

# Assignment 5

## Question 1

$$\text{GFC}(32) \rightarrow d^5 = d^2 + 1$$

$$\begin{aligned}g(x) &= (x+d)(x+d^2)(x+d^3)(x+d^4)(x+d^5) \\&= (x^2+(d+d^2)x+d^3)(x^2+(d^3+d^4)x+d^7)(x+d^5) \\&= (x^2+d^{19}x+d^3)(x^2+d^{24}x+d^7)(x+d^5) \\&= (x^4+d^{21}x^3+d^3x^2+d^{19}x^3+d^9x^2+d^{26}x+d^3x^2 \\&\quad +d^{24}x+d^{10})(x+d^5) \\&= (x^4+(d^{21}+d^{19})x^3+(d^7+d^9+d^3)x^2+(d^{26}+d^{24})x \\&\quad +d^{10})(x+d^5) \\&= (x^4+d^{24}x^3+d^{19}x^2+d^{26}x+d^{10})(x+d^5) \\&= x^5+d^5x^4+d^{24}x^4+d^{29}x^3+d^{19}x^3+d^{24}x^2 \\&\quad +d^{29}x^2+d^3x+d^{10}x+d^{15} \\&= x^5+d^{16}x^4+d^{23}x^3+d^{26}x^2+d^{27}x+d^{15}\end{aligned}$$

$$\text{(a)} \quad n-k=5 \quad 31-k=5 \quad k=31-5=26$$

$$r = \frac{k}{n} = \frac{26}{31}$$

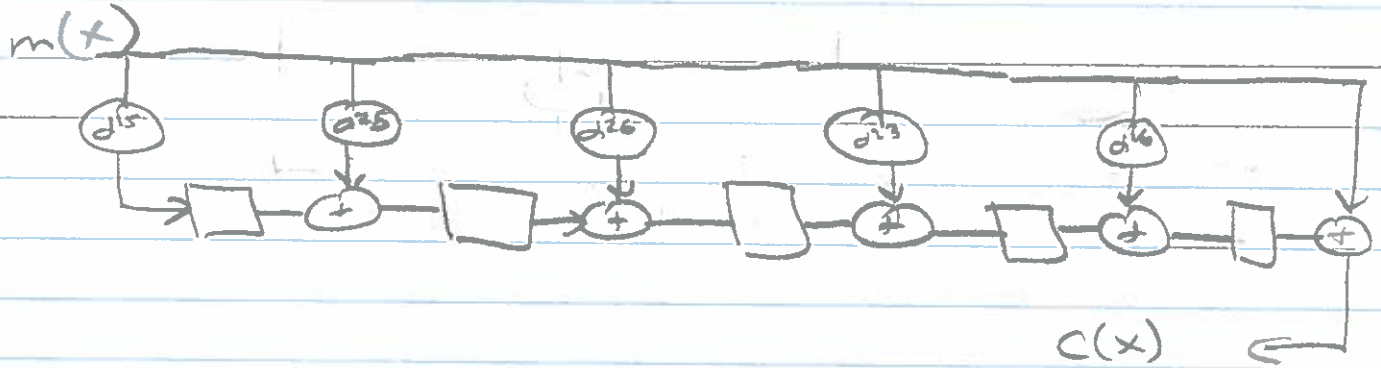
$$\text{(b)} \quad c(x) = m(x)g(x)$$

$$= (1+d^5x+d^{10}x^2+d^{15}x^3)(d^{15}+d^{20}x+d^{25}x^2+d^{30}x^3+d^{35}x^4+x^5)$$

$$\begin{aligned}&= d^{15} + d^{20}x + d^{25}x^2 + d^{30}x^3 + d^{35}x^4 + x^5 + \\&\quad d^{16}x + d^{26}x^2 + d^{27}x^3 + d^{24}x^4 + d^{17}x^5 + d^6x^6 \\&\quad + d^{20}x^2 + d^{30}x^3 + x^4 + d^{28}x^5 + d^{21}x^6 + d^5x^7 \\&\quad + d^{26}x^{13} + d^5x^{14} + d^6x^{15} + d^3x^{16} + d^{24}x^{17} + d^{11}x^{18}\end{aligned}$$

$$c(x) = d^{15} + d^1 x + d^{20} x^2 + d^{20} x^3 + d^2 x^4 + d^2 x^5 + d^9 x^6 + d^5 x^7 + d^{26} x^{13} + d^5 x^{14} + d^6 x^{15} + d^3 x^{16} + d^{27} x^{17} + d^{11} x^{18}$$

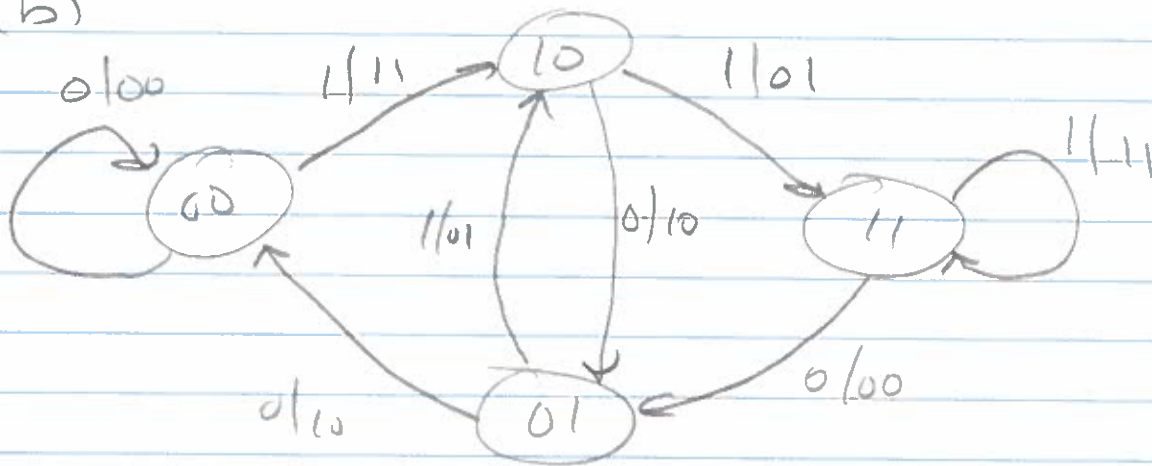
(c)



## Question 2

a)  $r = \frac{1}{2}$  (one input, two outputs)

(b)

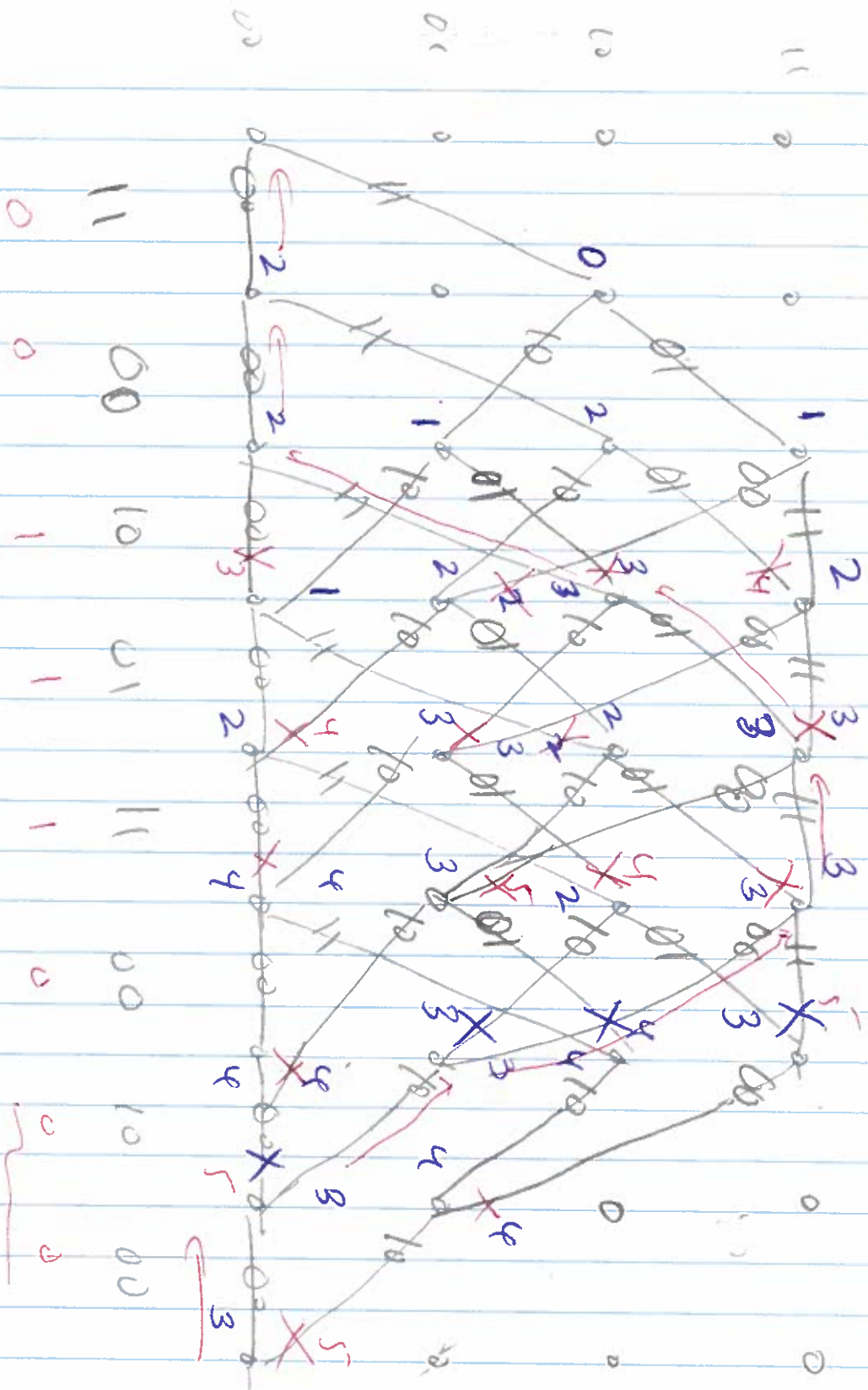


(c) input 000  $\rightarrow$  000000

input 100  $\rightarrow$  111010  $\rightarrow$  weight of 4

$\downarrow$  free = 4

input 1100 also produces a path of weight 4.



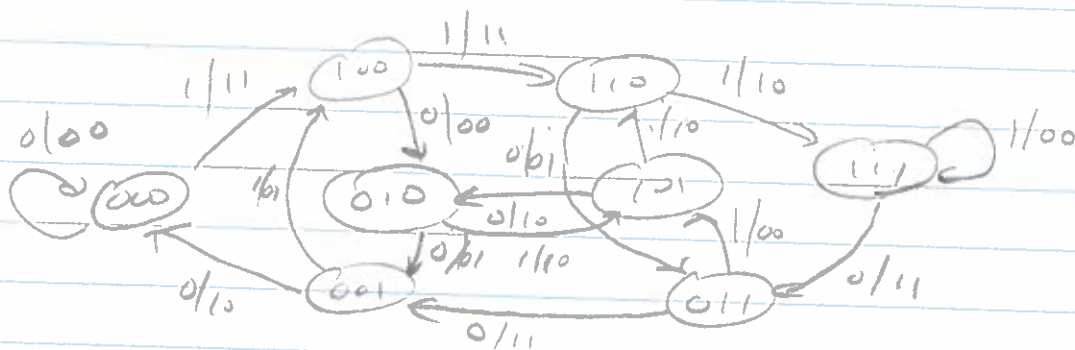
There are many possible solutions because there were many ties throughout the decoding process

### Question 3

$$G(D) = [1 + D^3 \quad 1 + D + D^2 \quad 1]$$

### Question 4

$$G(D) = [1 + D^3 \quad 1 + D^2]$$



The code is catastrophic if there is a loop that corresponds to an input that is not all zeros that results in an all zero output. A loop starts and ends in the same state. Loop  $111 \rightarrow 111$  has an input of 1 with an output of 00. Therefore the code is catastrophic.

$$\text{Let } G_2(D) = \frac{1}{D+1} G(D) = [D^2 + 1 \quad D + 1]$$

$G_2(D)$  should be equivalent to  $G(D)$



but this code still produces the same "11" state dense output is 00



$$G(s) = \frac{1}{s^2}, B(s) = [D+1 \quad 1]$$



this code is simpler and is not catastrophic