CSI 2101 Discrete Structures — Winter 2012
Prof. Lucia Moura — University of Ottawa

**Homework Assignment #1** (100 points, weight 5%)
Due: Thursday Feb 9, at 1:00 p.m. (in lecture);
*assignments with lateness between 1min-24hs will have a discount of 10%; after 24hs, not accepted; please drop off late assignments under my office door (STE5027).*

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## Propositional Logic

1. (12 points) Use logical equivalences to show that $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology.

2. (12 points; each 2+2+2 points=truth table+DNF+CNF)
For each of the following compound propositions give its truth table and derive an equivalent compound proposition in disjunctive normal formal (DNF) and in conjunctive normal form (CNF).

   (a) $(p \rightarrow q) \rightarrow r$

   (b) $(p \wedge \neg q) \vee (p \leftrightarrow r)$

## Predicate Logic

3. (15 points) For each of the given statements:
1 - Express each of the statements using quantifiers and propositional functions.
2 - Form the negation of the statement so that no negation is to the left of the quantifier.
3 - Express the negation in simple English. (Do not simply use the words "it is not the case that...").

   (a) Some drivers do not obey the speed limit.

   (b) All Swedish movies are serious.

   (c) No one can keep a secret.

   (d) No monkey can speak French.

   (e) There is someone in the class who does not have a good attitude.

4. (10 points) Translate these system specifications into English where the predicate $S(x, y)$ is "$x$ is in state $y$" and where the domain for $x$ and $y$ consists of all systems and all possible states, respectively.

   (a) $\exists S(x, \text{open})$

   (b) $\forall x(S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$

   (c) $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$

(d) $\exists x \neg S(x, \text{available})$

(e) $\forall x \neg S(x, \text{working})$

5. (3+3+3+3=12 marks) Rewrite the following statments statements so that all negation symbols immediately precede predicates (that is, no negation is outside a quantifier or an expression involving logical connectives). Show al the steps in your derivation.

   (a) $\neg \forall x \exists y P(x, y)$

   (b) $\neg \exists y (Q(y) \wedge \forall x \neg R(x, y))$

   (c) $\neg \exists y (\exists x R(x, y) \vee \forall x S(x, y))$

   (d) $\neg \exists y (\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

6. (10 points) Prove these logical equivalences, assuming that the domain is nonempty. You will probably have to use a proof by cases on the two possible values of proposition $\forall y Q(y)$ and $\exists y Q(y)$ respectively. This proof will use word arguments (not symbolic formula manipulation).

   (a) $\forall x (\forall y Q(y) \rightarrow P(x)) \equiv \forall y Q(y) \rightarrow \forall x P(x)$

   (b) $\exists x (\exists y Q(y) \rightarrow P(x)) \equiv \exists y Q(y) \rightarrow \exists x P(x)$

7. (10 points) A statement is in prenex normal form (PNF) if and only if all quantifiers occur at the beginning of the statement (without negations), followed by a predicate involving no quantifiers. Put the following statement in prenex normal form:
   (Hint: your first step should rename one of the two $x$'s as $y$; check useful valid equivalences in exercises 48,49 page 62 of 6th edition)

   (a) $\exists x P(x) \vee \exists x Q(x) \vee A$, where $A$ is a proposition not involving any quantifiers.

   (b) $\exists x P(x) \rightarrow \exists x Q(x)$

## Rules of Inference

8. (9 points) For each of these arguments, determine whether the argument is correct or incorrect and explain why.

   (a) Everyone born in Ottawa has eaten a beaver tail. Susan has never eaten a beaver tail. Therefore Susan was not born in Ottawa.

   (b) A convertible car is fun to drive. Joe's car is not a convertible. Therefore, Joe's car is not fun to drive.

   (c) Emma likes all fine restaurants. Emma likes the restaurant "Le Cordon Bleu". Therefore, "Le Cordon Bleu" is a fine restaurant.

9. (10 points) Give a formal proof, using known rules of inference, to establish the conclusion of the argument (3rd statement) using the first 2 statements as premises, where the domain of all quantifiers is the same.
Remember that a formal proof is a sequence of steps, each with a reason noted beside it; each step is either a premise, or is obtained from previous steps using inference rules.

- premise: $\forall x(P(x) \lor Q(x))$
- premise: $\forall x((\neg P(x) \land Q(x)) \to R(x))$
- conclusion: $\forall x(\neg R(x) \to P(x))$