CSI 5165 Combinatorial Algorithms | Fall 2014
Computer Science | University of Ottawa

Homework Assignment #1 (100 points, weight 15%)
Due: October 15, 2014 at 11:30PM, via blackboard (uottawa virtual campus), details to follow.

Generating elementary combinatorial objects

1. (30 points) **Correctness of Successor algorithm for Graycodes**
Prove Theorem 2.2 of the textbook which states that Algorithm 2.3 (also given in class) correctly computes successor for the binary reflected Gray code. You need to state and prove several facts, which will formalize the informal statements given in page 38 of the textbook:

> Algorithm 2.3 works as follows. If $w(A)$ is even, then the last bit of A (namely a_0) is flipped; if $w(A)$ is odd, then we find the first "1" from the right, and flip the next bit (to the left). The last vector in G^n, which has no successor (added by editor: or sucessor $[0, \ldots, 0]$ in circular order) is $[1, 0, \ldots, 0]$ This corresponds to the set $\{1\}$.

Hint: Prove the facts by induction on n.

2. (30 points) **Generating all variations of a multiset**
The variations of a multiset are the permutations of all its sub-multisets.
Example: the variations of multiset $\{1, 2, 2, 3\}$ are:

ϵ, 1, 12, 122, 1223, 123, 1232, 13, 132, 1322
2, 21, 212, 2123, 213, 2132, 22, 221, 2213, 223, 2231, 23, 231, 2312, 232, 2321
3, 31, 312, 3122, 32, 321, 3212, 322, 3221

Give an algorithm (pseudocode) to generate all variations of a multiset.
To verify that your algorithm works, you may implement it and test it; however the implementation is not required for the assignment.

Hint: You may use a similar idea as the one for lexicographical order of permutations.

3. (40 points) **Generalized Gray codes**

   (a) Let $m_0, m_1, \ldots m_{n-1}$ be integer numbers greater than or equal 2. In this exercise we want to generate all n-tuples $(a_{n-1}, a_{n-2}, \ldots, a_1, a_0)$ where $0 \leq a_j < m_j$ for all j, $0 \leq j < n$, according to the following minimal change ordering: two successive tuples differ in exactly one component with the absolute value of their difference equals to 1 (i.e. the component is either incremented or decremented by 1). Adapt the binary reflected Gray code successor algorithm to the case of this generalized Gray code. Give your algorithm in pseudocode form.

   (b) Given the prime factorization of a number $p_1^{e_1} p_2^{e_2} \cdots p_t^{e_t}$, give an algorithm to run through all divisors of the number, by repeatedly multiplying or dividing by a single prime at each step.
   Hint: Use the algorithm developed in part 1.