CSI 2101 Discrete Structures — Winter 2012
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Quiz #2

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1. **S1.4, Exercise 9:** Let $L(x, y)$ be the statement "x loves y", where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements:

   a. Everybody loves Jerry:
   $$\forall x L(x, \text{Jerry})$$

   b. Everybody loves somebody:
   $$\forall x \exists y L(x, y)$$

   c. There is somebody whom everybody loves:
   $$\exists y \forall x L(x, y)$$

   d. Nobody loves everybody:
   $$\neg \exists x \forall y L(x, y) \equiv \forall x \exists y \neg L(x, y)$$

   e. There is somebody whom Lydia does not love:
   $$\exists y \neg L(\text{Lydia}, y)$$

   f. There is somebody whom no one loves:
   $$\exists y \forall x \neg L(x, y)$$

   g. There is exactly one person whom everybody loves:
   $$\exists y (\forall x L(x, y) \wedge \forall z ((\forall w L(w, z)) \rightarrow z = y))$$

   h. There are exactly two people whom Lynn loves:
   $$\exists x \exists y (L(\text{Lynn}, x) \wedge L(\text{Lynn}, y) \wedge x \neq y \wedge \forall z (L(\text{Lynn}, z) \rightarrow (z = x \vee z = y)))$$

   i. Everyone loves him or herself:
   $$\forall x L(x, x)$$

2. **S1.4, Exercise 33:** Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a.

$$\neg\forall x\forall y P(x,y) \equiv \exists x\exists y\neg P(x,y)$$

b.

$$\begin{aligned}\neg\forall y\forall x(P(x,y)\vee Q(x,y)) &\equiv \exists y\exists x\neg(P(x,y)\vee Q(x,y))\\ &\equiv \exists y\exists x(\neg P(x,y)\wedge\neg Q(x,y))\end{aligned}$$

c.

$$\begin{aligned}\neg(\exists x\exists y\neg P(x,y)\wedge\forall x\forall y Q(x,y)) &\equiv \neg\exists x\exists y\neg P(x,y)\vee\neg\forall x\forall y Q(x,y)\\ &\equiv \forall x\forall y\neg\neg P(x,y)\vee\exists x\exists y\neg Q(x,y)\\ &\equiv \forall x\forall y P(x,y)\vee\exists x\exists y\neg Q(x,y)\end{aligned}$$

d.

$$\begin{aligned}\neg\forall x(\exists y\forall z P(x,y,z)\wedge\exists z\forall y P(x,y,z)) &\equiv \exists x\neg(\exists y\forall z P(x,y,z)\wedge\exists z\forall y P(x,y,z))\\ &\equiv \exists x(\neg\exists y\forall z P(x,y,z)\vee\neg\exists z\forall y P(x,y,z))\\ &\equiv \exists x(\forall y\exists z\neg P(x,y,z)\vee\forall z\exists y\neg P(x,y,z))\end{aligned}$$