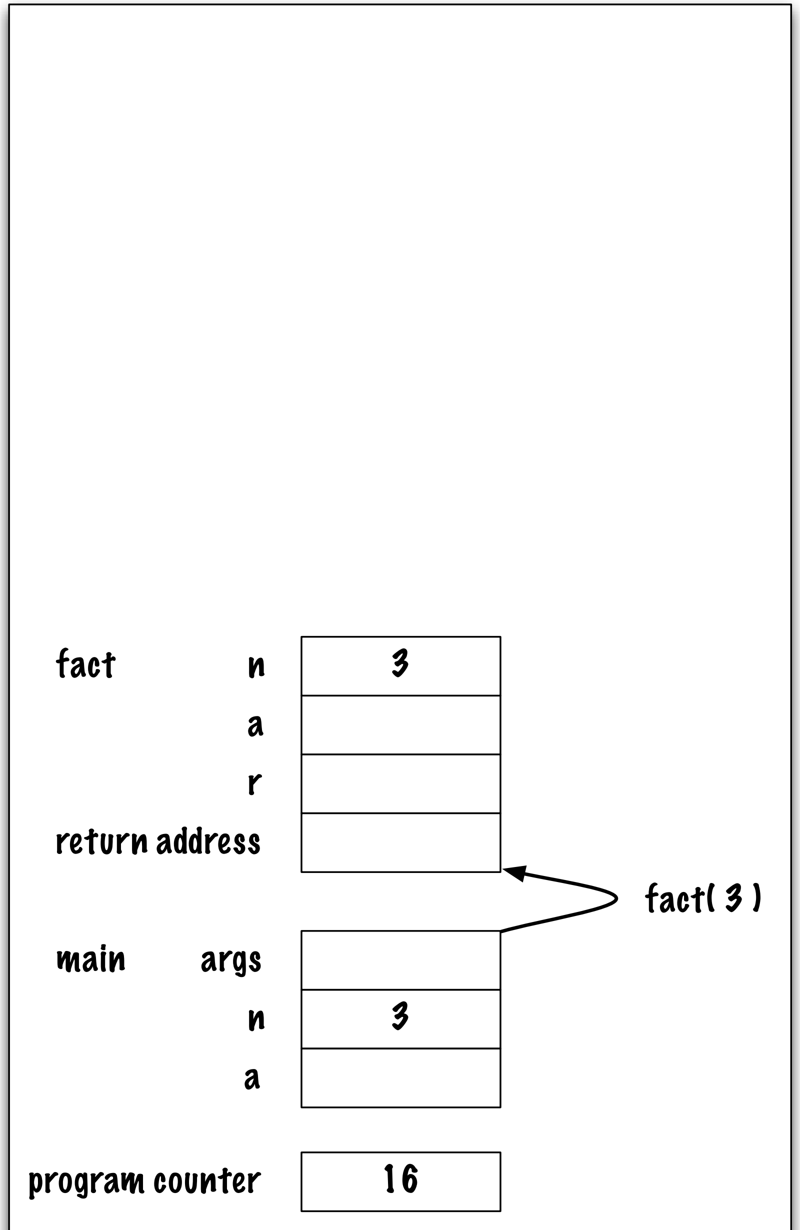
3
a
program counter
16


fact
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fact(3)
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program counter
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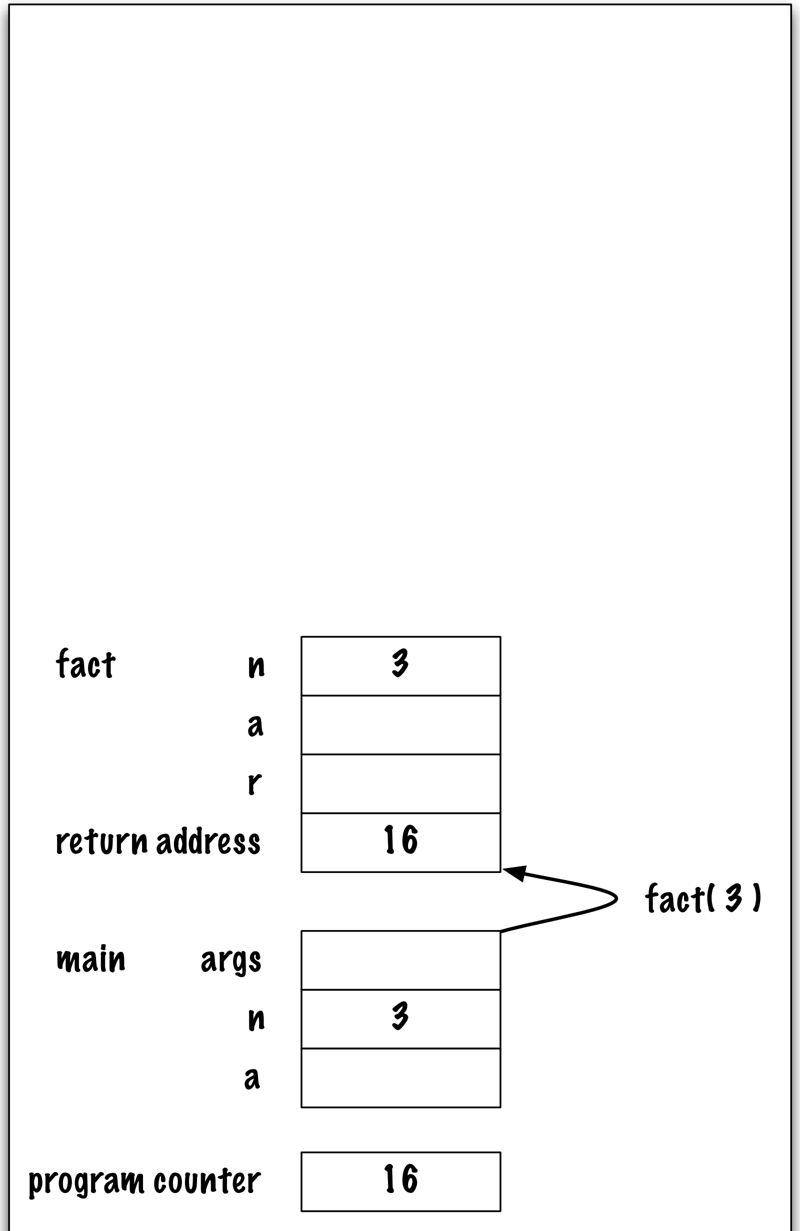
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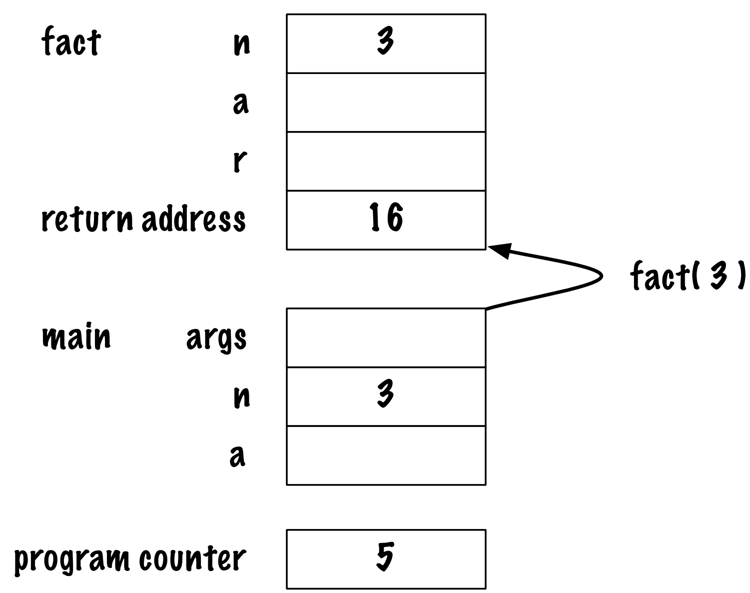
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main
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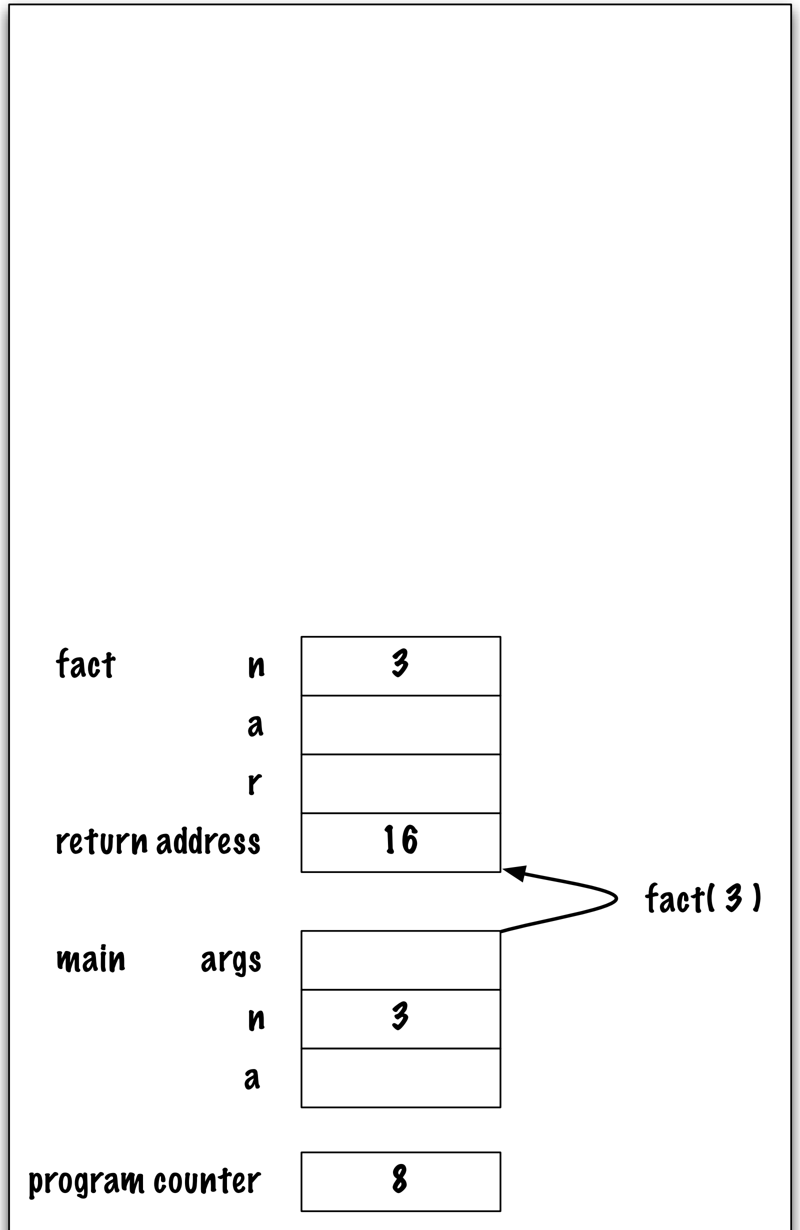
a
program counter
5


fact
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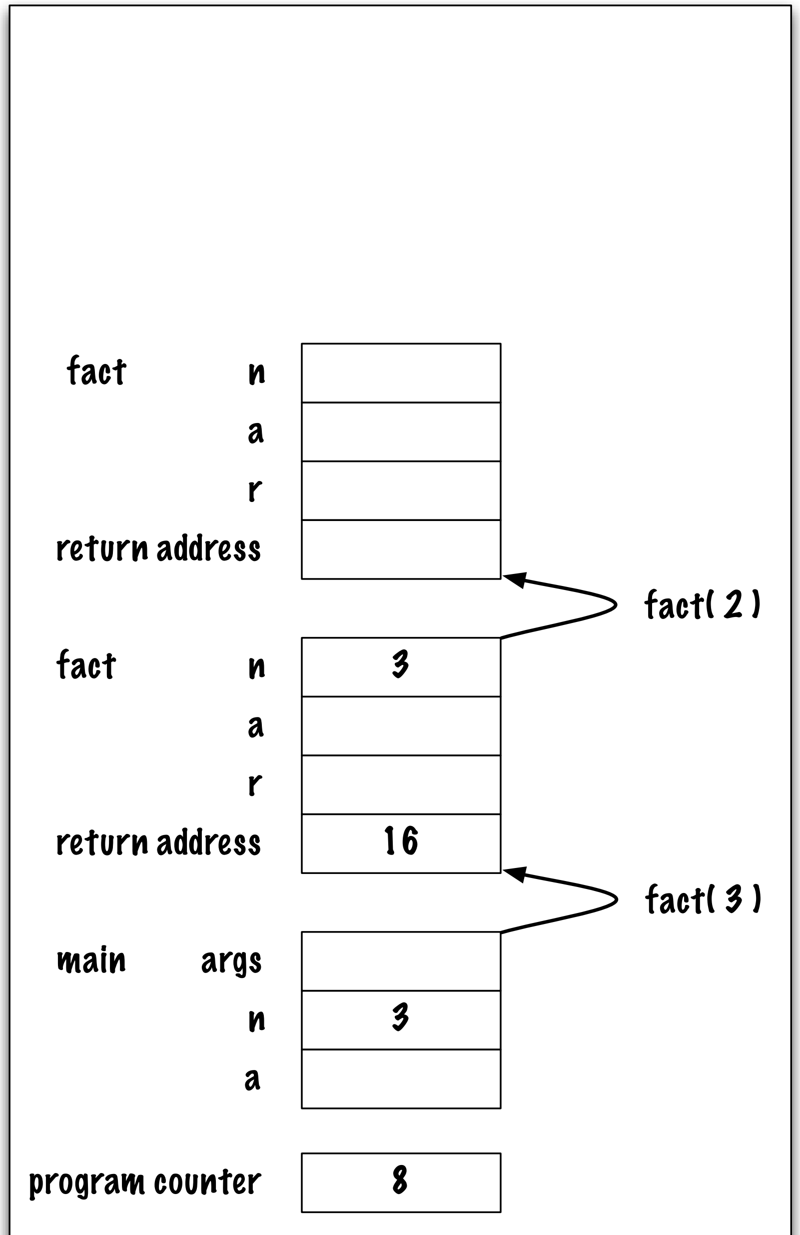
16
fact(3)
main
args
n
3
a
program counter
8


fact
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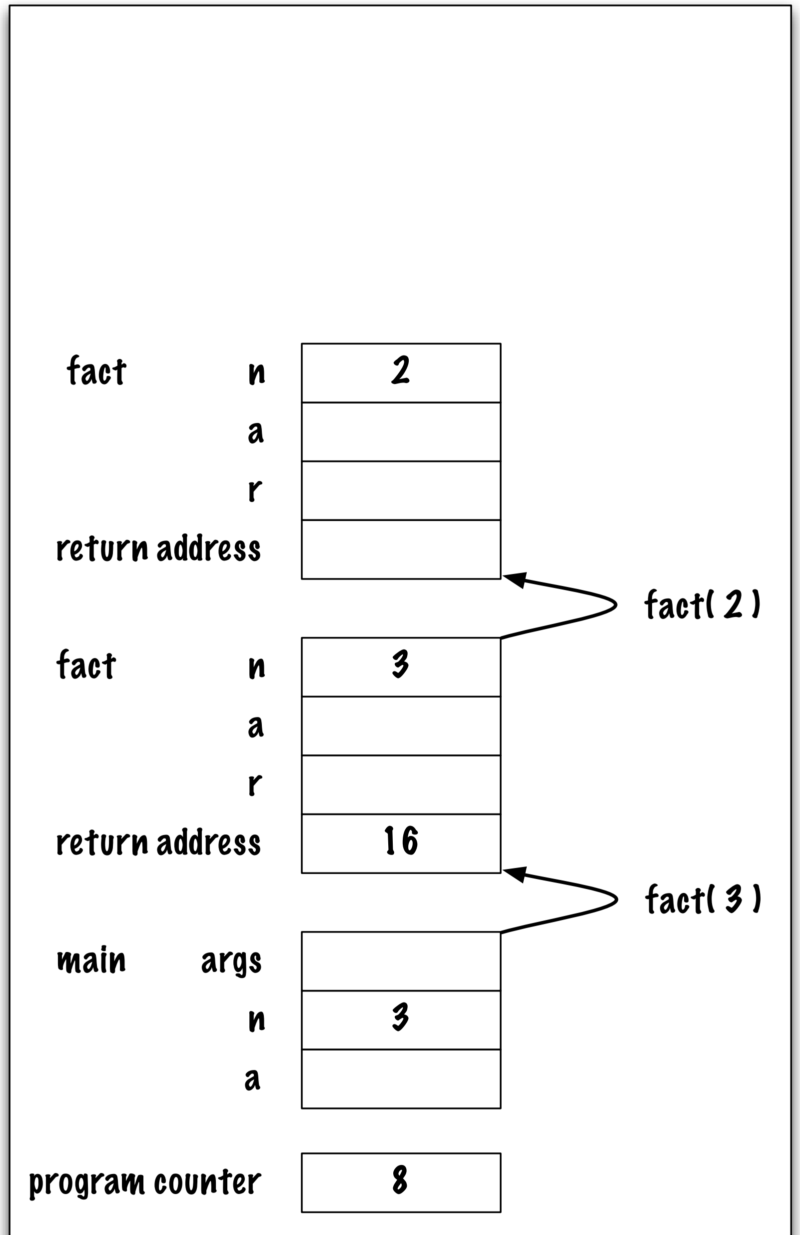
program counter
8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16
fact(3)
main
args
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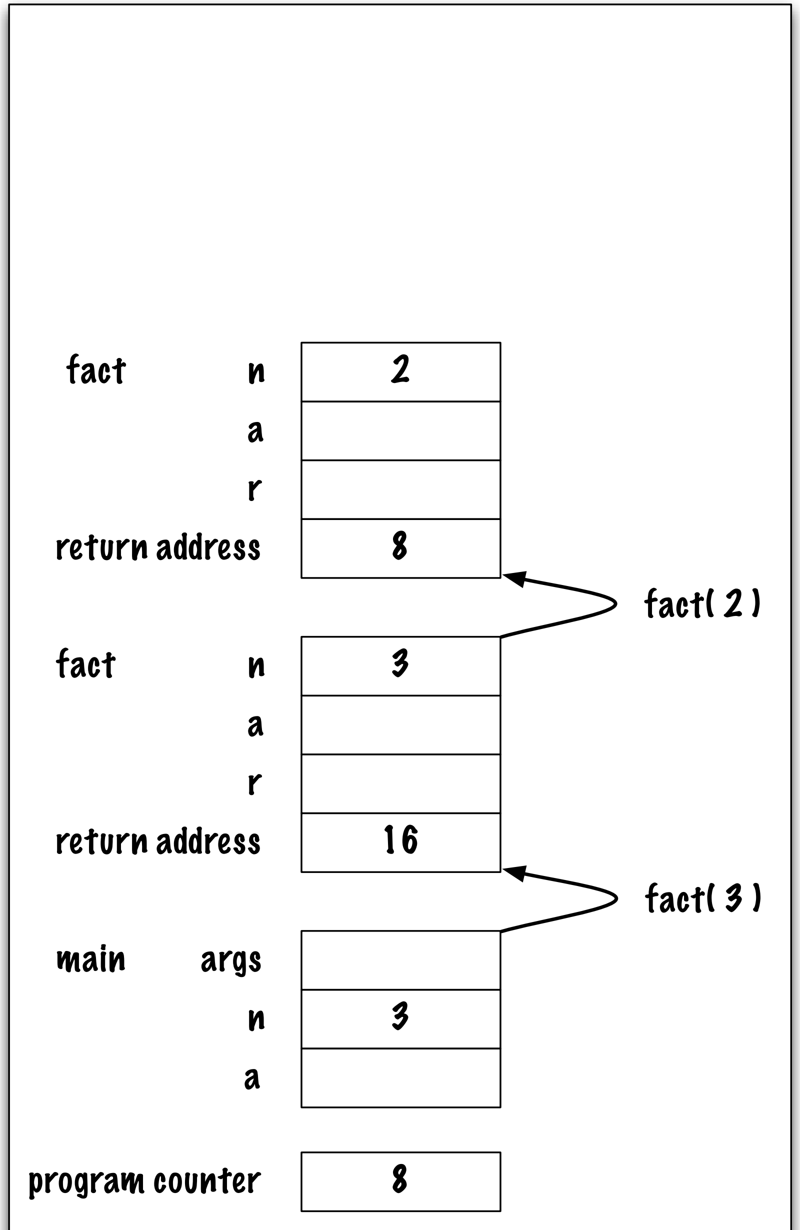
3
a
program counter
8



fact
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return address
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fact(2)
fact
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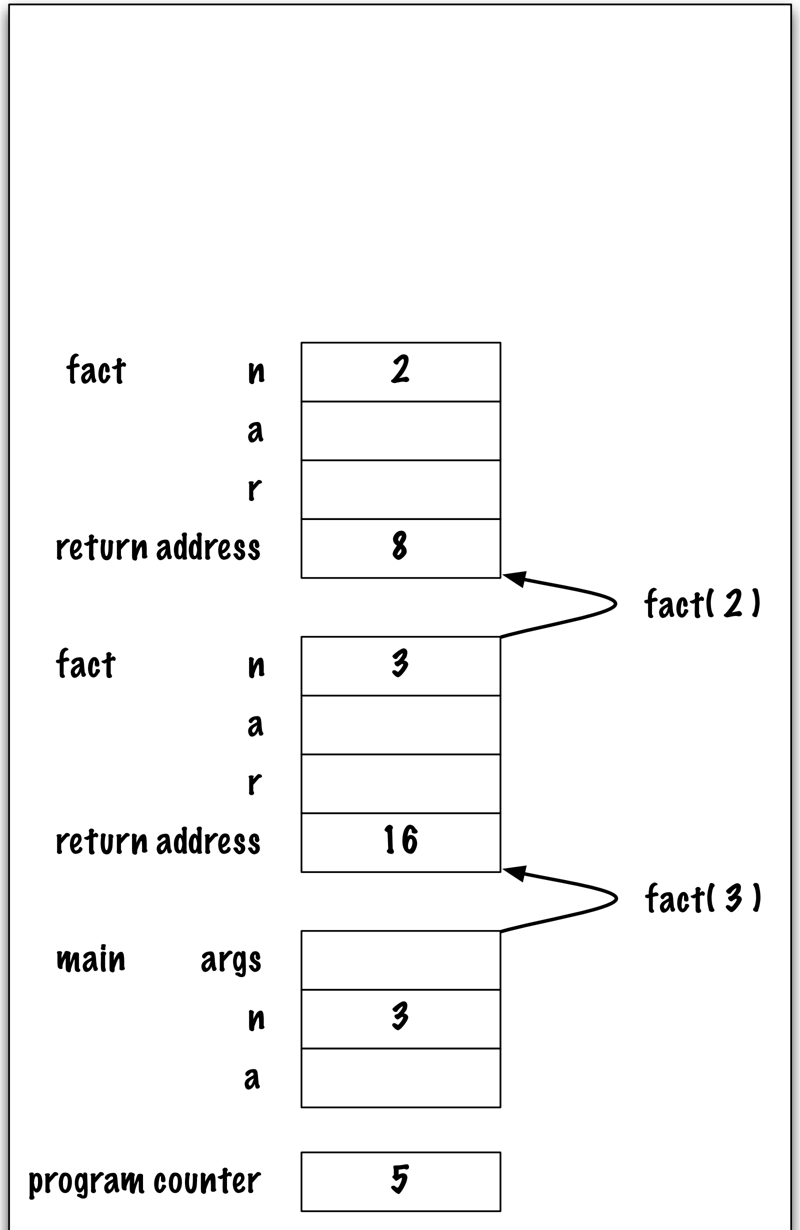
return address
16
fact(3)
main
args
n
3
a
program counter
8



fact
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fact(3)
main
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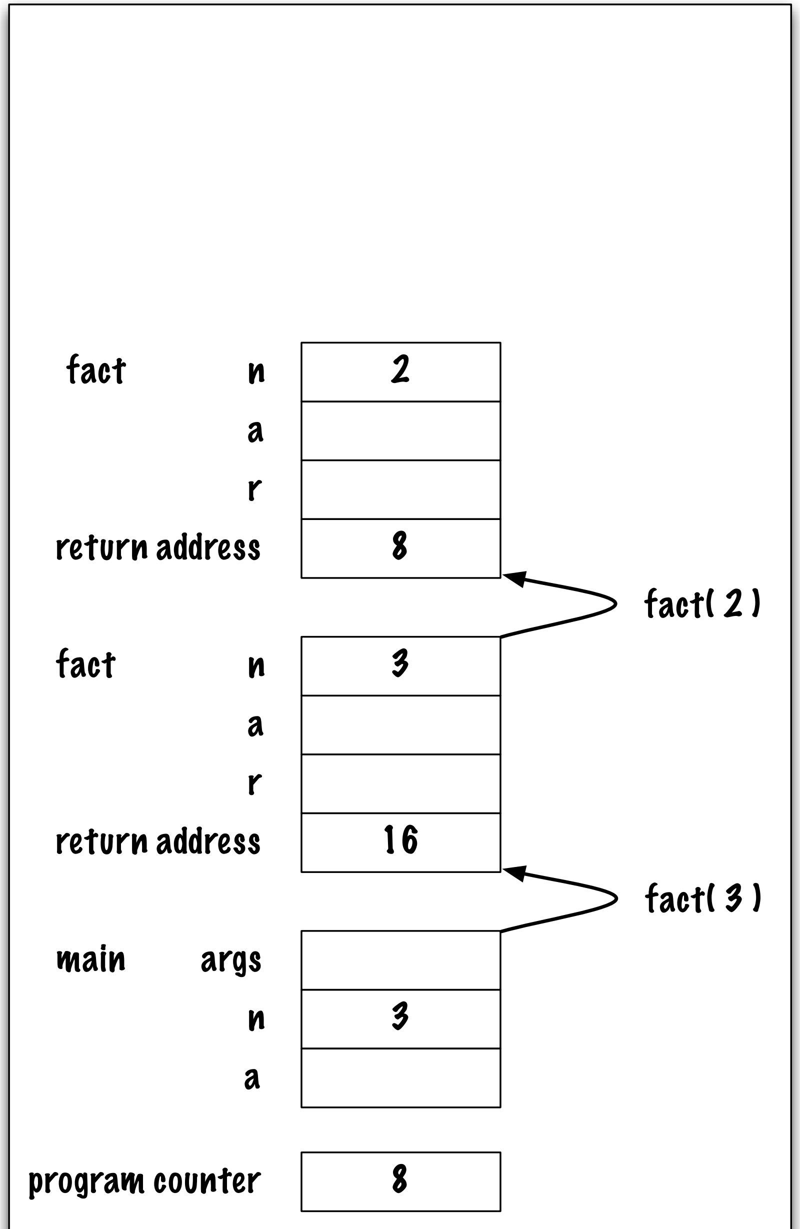
program counter
5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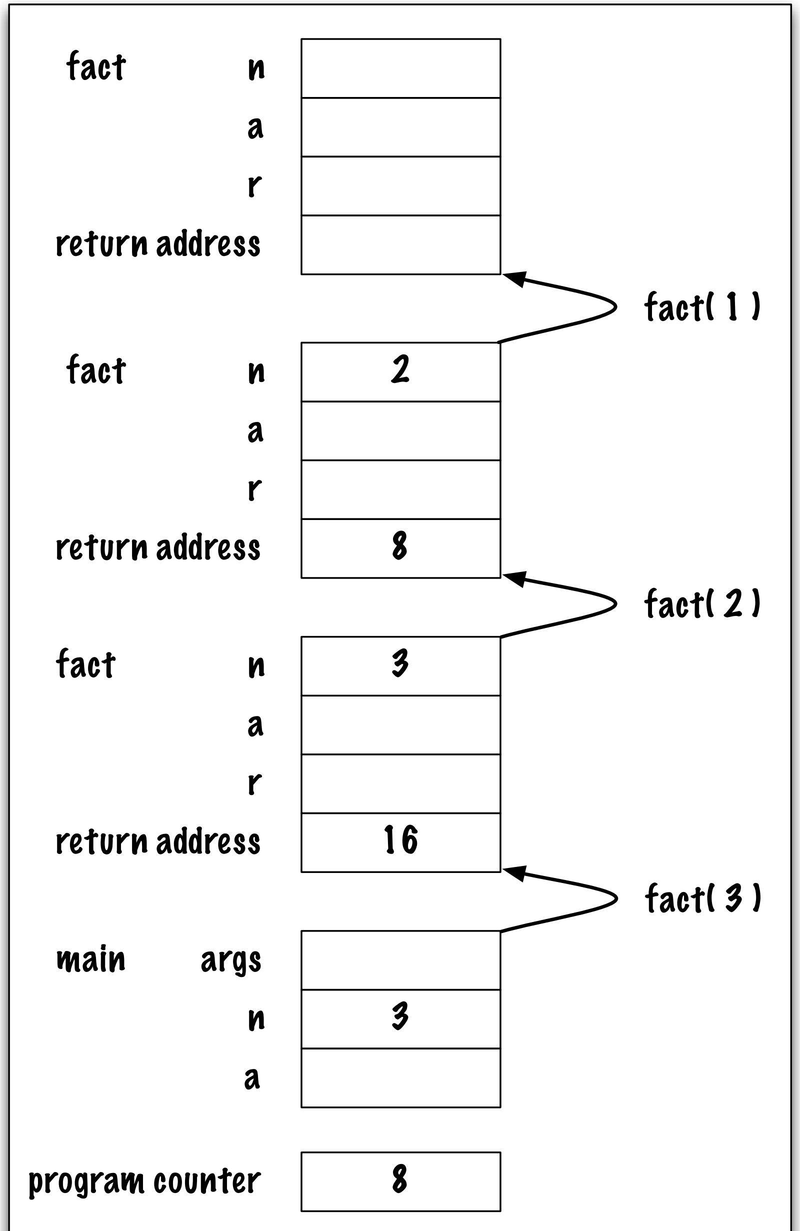
program counter
8


fact
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fact(3)
main
args
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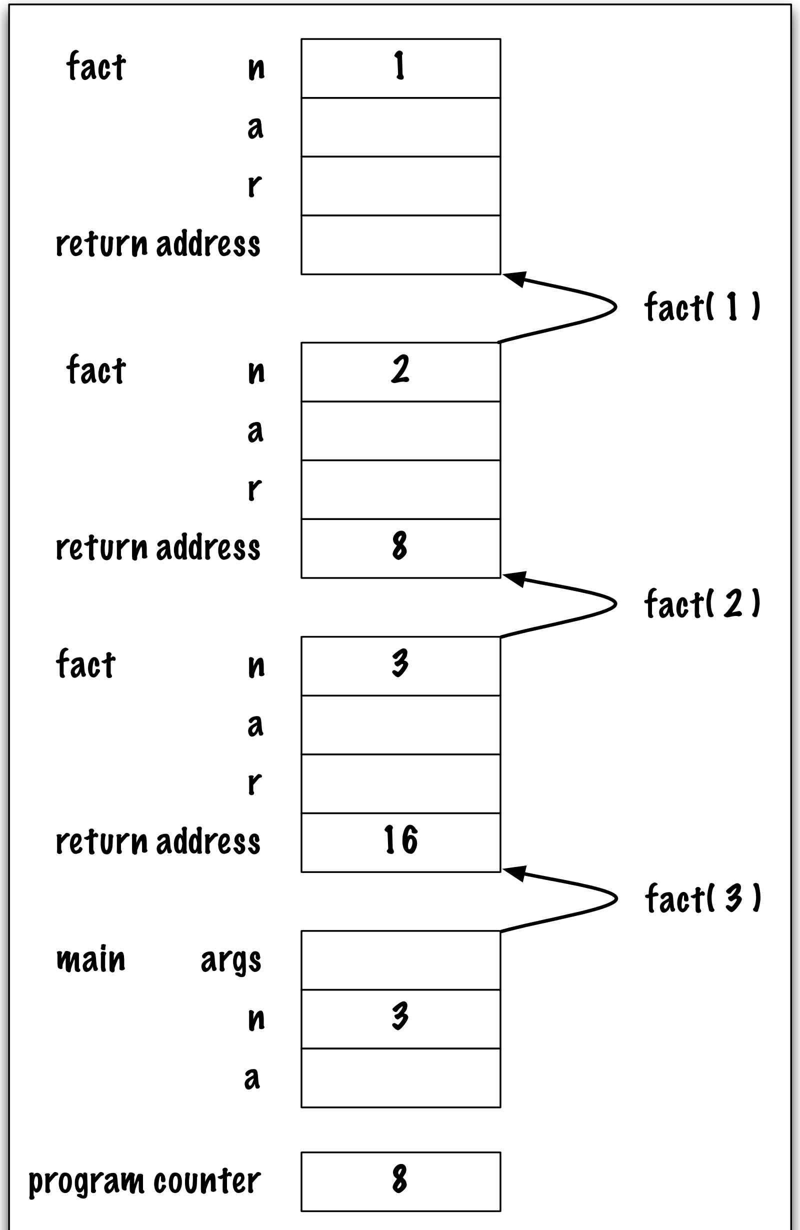
program counter
8


fact
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16
fact(3)
main
args
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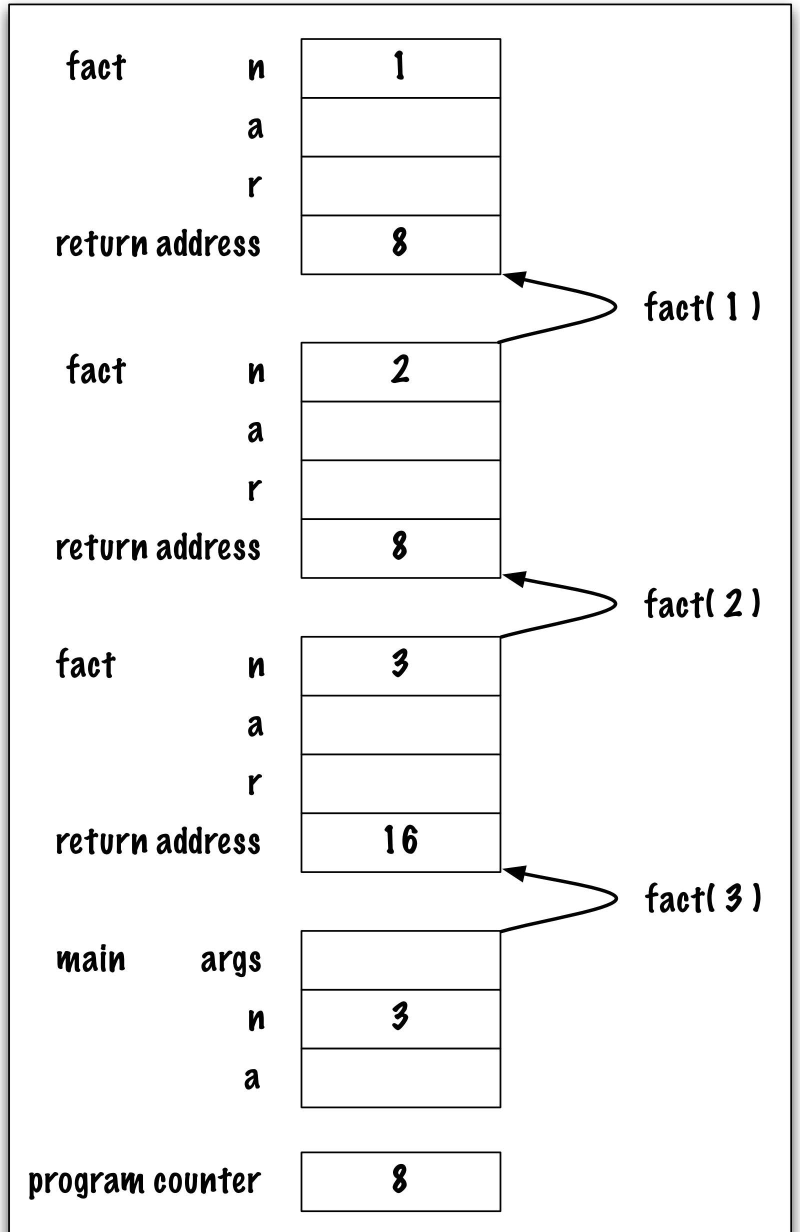
a
program counter
8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args
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program counter
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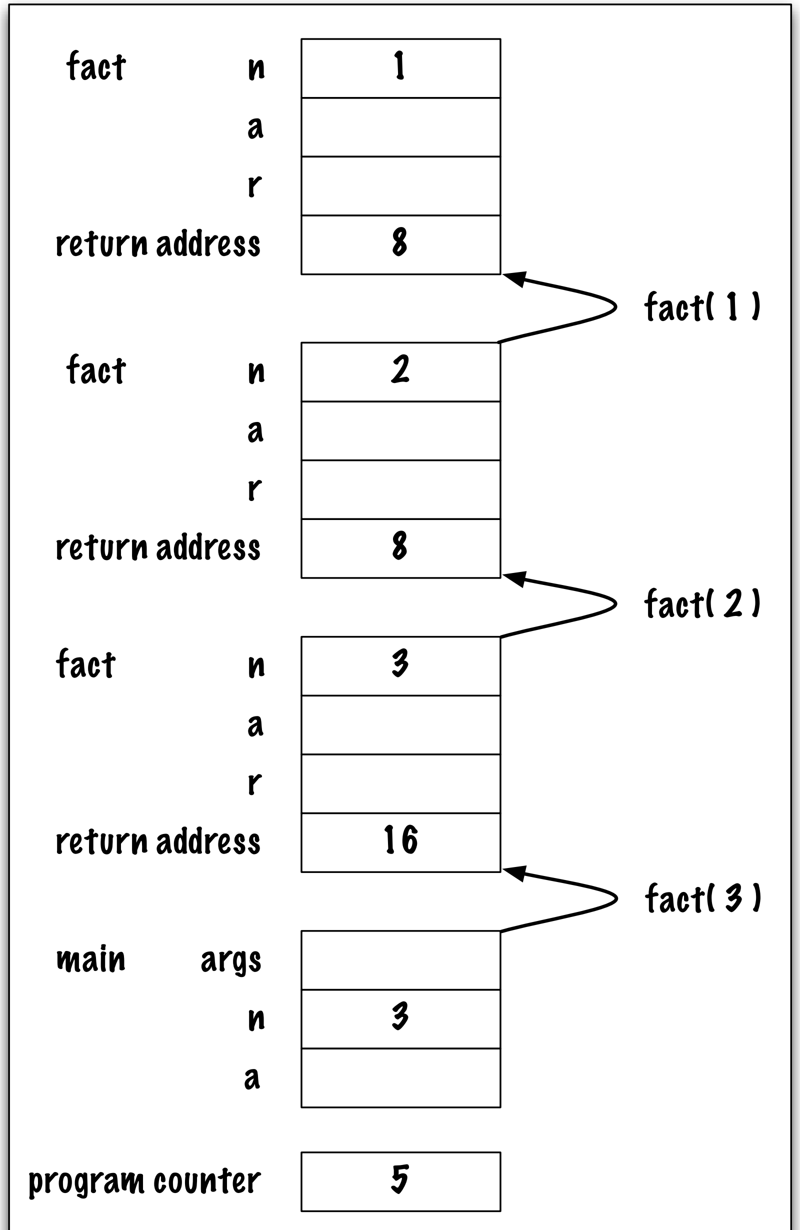
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main
args
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program counter
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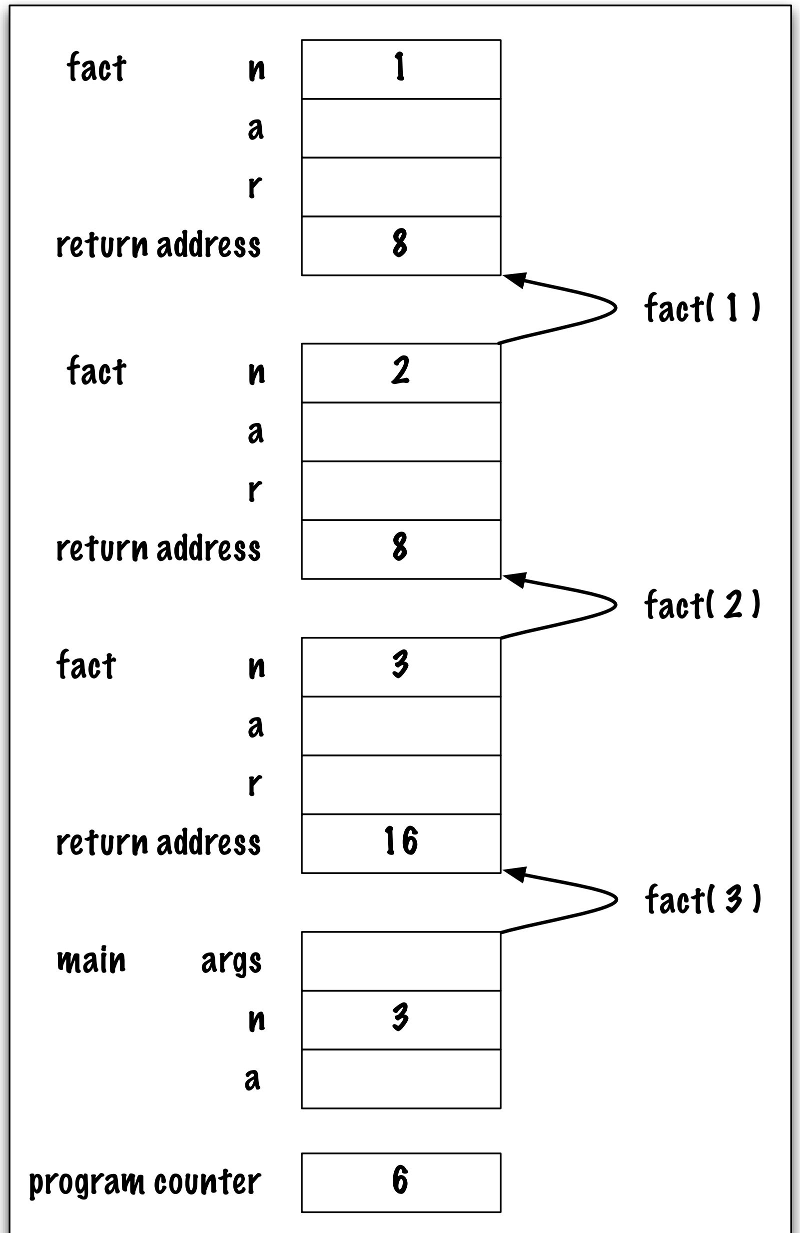

fact
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16
fact(3)
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args
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3
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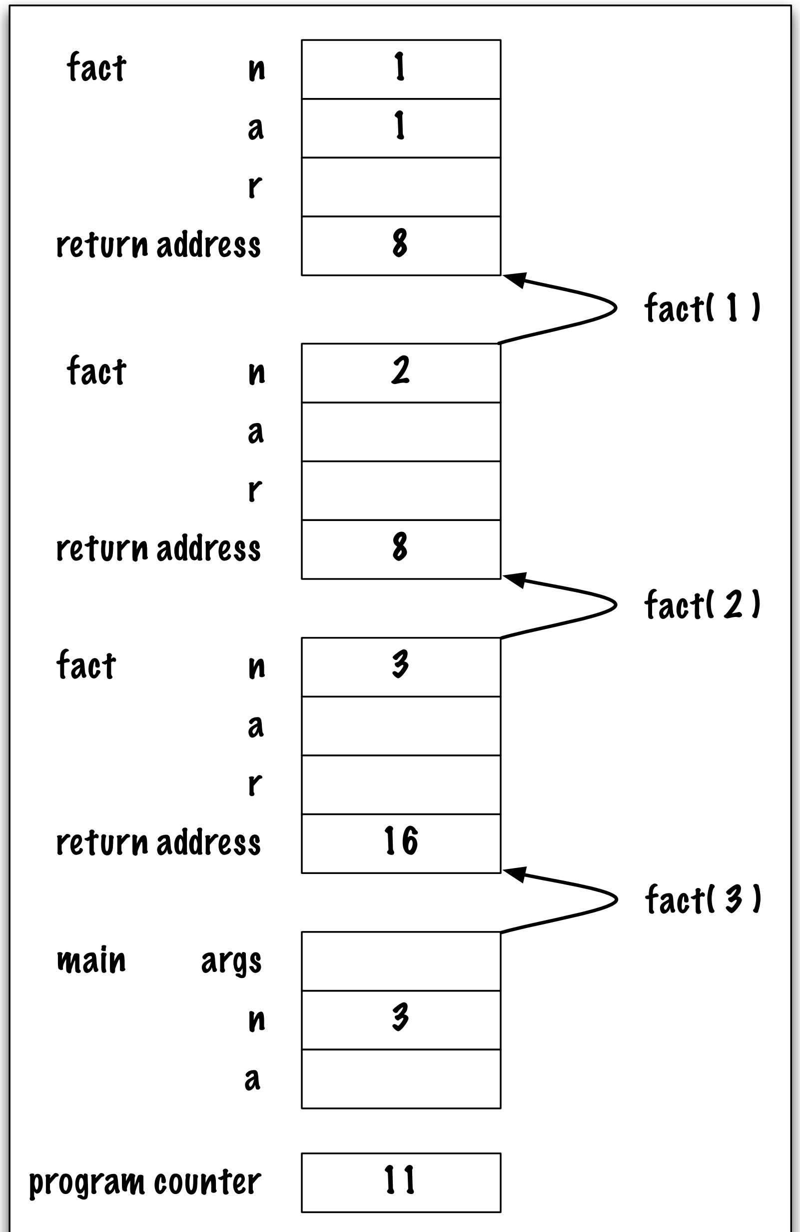
program counter
6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fact(2)
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main
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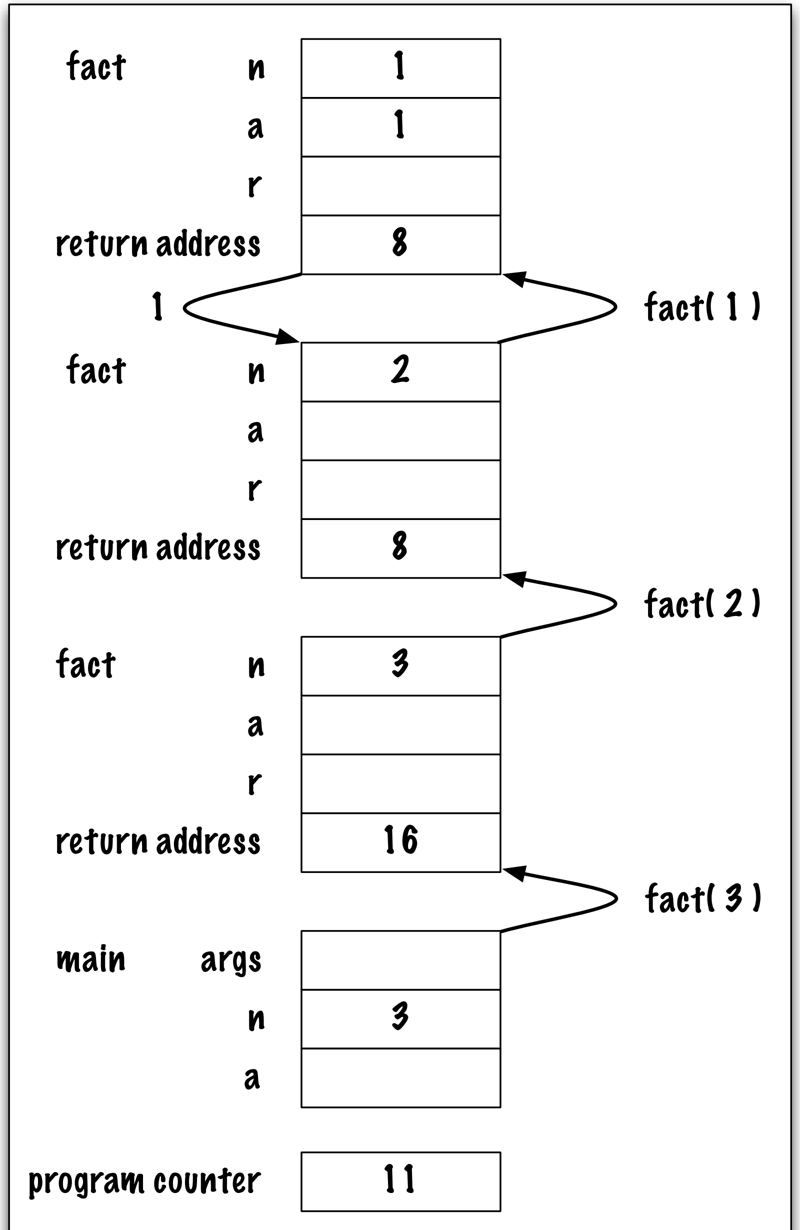
n
3
a
program counter
11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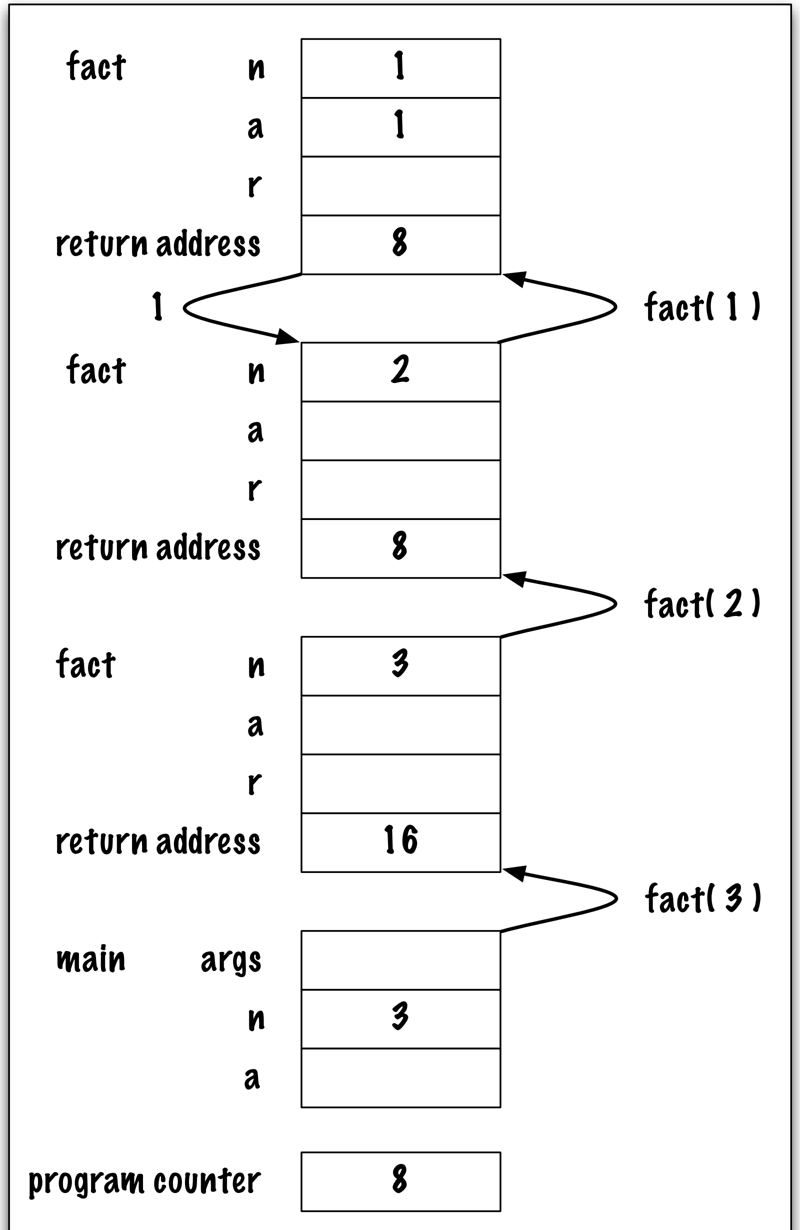
return address
16
fact(3)
main
args
n
3
a
program counter
11


fact
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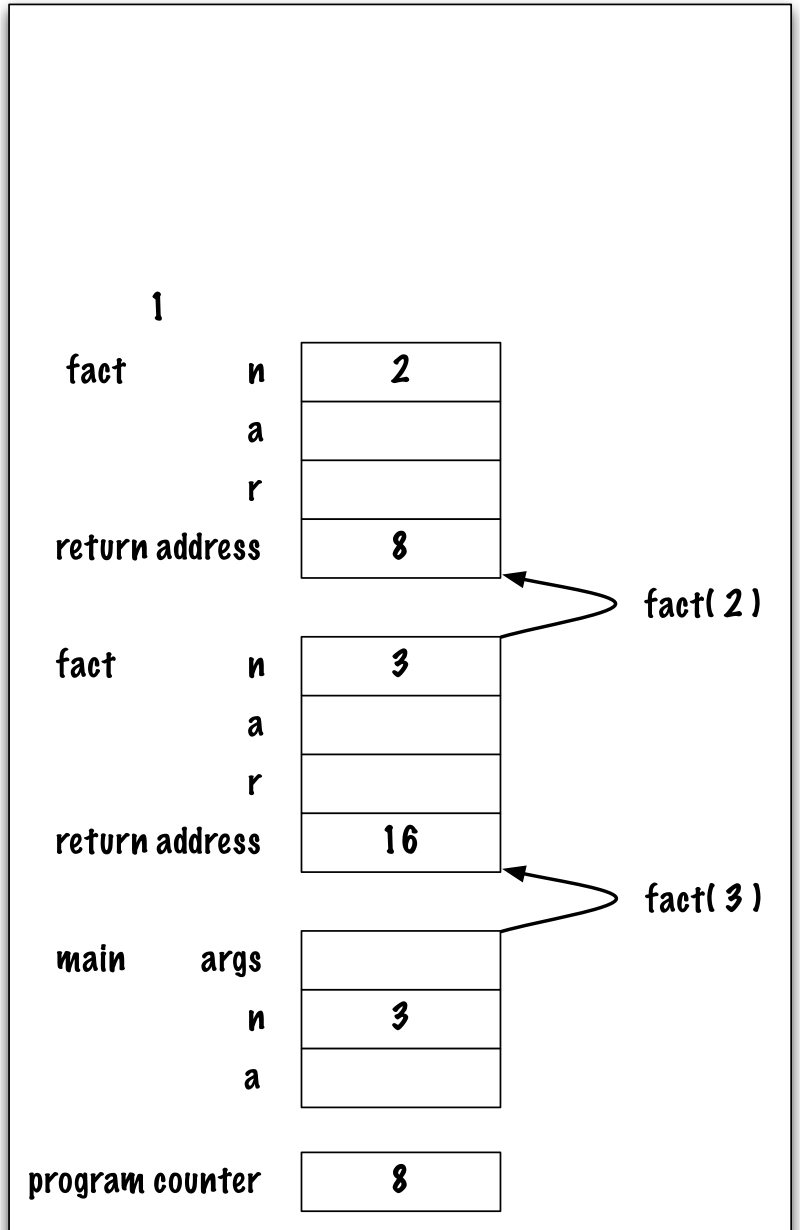
r
return address
8
1
fact(1)
fact
n
2
a
r
return address
8
fact(2)
fact
n
3
a
r
return address
16
fact(3)
main
args
n
3
a
program counter
8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r
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n
3
a
program counter
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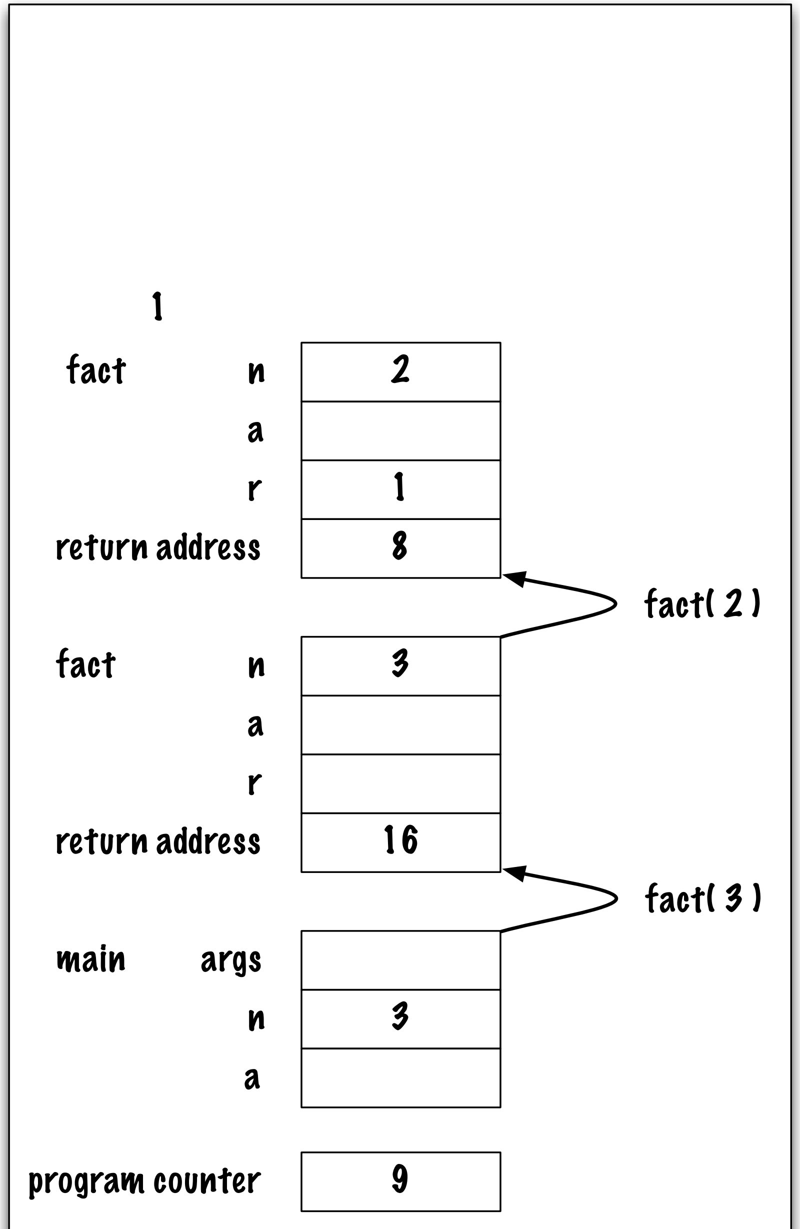
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n
2
a
r
1
return address
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a
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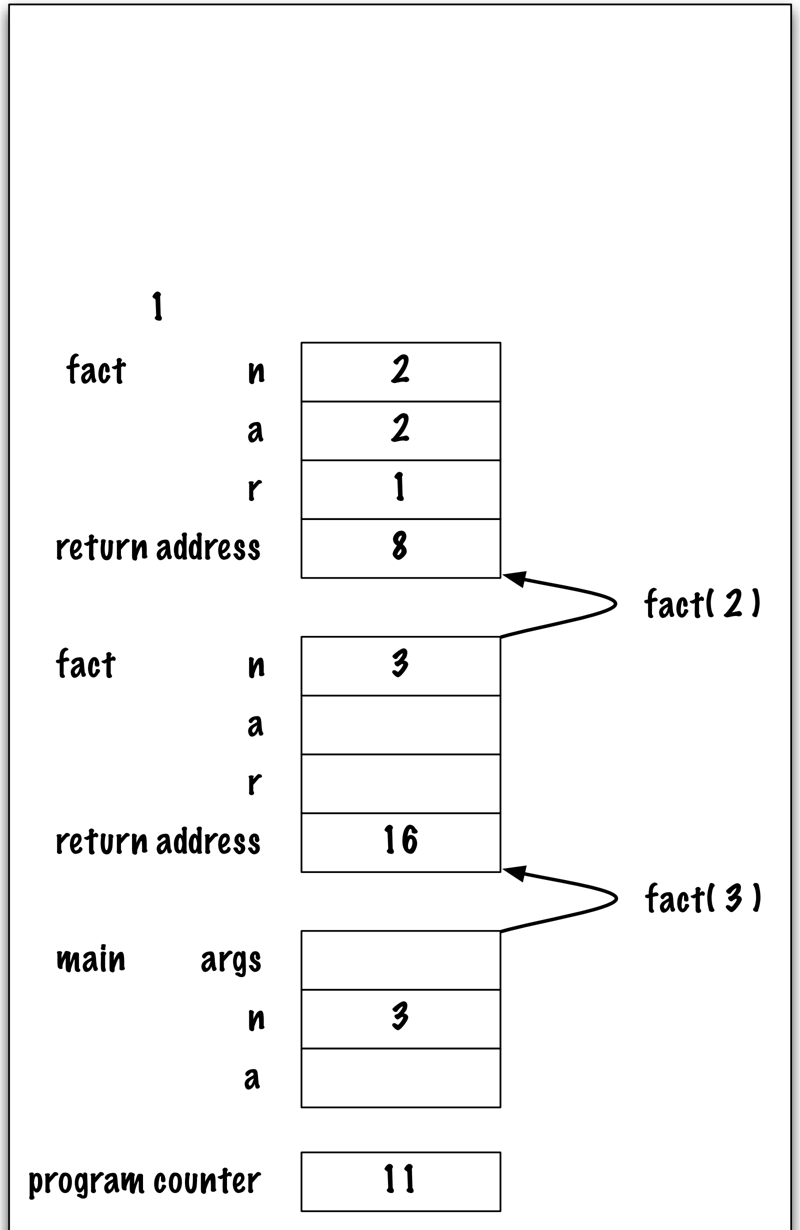
fact(3)
main
args
n
3
a
program counter
9



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2
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1
return address
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fact(2)
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a
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return address
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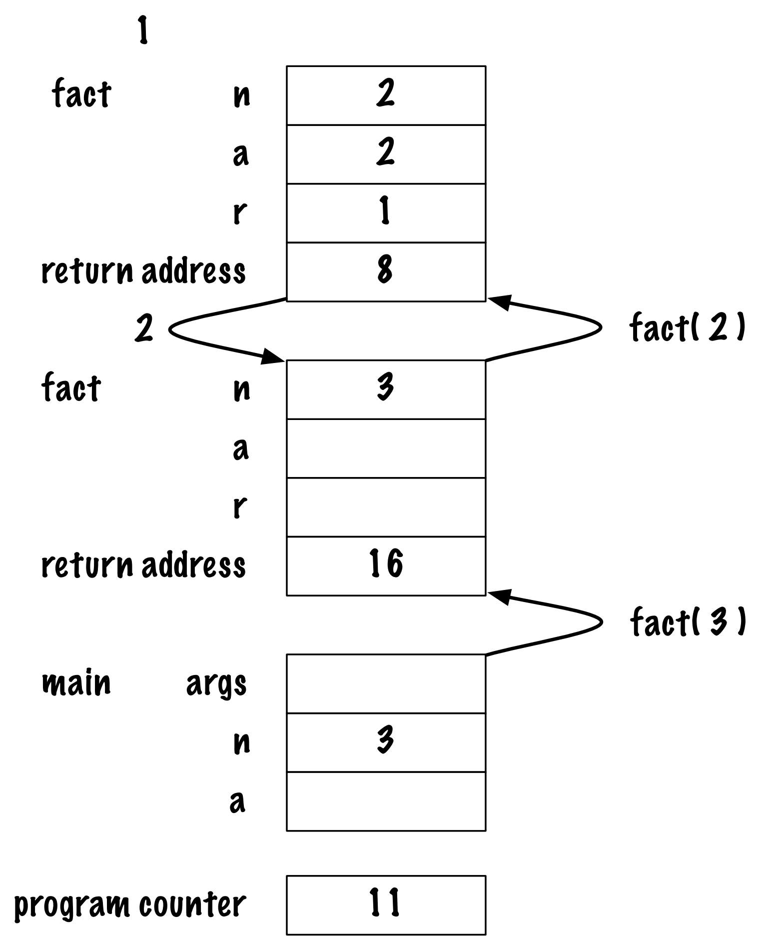
fact(3)
main
args
n
3
a
program counter
11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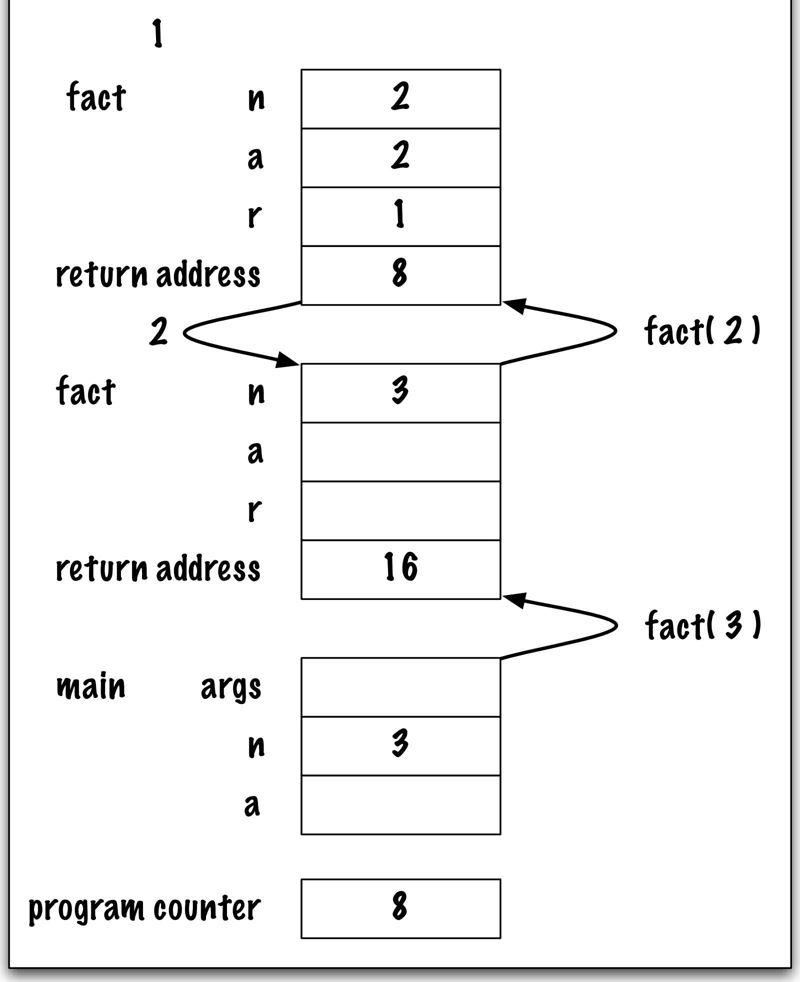
main
args
n
3
a
program counter
11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fact(3)
main
args
n
3
a
program counter
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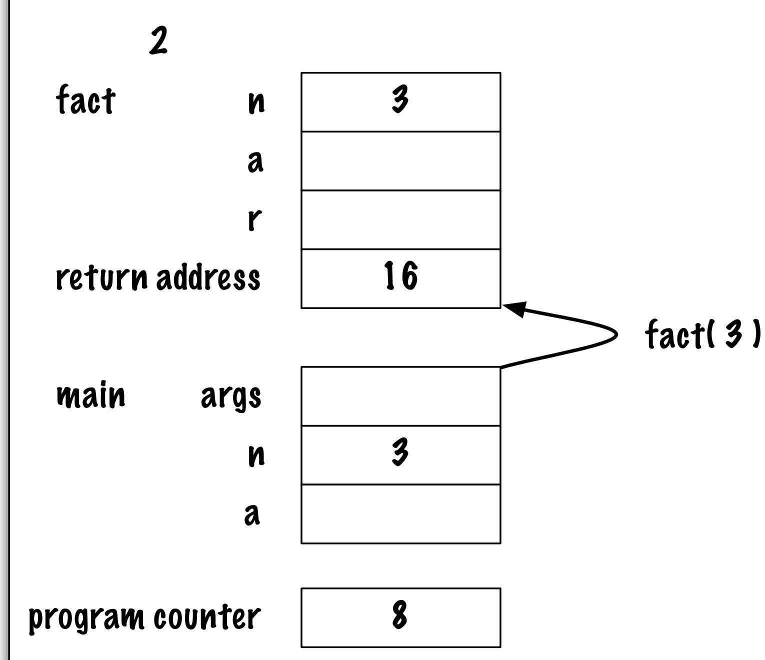
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return address
16
fact(3)
main
args
n
3
a
program counter
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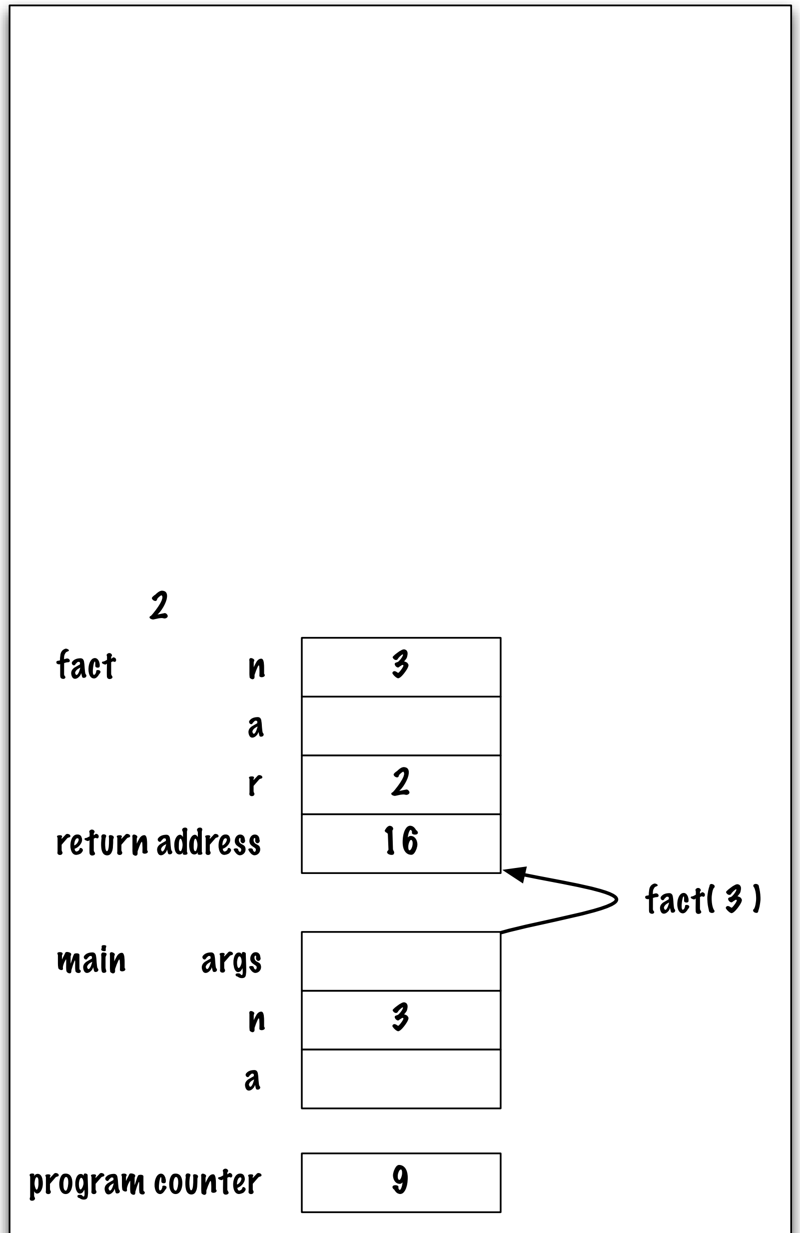
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r
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return address
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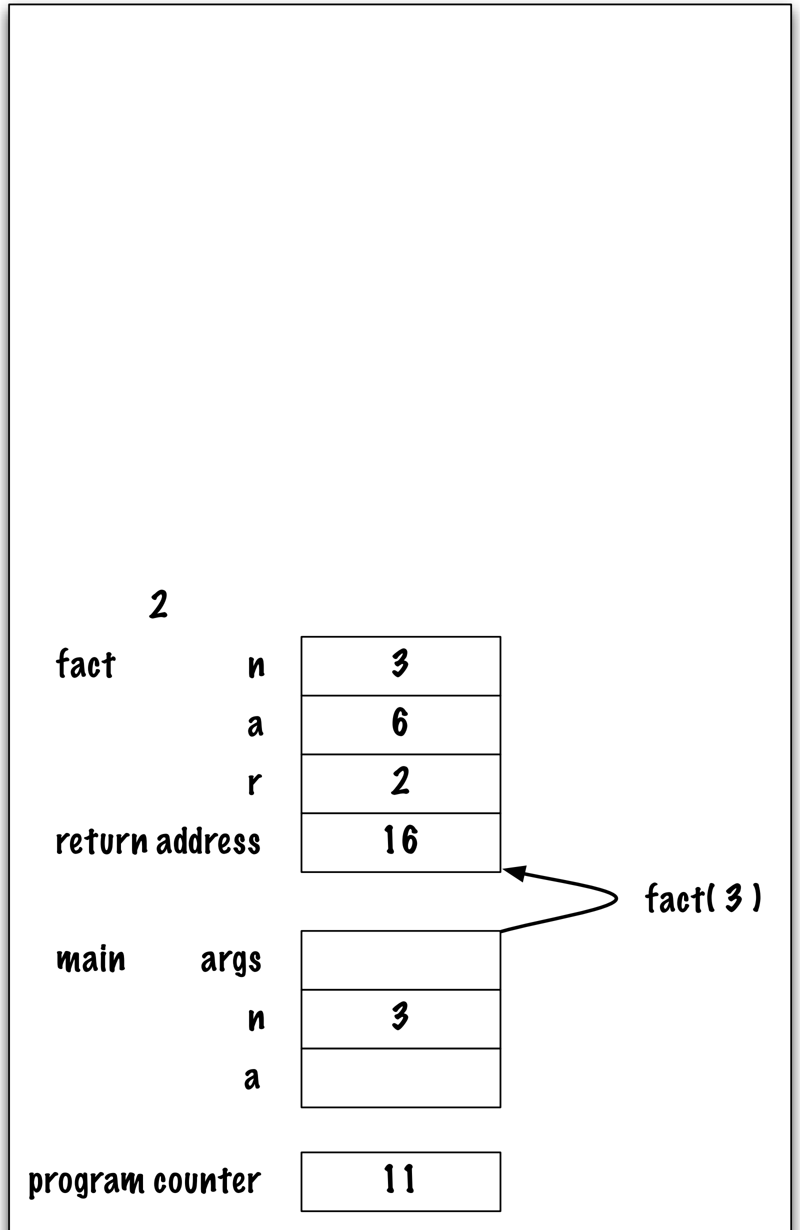
fact(3)
main
args
n
3
a
program counter
9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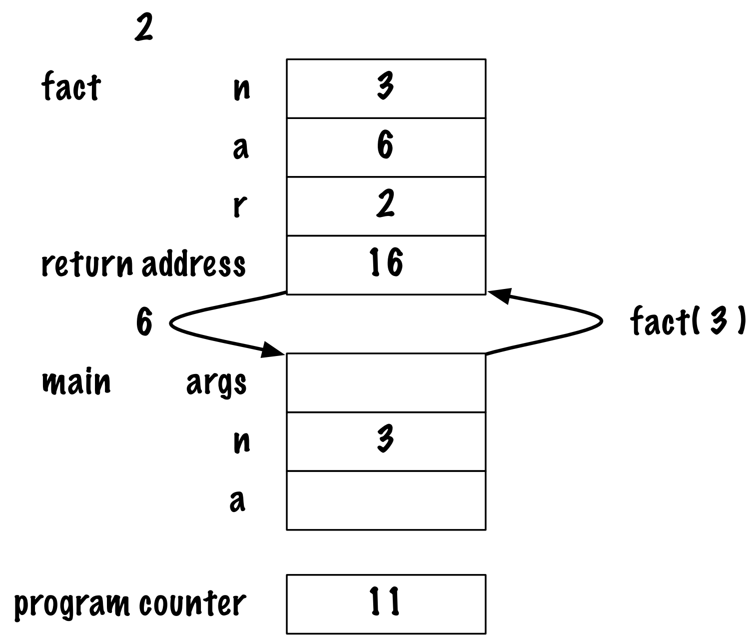
a
6
r
2
return address
16
fact(3)
main
args
n
3
a
program counter
11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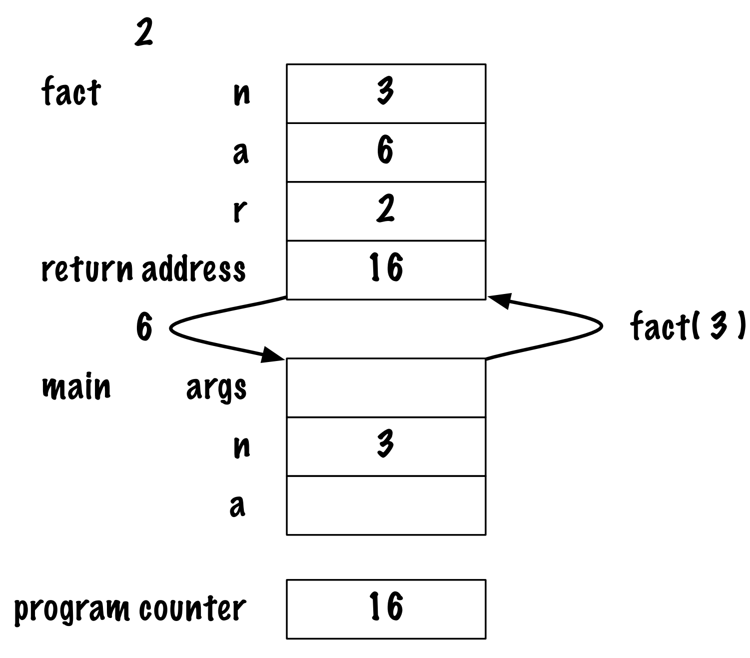
n
3
a
6
r
2
return address
16
6
fact(3)
main
args
n
3
a
program counter
11


2
fact
n
3
a
6
r
2
return address
16
6
fact(3)
main
args
n
3
a
program counter
16

6

main	args	
	n	3
	a	

program counter	16

6

main	args	
	n	3
	a	6

program counter	17