Parallel Tabu Search for Robust Image Filtering*

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ABSTRACT — **We propose to perform detail-preserving filtering by minimizing an objective function that uses a fairly simple regularization function to control smoothing interaction of pixel neighbors where discontinuities are implicitly addressed. To accomplish the minimization of this objective function, we use Tabu search. Tabu search adapts to the particular structure of the problem it tries to solve, and thus preforms an intelligent exploration of the state space.**

1 Introduction

Robust image restoration consists of suppressing image noise while preserving significant details such as sharp edges. Given a noisy image with pixel intensities $\{d_1, ..., d_N\}$, the goal is to find the filtered version of this image $\{x_1, ...x_N\}$ in which noise has been removed. A common approach to this problem is to express the problem of image filtering with a functionnal whose global minimum corresponds to the desired image [1]. Typically, this objective function is of the form:

$$F(\mathbf{x}) = \sum_{i=1}^{N} \alpha(x_i - d_i)^2 + \lambda \sum_{j \in \mathcal{N}_i} g(x_i - x_j)$$

where $g(s)$ is the regularization function that defines how the neighborhood $\mathcal{N}_i$ of pixel i interacts with the candidate solution x_i. In this context, suitable filtering will be achieved if 1) $F(\mathbf{x})$ is easily computable, 2) smoothing properties of $g(s)$ preserve discontinuities and 3) minimization can be performed at a reasonable computational cost.

A common solution to realize an edge-preserving regularization is to introduce in the defintion of $g(s)$ a line process to delineate regions on which smoothing can applied [2]. But this formulation renders the objective function too complex by considerably increasing the dimensionality of the search space.

Under the assumption that the restored image must correspond to the state of minimum energy of the objective function, it is essential to proceed to this global minimization. Simulated annealing in the context of MRF modeling has been widely used to perform this task [2][3]. This technique is known for its high computational cost. More recently, a technique called graduated non convexity (GNC) has been proposed [4][5]. GNC can be seen as a deterministic annealing approach and its complexity strongly depends on the adopted mathematical formulation.

We propose to perform detail-preserving filtering by minimizing an objective function that uses a fairly simple regularization function to control smoothing interaction of pixel neighbors where discontinuities are implicitly addressed as proposed in [6]. Minimization of the objective function is performed by Tabu search [7] -[11],

2 Tabu search

Let $F(\mathbf{x})$, $\mathbf{x} = \{x_1, ..., x_N\}$, be an objective function that one whishes to minimize. And let the state space $\mathcal{X}$ be the set of all feasible

solutions (i.e. $\mathbf{x} \in \mathcal{X}$). The goal is to move from one state to another in order to iteratively reach the global minimum of the function. The set of all allowed moves from a state $\mathbf{x}$ is designated by $\mathcal{M}(\mathbf{x}) \subset \mathcal{X}$. The Tabu procedure consists in choosing the move $\mathcal{M}(\mathbf{x})$ that will produce the highest reduction of the objective function. From here this is similar to the usual gradient descent: iteratively, the procedure will eventually bring the objective function to a local minimum. In such a situation, the *best* move will cause an augmentation of the objective function. The Tabu search technique allows such a move when it is not possibe to do better. The key aspect of the Tabu Search is that, in order to avoid cycling (i.e. to come back to an already visited state), each time a move to a new state is performed, all complementary moves (i.e. those that cancelled this move) enter into a Tabu status. This Tabu status remains valid for a certain amount of time. In general, a move that is in a Tabu status will be forbidden. This is the short term memory that forbid backward moves and thus allows to escape from a local minimum. A long-term memory also exists, expressed by an aspiration level condition that allows, under some circumstances, a move despite its Tabu status, if this move is considered as very advantageous in terms of the objective function. The stopping criteria of the search is generally based on a maximum number of iteration until no improvement of the current best minimum is obtained.

3 Image restoration

Let us assume that, in a small neighborhood, all pixel intensity values are realizations of the same gaussian distribution with mean depending on the original image and variance proportional to the level of noise. Then the optimal estimate is the mean of all observations $\{x_j\}$ which corresponds to the minimum of $\lambda(x-x_j)^2$. The problem here is that some of the neighbors x_j may not belong to the same distribution as x. In fact, if the difference between x and x_j becomes important then the probability that these two observations come from the same distribution decreases. As a consequence, when $(x-xj)^2$ is above a certain threshold, x_j should not contribute to the estimation of x. This suggests the following form of $g(s)$ (see Figure 1):

$$g(s) = \gamma + (s^2 - \gamma)e^{-\beta s^2}$$

For small differences, $g(s)$ is similar to s^2 (the dotted curve in Figure 1) and this remains true until a certain threshold from which the contribution decreases. Note that the residual cost γ is required to penalize situations where most of the neighbors would be discarded as outliers. Similar functions have also been proposed in [12] and [6].

Minimization of $F(\mathbf{x})$ can be formulated as an integer programming problem: each x_i can take integer values from 0 to 255. Let us define the contribution C_i of a pixel i: this is the sum of all terms in the objective function where the term x_i appears:

$$C_i = \alpha(x_i - d_i)^2 + 2\lambda \sum_{j \in \mathcal{N}_i} g(x_i - x_j)$$

Note that the sum for $i = 1, .., N$ of the values all contributions is not equal to the value of the objective function. Now suppose that we are in a given state $\mathbf{x}$. We define an allowed move as one for which there is an increase or a decrease of a pixel intensity of 1. This gives $2N$ possible moves. We select the one having the smallest contribution C_i' that is not in a Tabu status. The corresponding reduction in the objective function is simply $C_i' - C_i$. When a move is done, the complementary move becomes Tabu. This means that a backward move to a previous pixel value cannot occur during its Tabu period except if this move makes the objective function smaller than the best minimum found so far.

4 Results

Minimization should be done over the whole image. However, considering the fact that a single pixel has a limited influence on the global objective function, local minimization is appropriate. We chose to perform minimization over

3×3 windows. Moreover, if we carefully choose a non-overlaping window configuration, then parallel computing become possible. In order to propagate results of a local minimization these parallel minimizations have been repeated with 16 different window configurations such that each pixel is once the central pixel of one window. With such procedure, most pixels are subjected to 9 minimization procedures.

Figure 3 shows results that we obtained when filtering an artificial image corrupted with a noise of variance $\sigma^2 = 4$. Result of the minimization of the proposed objective function using Tabu search is compared to simple mean and median filtering, and with result obtained from the minimization with an objective function where $g(s)$ is simply s^2. Evolution of the objective function when applying our parallel Tabu search algorithm is shown in Figure 2. Each iteration corresponds to the application of a Tabu search sequence made of 25 moves with Tabu status of duration 10. Curve 2(b) illustrates how objective function value is temporarly increases during minimization in order to escape from local minimum and thus potentially reach a more interesting minimum. Figure 4 shows results obtained for different level of noise. Figure 5 shows results with a real image.

References

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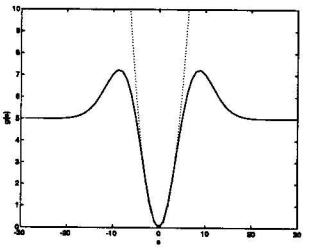
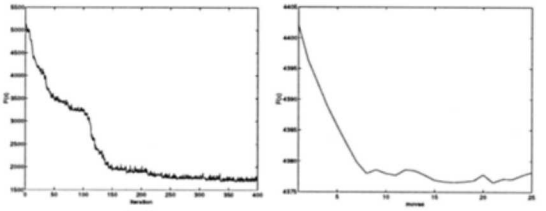
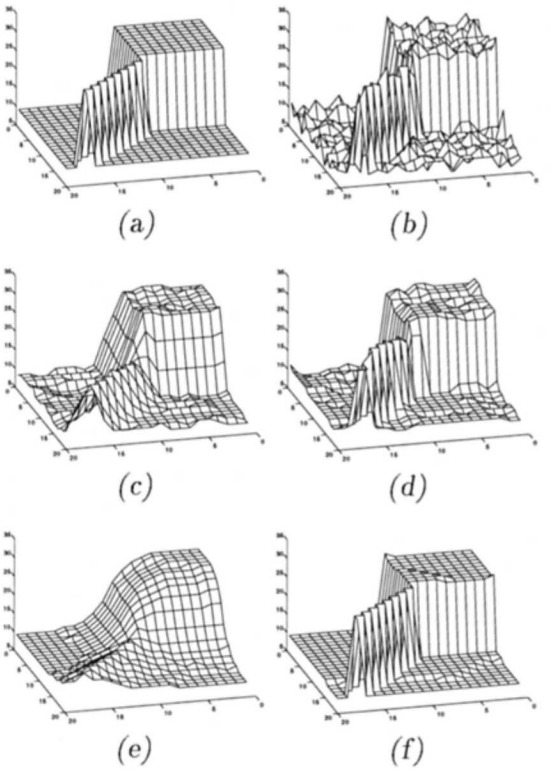


Figure 1. Graphical representation of $\lambda g(s)$ with $\lambda = 0.2$, $\gamma = 25$ and $\beta = 0.02$. The dotted curve is $g(s) = 0.25s^2$.



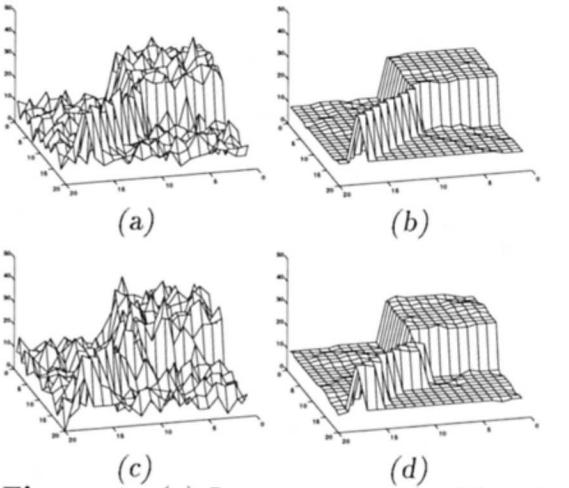
(a) (b)

Figure 2. (a) Evolution of the objective function during minimization that produced result of figure 4(f). One iteration corresponds to the application of a Tabu search procedure over one window. (b) Evolution of the objective function for a particular window.



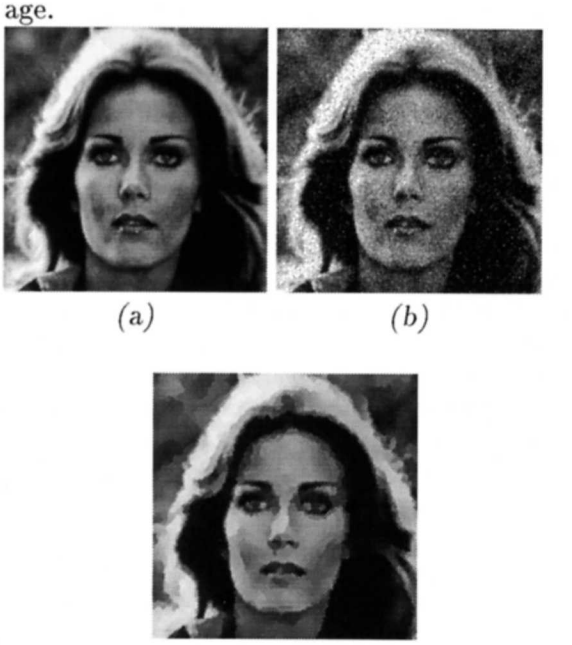
(a) (b) (c) (d) (e) (f)

Figure 3. (a) Original image. (b) Image corrupted with noise $\sigma^2 = 4$. (c) Mean filtering. (d) Median filtering. (e) Minimization of $F(\mathbf{x})$ with $g(s) = s^2$, $\lambda = 0.25$ and $\alpha = 0.5$. (f) Minimization of $F(\mathbf{x})$ with $g(s)$ of Figure 1 and $\alpha = 0.5$.



(a) (b) (c) (d)

Figure 4. (a) Image corrupted with noise $\sigma^2 = 16$. (b) Filtered image. (c) Image corrupted with noise $\sigma^2 = 25$. (d) Filtered image.



(a) (b) (c)

Figure 5. (a) Original image. (b) Image corrupted with noise $\sigma^2 = 400$. (c) Resulting filtered image with $\lambda = 0.015$, $\alpha = 0.01$, $\beta = 0.0056$ and $\gamma = 500$.