Question 1

(a) $k=3$ $n=6$ (dimensions of G)

and $r = \frac{1}{2} = \frac{3}{6}$

(b)

000	→	000000	
001	→	101100	(row 3 of G)
010	→	011001	(row 2 of G)
011	→	110101	(row 2+3 of G)
100	→	110010	(row 1 of G)
101	→	011110	(row 1+3 of G)
110	→	101011	(row 2+3 of G)
111	→	000111	(row 1+2+3 of G)

$d_{min} = 3$

(c) t = # of correctable errors

$= \frac{d_{min} - 1}{2} = 1$

(d) there are no codewords that start with

001, 010 or 100

but there are that end with them

Therefore $G_{syst} = \begin{bmatrix} 101100 \\ 110010 \\ 011001 \end{bmatrix}$

↑ I is here

When $\bar{G}_{syst}$ takes form $[P \vdots I_k]$
then H take form $[I_{n-k} \vdots P^T]$

Therefore $H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

check $GH^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$GH^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(e)

000000	101100	011001	110101	110010	011110	101011	000111
000001	101101	011000	110100	110011	011111	101010	000110
000010	101110	011011	110111	110000	011100	101001	000101
000100	101000	011101	110001	110110	011010	**101111**	000011
001000	100100	010001	111101	111010	010110	100011	001111
010000	(*) 111100	001001	100101	100010	001110	111011	010111
my choice → 100000	001100	111001	010101	010010	111110	001011	100111
100001	001101	111000	010100	010011	111111	001010	100110

(f) $\vec{r} = 101111 \rightarrow \hat{c} = 101011$ from standard array

$\hat{m} = 110$

(g) is the same question as (d).

(h) $\vec{r} = 111100$

$$S = \vec{r} H^T = (111100) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$= (0\ 1\ 0) =$ row 2 of H^T

Therefore $\vec{e} = (0\underline{1}0000)$

$\hat{c} = \vec{r} + \hat{e} = \underline{\underline{101100}} \rightarrow \hat{m} = 001$

checking standard array 111100 is in the column that is headed by 101100

i) $P_u(E)$ = probability that error pattern is a code word except all zero code word.

there are 4 code words of weight 3 and 3 of weight 4

$$P_u(E) = 4p^3(1-p)^4 + 3p^4(1-p)^3$$
$$= 4(0.05)^3(0.95)^4 + 3(0.05)^4(0.95)^3$$
$$\underline{\underline{= 4.23 \times 10^{-4}}}$$

$P(E)$ = probability that $\vec{e}$ is not one of the vectors in the leftmost column of your standard array

$\therefore P(E) = 1 - P(c)$

where $P(c)$ is the probability that the error pattern $\vec{e}$ is one of the vectors in the leftmost column of your standard array

$p(ei) = 0 = 0.95$

$p(ei) = 1 = 0.05$

$\therefore \quad p(c) = 0.95^6 + 6(0.05)(0.95)^5 + 1\cdot(0.05)^2(0.95)^4$

$= 0.9692$

$P(E) = 1 - P(c) = 0.0307 = 3.07 \times 10^{-2}$

Question 2

(a) $k=2$ $n=4$ $r=\frac{1}{2}$

b) $00 \to 0000$
$01 \to 01\alpha^2 1$
$0\alpha \to 0\alpha 1\alpha$
$0\alpha^2 \to 0\alpha^2 1\alpha^2$
$10 \to 101\alpha$
$11 \to 11\alpha\alpha^2$
$1\alpha \to 1\alpha 00$
$1\alpha^2 \to 1\alpha^2\alpha^2 1$
$\alpha 0 \to \alpha 0\alpha\alpha^2$
$\alpha 1 \to \alpha 11\alpha$
$\alpha\alpha \to \alpha\alpha\alpha^2 1$
$\alpha\alpha^2 \to \alpha\alpha^2 00$
$\alpha^2 0 \to \alpha^2 0\alpha^2 1$
$\alpha^2 1 \to \alpha^2 100$
$\alpha^2\alpha \to \alpha^2\alpha\alpha\alpha^2$
$\alpha^2\alpha^2 \to \alpha^2\alpha^2 1\alpha$

$d_{min} = 2$

(c) This code cannot even correct all error patterns of weight 1

(d) $H = \begin{bmatrix} 1 & \alpha^2 & 1 & 0 \\ \alpha & 1 & 0 & 1 \end{bmatrix}$

The rest of question 2 can't be simply repeated using the information from question 1
The standard array is too big to generate

Here is an example of syndrome decoding using this code

$\vec{r} = 0\ 1\ \alpha\ 1$

$$S = \vec{r} H^T = (0\ 1\ \alpha\ 1)\begin{bmatrix} 1 & \alpha \\ \alpha^2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= (\alpha^2+\alpha \quad 0)$$

$$= (1 \quad 0) = 1 \cdot \text{row } 3$$

therefore $\vec{e} = (0\ 0\ 1\ 0)$

$$\hat{c} = \vec{r} + \vec{e} = (0\ 1\ \alpha\ 1) + (0\ 0\ 1\ 0)$$

$$= (0\ 1\ \alpha^2\ 1)$$

example 2 $\vec{r} = \alpha\ \alpha^2\ \alpha^2\ \alpha^2$

$$S = (\alpha\ \alpha^2\ \alpha^2\ \alpha^2)\begin{bmatrix} 1 & \alpha \\ \alpha^2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = (\alpha^2\ \alpha^2)$$

there is no way to multiply any of $0, 1, \alpha, \text{or } \alpha^2$ with any row to get (α^2, α^2)

Therefore there are at least two errors in the codeword

$(\alpha^2 \ \alpha^2) = \alpha^2 \text{ row} 3 + \alpha^2 \text{ row} 4$

$$\Rightarrow \hat{e} = (0 \ 0 \ \alpha^2 \ \alpha^2)$$

or $\alpha^2 \ \alpha^2 = 1 \cdot \text{row } 2 + \alpha \text{ row } 4$

$$\hat{e} = (0 \ 1 \ 0 \ \alpha)$$

which is equally likely.

Therefore we can't decode this one.