Question 1

$\alpha^5 = \alpha^2 + 1$

0	$\alpha^{10} = \alpha^4 + 1$	$\alpha^{21} = \alpha^4 + \alpha^3$
1	$\alpha^{11} = \alpha^2 + \alpha + 1$	$\alpha^{22} = \alpha^4 + \alpha^2 + 1$
α	$\alpha^{12} = \alpha^3 + \alpha^2 + \alpha$	$\alpha^{23} = \alpha^3 + \alpha^2 + \alpha + 1$
α^2	$\alpha^{13} = \alpha^4 + \alpha^3 + \alpha^2$	$\alpha^{24} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha$
α^3	$\alpha^{14} = \alpha^4 + \alpha^3 + \alpha^2 + 1$	$\alpha^{25} = \alpha^4 + \alpha^3 + 1$
α^4	$\alpha^{15} = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$	$\alpha^{26} = \alpha^4 + \alpha^2 + \alpha + 1$
$\alpha^5 = \alpha^2 + 1$	$\alpha^{16} = \alpha^4 + \alpha^3 + \alpha + 1$	$\alpha^{27} = \alpha^3 + \alpha + 1$
$\alpha^6 = \alpha^3 + \alpha$	$\alpha^{17} = \alpha^4 + \alpha + 1$	$\alpha^{28} = \alpha^4 + \alpha^2 + \alpha$
$\alpha^7 = \alpha^4 + \alpha^2$	$\alpha^{18} = \alpha + 1$	$\alpha^{29} = \alpha^3 + 1$
$\alpha^8 = \alpha^3 + \alpha^2 + 1$	$\alpha^{19} = \alpha^2 + \alpha$	$\alpha^{30} = \alpha^4 + \alpha$
$\alpha^9 = \alpha^4 + \alpha^3 + \alpha$	$\alpha^{20} = \alpha^3 + \alpha^2$	

I won't put the whole addition table due to space but I'll include some to enable you to verify your table

$1 + \alpha = \alpha^{18}$ $\qquad$ $\alpha^9 + \alpha^{19} = \alpha^{13}$ *

$\alpha^3 + \alpha^4 = \alpha^{21}$ $\qquad$ $\alpha^{28} + \alpha^{16} = \alpha^8$

$\alpha^5 + \alpha^{19} = \alpha^{18}$ $\qquad$ etc.

$\alpha^{10} + \alpha^{20} = \alpha^{14}$

$\alpha^{27} + \alpha^7 = \alpha^{15}$

$\alpha^{18} = \alpha + 1$

$\alpha^3 + \alpha^4 = \alpha^{21}$

$\alpha^5 + \alpha^{19} = \alpha^2 + 1 + \alpha^2 + \alpha = \alpha + 1 = \alpha^{18}$

* $\alpha^9 + \alpha^{19} = \alpha^4 + \alpha^3 + \alpha + \alpha^2 + \alpha = \alpha^4 + \alpha^3 + \alpha^2 = \alpha^{13}$

Question 2

conjugacy classes

$\{1\} \longrightarrow x+1$

$\{\alpha, \alpha^2, \alpha^4, \alpha^8, \alpha^{16}\} \longrightarrow (x+\alpha)(x+\alpha^2)(x+\alpha^4)(x+\alpha^8) \cdot (x+\alpha^{16})$

$\{\alpha^3, \alpha^6, \alpha^{12}, \alpha^{24}, \alpha^{17}\} \rightarrow (x+\alpha^3)(x+\alpha^6)(x+\alpha^{12}) \cdot (x+\alpha^{24})(x+\alpha^{17})$

$\{\alpha^5, \alpha^{10}, \alpha^{20}, \alpha^9, \alpha^{18}\} \rightarrow (x+\alpha^5)(x+\alpha^{10})(x+\alpha^{20}) \cdot (x+\alpha^9)(x+\alpha^{18})$

$\{\alpha^7, \alpha^{14}, \alpha^{28}, \alpha^{25}, \alpha^{19}\} \rightarrow (x+\alpha^7)(x+\alpha^{14})(x+\alpha^{28}) \cdot (x+\alpha^{25})(x+\alpha^{19})$

$\{\alpha^{11}, \alpha^{22}, \alpha^{13}, \alpha^{26}, \alpha^{21}\} \rightarrow (x+\alpha^{11})(x+\alpha^{22})(x+\alpha^{13}) \cdot (x+\alpha^{26})(x+\alpha^{21})$

$\{\alpha^{15}, \alpha^{30}, \alpha^{29}, \alpha^{27}, \alpha^{23}\} \rightarrow (x+\alpha^{15})(x+\alpha^{30})(x+\alpha^{29}) \cdot (x+\alpha^{27})(x+\alpha^{23})$

$$(x+\alpha)(x+\alpha^2)(x+\alpha^4)(x+\alpha^8)(x+\alpha^{16}) =$$
$$(x^2+(\alpha+\alpha^2)x+\alpha^3)(x+\alpha^4)(x+\alpha^8)(x+\alpha^{16}) =$$
$$(x^2+\alpha^{19}x+\alpha^3)(x+\alpha^4)(x+\alpha^8)(x+\alpha^{16}) =$$
$$(x^3+(\alpha^4+\alpha^{19})x^2+(\alpha^{23}+\alpha^3)x+\alpha^7)(x+\alpha^8)(x+\alpha^{16}) =$$
$$(x^3+\alpha^{28}x^2+\alpha^{11}x+\alpha^7)(x+\alpha^8)(x+\alpha^{16}) =$$
$$(x^4+(\alpha^8+\alpha^{28})x^3+(\alpha^5+\alpha^{11})x^2+(\alpha^7+\alpha^{19})x+\alpha^{15}) \cdot (x+\alpha^{16}) =$$
$$(x^4+\alpha^{16}x^3+\alpha x^2+\alpha^{30}x+\alpha^{15})(x+\alpha^{16}) =$$
$$(x^5+(\alpha^{16}+\alpha^{16})x^4+(\alpha^{32}+\alpha)x^3+(\alpha^{30}+\alpha^{17})x^2 + (\alpha^{15}+\alpha^{15})x+\alpha^{31})$$
$$= (x^5+x^2+1)$$

similarly $(x+\alpha^3)(x+\alpha^6)(x+\alpha^{12})(x+\alpha^{24})(x+\alpha^{17})$
$$= x^5+x^4+x^3+x^2+1$$

$(x+\alpha^5)(x+\alpha^{10})(x+\alpha^{20})(x+\alpha^9)(x+\alpha^{18})$
$$= x^5+x^4+x^2+x+1$$

$(x+\alpha^7)(x+\alpha^{14})(x+\alpha^{28})(x+\alpha^{25})(x+\alpha^{19})$
$$= x^5+x^3+x^2+x+1$$

$(x+\alpha^{11})(x+\alpha^{22})(x+\alpha^{13})(x+\alpha^{26})(x+\alpha^{21})$
$$= x^5+x^4+x^3+x+1$$

and

$(x+\alpha^{15})(x+\alpha^{30})(x+\alpha^{29})(x+\alpha^{27})(x+\alpha^{23})$
$$= x^5+x^3+1$$

$$x^{31}+1 = (x+1)(x^5+x^2+1)(x^5+x^4+x^2+x+1)$$
$$\cdot(x^5+x^3+x^2+x+1)(x^5+x^4+x^3+x+1)$$
$$\cdot(x^5+x^3+1)(x^5+x^4+x^3+x^2+1)$$

Question 3

(a) $k=3$ $n=5$ $r=\frac{3}{5}$

(b)
$000 \rightarrow 00000$
$001 \rightarrow 01101$
$010 \rightarrow 11111$
$011 \rightarrow 10010$
$100 \rightarrow 11001$
$101 \rightarrow 10100$
$110 \rightarrow 00110$
$111 \rightarrow 01010$

(c) $d_{min} = \min(HW) = 2$

(d) need to put G in systematic form

$\rightarrow$ only need to find the code words in (a) that start with 100, 010 and 001

$$G_{syst} = \begin{bmatrix} 1&0&0&1&0 \\ 0&1&0&0&1 \\ 0&0&1&1&0 \end{bmatrix} \qquad H = \begin{bmatrix} 1&0&1&1&0 \\ 0&1&0&0&1 \end{bmatrix}$$

Question 4

$$G = \begin{bmatrix} 1 & 0 & \alpha & 1 & \alpha^2 \\ 0 & 1 & \alpha^2 & \alpha & 1 \end{bmatrix}$$

G is in systematic form

b) $$H = \begin{bmatrix} \alpha & \alpha^2 & 1 & 0 & 0 \\ 1 & \alpha & 0 & 1 & 0 \\ \alpha^2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) G is $2 \times 5 = k \times n$ therefore

$k = 2 \quad n = 5 \quad r = \frac{2}{5}$

(c) d_{min} = # of rows that ~~combined to form~~ can be linearly combined to form $\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$$\Rightarrow \text{row}1 + \alpha^2 \text{row}2 = \begin{bmatrix} \alpha \\ 1 \\ \alpha^2 \end{bmatrix} + \alpha^2 \begin{bmatrix} \alpha^2 \\ \alpha \\ 1 \end{bmatrix}$$

$$= \begin{matrix} \alpha \\ 1 \\ \alpha^2 \end{matrix} + \begin{matrix} \alpha \\ 1 \\ \alpha^2 \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Therefore $d_{min} = 2$

also, you can solve this problem by finding all codewords.