Appendix of A Logical Framework for Systems Biology

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A Sequent Calculus for HyLL

Judgemental rules

$$\Gamma; p(\boldsymbol{t}) \ @\ w \vdash p(\boldsymbol{t}) \ @\ w \ [init] \qquad \frac{\Gamma, A \ @\ u; \Delta, A \ @\ u \vdash C \ @\ w}{\Gamma, A \ @\ u; \Delta \vdash C \ @\ w}\ copy$$

Multiplicative

$$\frac{\Gamma; \Delta \vdash A \ @\ w \quad \Gamma; \Delta' \vdash B \ @\ w}{\Gamma; \Delta, \Delta' \vdash A \otimes B \ @\ w} \otimes R \qquad \frac{\Gamma; \Delta, A \ @\ u, B \ @\ u \vdash C \ @\ w}{\Gamma; \Delta, A \otimes B \ @\ u \vdash C \ @\ w} \otimes L$$

$$\Gamma; . \vdash \mathbf{1} \ @\ w \ [\mathbf{1}R] \qquad \frac{\Gamma; \Delta \vdash C \ @\ w}{\Gamma; \Delta, 1 \ @\ u \vdash C \ @\ w}\, 1\, L$$

$$\frac{\Gamma; \Delta,\ A \ @\ w \vdash B \ @\ w}{\Gamma; \Delta \vdash A \rightarrow B \ @\ w} \rightarrow R \qquad \frac{\Gamma; \Delta \vdash A \ @\ u \quad \Gamma; \Delta', B \ @\ u \vdash C \ @\ w}{\Gamma; \Delta, \Delta', A \rightarrow B \ @\ u \vdash C \ @\ w} \rightarrow L$$

Additive

$$\Gamma; \Delta \vdash T \ @\ w \ [T\ R] \qquad \Gamma; \Delta, \mathbf{0} \ @\ u \vdash C \ @\ w \ [\mathbf{0}L]$$

$$\frac{\Gamma; \Delta \vdash A \ @\ w \qquad \Gamma; \Delta \vdash B \ @\ w}{\Gamma; \Delta \vdash A \,\&\, B \ @\ w}\ \&\, R \qquad \frac{\Gamma; \Delta, A_i \ @\ u \vdash C \ @\ w}{\Gamma; \Delta, A_1 \,\&\, A_2 \ @\ u \vdash C \ @\ w}\ \&\, L_i$$

$$\frac{\Gamma; \Delta \vdash A_i \ @\ w}{\Gamma; \Delta \vdash A_1 \oplus A_2 \ @\ w} \oplus R_i \qquad \frac{\Gamma; \Delta, A \ @\ u \vdash C \ @\ w \quad \Gamma; \Delta, B \ @\ u \vdash C \ @\ w}{\Gamma; \Delta, A \oplus B \ @\ u \vdash C \ @\ w} \oplus L$$

Quantifiers

$$\frac{\Gamma; \Delta \vdash A \ @\ w}{\Gamma; \Delta \vdash \forall \alpha.\ A \ @\ w}\ [\forall R^{\alpha}] \qquad \frac{\Gamma; \Delta, A[\tau/\alpha] \ @\ u \vdash C \ @\ w}{\Gamma; \Delta, \forall \alpha.\ A \ @\ u \vdash C \ @\ w}\ [\forall L]$$

$$\frac{\Gamma; \Delta \vdash A[\tau/\alpha] \ @\ w}{\Gamma; \Delta \vdash \exists \alpha.\ A \ @\ w}\ [\exists R] \qquad \frac{\Gamma; \Delta, A \ @\ u \vdash C \ @\ w}{\Gamma; \Delta, \exists \alpha.\ A \ @\ u \vdash C \ @\ w}\ [\exists L^{\alpha}]$$

For $\forall R^{\alpha}$ and $\exists L^{\alpha}$, α is assumed to be fresh with respect to Γ, Δ, and C.
For $\exists R$ and $\forall L$, τ stands for a term or world, as appropriate.

Exponentials rules

$$\frac{\Gamma; . \vdash A \;@\; w}{\Gamma; . \vdash !A \;@\; w}\; !R \qquad \frac{\Gamma, A \;@\; u; \Delta \vdash C \;@\; w}{\Gamma; \Delta, !A \;@\; u \vdash C \;@\; w}\; !L$$

Hybrid connectives

$$\frac{\Gamma; \Delta \vdash A \;@\; u}{\Gamma; \Delta \vdash (A \;\mathtt{at}\; u) \;@\; w}\; [\mathtt{at}\; R] \qquad \frac{\Gamma; \Delta, A \;@\; u \vdash C \;@\; w}{\Gamma; \Delta, (A \;\mathtt{at}\; u) \;@\; v \vdash C \;@\; w}\; [\mathtt{at}\; L]$$

$$\frac{\Gamma; \Delta \vdash A[w/u] \;@\; w}{\Gamma; \Delta \vdash \downarrow u.A \;@\; w}\; [\downarrow R] \qquad \frac{\Gamma; \Delta, A[v/u] \;@\; v \vdash C \;@\; w}{\Gamma; \Delta, \downarrow u.A \;@\; v \vdash C \;@\; w}\; [\downarrow L]$$

B Example Specification in HyLL

B.1 Strong Rules

- *Variables*:

  $\mathtt{unchanged}(x, w) \stackrel{\text{def}}{=} \;!\; [(\mathtt{pres}(x) \;\mathtt{at}\; w \to \mathtt{pres}(x) \;\mathtt{at}\; w.1) \;\;\&\;\; (\mathtt{abs}(x) \;\mathtt{at}\; w \to \mathtt{abs}(x) \;\mathtt{at}\; w.1)]$.
  $\mathtt{unchanged}(V, w) \stackrel{\text{def}}{=} \otimes_{x \in V} \mathtt{unchanged}(x, w)$.
- *Activation*:

  $s\_\mathtt{active}(V, a, b) \stackrel{\text{def}}{=} \mathtt{pres}(a) \otimes \mathtt{abs}(b) \to \delta_1(\mathtt{pres}(a) \otimes \mathtt{pres}(b)) \otimes \downarrow u.\; \mathtt{unchanged}(V \setminus \{a, b\}, u))$.
- *Activation with consumption*:

  $s\_\mathtt{active}_c(V, a, b) \stackrel{\text{def}}{=} \mathtt{pres}(a) \otimes \mathtt{abs}(b) \to \delta_1(\mathtt{abs}(a) \otimes \mathtt{pres}(b))) \otimes \downarrow u.\; \mathtt{unchanged}(V \setminus \{a, b\}, u))$.
- *Strong activation*:

  $s\_\mathtt{active}_s(V, a, b) \stackrel{\text{def}}{=} \mathtt{abs}(a) \otimes \mathtt{pres}(b) \to \delta_1(\mathtt{abs}(a) \otimes \mathtt{abs}(b)) \otimes \downarrow u.\; \mathtt{unchanged}(V \setminus \{a, b\}, u))$.
- *Inhibition*:

  $s\_\mathtt{inhib}(V, a, b) \stackrel{\text{def}}{=} \mathtt{pres}(a) \otimes \mathtt{pres}(b) \to \delta_1(\mathtt{pres}(a) \otimes \mathtt{abs}(b)) \otimes \downarrow u.\; \mathtt{unchanged}(V \setminus \{a, b\}, u))$.
- *Inhibition with consumption*:

  $s\_\mathtt{inhib}_c(V, a, b) \stackrel{\text{def}}{=} \mathtt{pres}(a) \otimes \mathtt{pres}(b) \to \delta_1(\mathtt{abs}(a) \otimes \mathtt{abs}(b)) \otimes \downarrow u.\; \mathtt{unchanged}(V \setminus \{a, b\}, u))$.
- *Strong inhibition*:

  $s\_\mathtt{inhib}_s(V, a, b) \stackrel{\text{def}}{=} \mathtt{abs}(a) \otimes \mathtt{abs}(b) \to \delta_1(\mathtt{abs}(a) \otimes \mathtt{pres}(b)) \otimes \downarrow u.\; \mathtt{unchanged}(V \setminus \{a, b\}, u))$.
- *Well definedness*:

  $\mathtt{well\_defined}_0(V) \stackrel{\text{def}}{=} \forall a \in V.\; [\mathtt{pres}(a) \otimes \mathtt{abs}(a) \to 0]$.
  $\mathtt{well\_defined}_1(V) \stackrel{\text{def}}{=} \forall a \in V.\; [\mathtt{pres}(a) \oplus \mathtt{abs}(a)]$.
  $\mathtt{well\_defined}(V) \stackrel{\text{def}}{=} \mathtt{well\_defined}_0(V), \mathtt{well\_defined}_1(V)$.
- *The system*

$$\begin{aligned}
\texttt{vars} &\stackrel{\text{def}}{=} \{\texttt{p53}, \texttt{Mdm2}, \texttt{DNAdam}\}\\
s_\texttt{rule}(1) &\stackrel{\text{def}}{=} s_\texttt{inhib}(\texttt{vars}, \texttt{DNAdam}, \texttt{Mdm2})\\
&\stackrel{\text{def}}{=} \texttt{pres}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2}) \to \delta_1(\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{p53}, u)\\
s_\texttt{rule}(2) &\stackrel{\text{def}}{=} \texttt{Inhib}_\texttt{s}(\texttt{vars}, \texttt{Mdm2}, \texttt{p53})\\
&\stackrel{\text{def}}{=} \texttt{abs}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53}) \to \delta_1(\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{DNAdam}, u))\\
s_\texttt{rule}(3) &\stackrel{\text{def}}{=} s_\texttt{active}(\texttt{vars}, \texttt{p53}, \texttt{Mdm2})\\
&\stackrel{\text{def}}{=} \texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{Mdm2}) \to \delta_1(\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{DNAdam}, u))\\
s_\texttt{rule}(4) &\stackrel{\text{def}}{=} s_\texttt{inhib}(\texttt{vars}, \texttt{Mdm2}, \texttt{p53})\\
&\stackrel{\text{def}}{=} \texttt{pres}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53}) \to \delta_1(\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{DNAdam}, u))\\
s_\texttt{rule}(5) &\stackrel{\text{def}}{=} \texttt{Inhib}_\texttt{c}(\texttt{vars}, \texttt{p53}, \texttt{DNAdam})\\
&\stackrel{\text{def}}{=} \texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{DNAdam}) \to \delta_1(\texttt{abs}(\texttt{p53}) \otimes \texttt{abs}(\texttt{DNAdam})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{Mdm2}, u))\\
s_\texttt{rule}(6) &\stackrel{\text{def}}{=} \texttt{Inhib}_\texttt{s}(\texttt{vars}, \texttt{DNAdam}, \texttt{Mdm2})\\
&\stackrel{\text{def}}{=} \texttt{abs}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2}) \to \delta_1(\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{p53}, u))
\end{aligned}$$

$$\texttt{system} \stackrel{\text{def}}{=} \texttt{vars}, s\_\texttt{rule}(1), s\_\texttt{rule}(2), s\_\texttt{rule}(3), s\_\texttt{rule}(4), s\_\texttt{rule}(5), s\_\texttt{rule}(6), \texttt{well\_defined}(\texttt{vars}).$$

– *Initial state*:

$$\texttt{initial\_state} \stackrel{\text{def}}{=} \texttt{abs}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2}),$$
$$\texttt{initial\_state at } 0.$$

– *Hypothesis (with strong rules)*:

$$\begin{aligned}
\texttt{dont_care}(x) &\stackrel{\text{def}}{=} \texttt{pres}(x) \oplus \texttt{abs}(x)\\
\texttt{dont_care}(V) &\stackrel{\text{def}}{=} \otimes_{x \in V} \texttt{dont_care}(x)
\end{aligned}$$

$$\begin{aligned}
s_\texttt{fireable}(1) &\stackrel{\text{def}}{=} \texttt{pres}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2}) \otimes \texttt{dont_care}(\texttt{p53})\\
s_\texttt{fireable}(2) &\stackrel{\text{def}}{=} \texttt{abs}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53}) \otimes \texttt{dont_care}(\texttt{DNAdam})\\
s_\texttt{fireable}(3) &\stackrel{\text{def}}{=} \texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{Mdm2}) \otimes \texttt{dont_care}(\texttt{DNAdam})\\
s_\texttt{fireable}(4) &\stackrel{\text{def}}{=} \texttt{pres}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53}) \otimes \texttt{dont_care}(\texttt{DNAdam})\\
s_\texttt{fireable}(5) &\stackrel{\text{def}}{=} \texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{DNAdam}) \otimes \texttt{dont_care}(\texttt{Mdm2})\\
s_\texttt{fireable}(6) &\stackrel{\text{def}}{=} \texttt{abs}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2}) \otimes \texttt{dont_care}(\texttt{p53})
\end{aligned}$$

$$\begin{aligned}
&s_\texttt{not_fireable}(1) \stackrel{\text{def}}{=}\\
&((\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2})) \oplus (\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2}))) \otimes \texttt{dont_care}(\texttt{p53})\\
&s_\texttt{not_fireable}(2) \stackrel{\text{def}}{=}\\
&((\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53})) \oplus (\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \oplus (\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \otimes \texttt{dont_care}(\texttt{DNAdam})\\
&s_\texttt{not_fireable}(3) \stackrel{\text{def}}{=}\\
&((\texttt{abs}(\texttt{p53}) \otimes \texttt{abs}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{abs}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2}))) \otimes \texttt{dont_care}(\texttt{DNAdam})\\
&s_\texttt{not_fireable}(4) \stackrel{\text{def}}{=}\\
&((\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \oplus (\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53})) \oplus (\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53}))) \otimes \texttt{dont_care}(\texttt{DNAdam})\\
&s_\texttt{not_fireable}(5) \stackrel{\text{def}}{=}\\
&((\texttt{abs}(\texttt{p53}) \otimes \texttt{pres}(\texttt{DNAdam})) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{DNAdam})) \oplus (\texttt{abs}(\texttt{p53}) \otimes \texttt{abs}(\texttt{DNAdam}))) \otimes \texttt{dont_care}(\texttt{Mdm2})\\
&s_\texttt{not_fireable}(6) \stackrel{\text{def}}{=}\\
&((\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2})) \oplus (\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2}))) \otimes \texttt{dont_care}(\texttt{p53})
\end{aligned}$$

B.2 General Rules

- *Variables*:

$$\texttt{unchanged}(x,w) \stackrel{\text{def}}{=} \ ! \ [(\texttt{pres}(x) \ \texttt{at} \ w \to \texttt{pres}(x) \ \texttt{at} \ w.1) \ \ \& \ \ (\texttt{abs}(x) \ \texttt{at} \ w \to \texttt{abs}(x) \ \texttt{at} \ w.1)].$$
$$\texttt{unchanged}(V,w) \stackrel{\text{def}}{=} \otimes_{x \in V} \texttt{unchanged}(x,w).$$

- *Activation*:

$$\begin{aligned}\texttt{active}(V,a,b) \stackrel{\text{def}}{=} & (\texttt{pres}(a) \oplus (\texttt{pres}(a) \otimes \texttt{pres}(b)) \oplus (\texttt{pres}(a) \otimes \texttt{abs}(b))) \\ & \to \delta_1 \ \ (\texttt{pres}(a) \otimes \texttt{pres}(b)) \otimes \downarrow u.\ \texttt{unchanged}(V \setminus \{a,b\},u)).\end{aligned}$$

- *Activation with consumption*:

$$\begin{aligned}\texttt{active}_c(V,a,b) \stackrel{\text{def}}{=} & (\texttt{pres}(a) \oplus (\texttt{pres}(a) \otimes \texttt{pres}(b)) \oplus (\texttt{pres}(a) \otimes \texttt{abs}(b))) \\ & \to \delta_1 \ \ (\texttt{abs}(a) \otimes \texttt{pres}(b)) \otimes \downarrow u.\ \texttt{unchanged}(V \setminus \{a,b\},u)).\end{aligned}$$

- *Strong activation*:

$$\begin{aligned}\texttt{active}_s(V,a,b) \stackrel{\text{def}}{=} & (\texttt{abs}(a) \oplus (\texttt{abs}(a) \otimes \texttt{pres}(b)) \oplus (\texttt{abs}(a) \otimes \texttt{abs}(b))) \\ & \to \delta_1 \ \ (\texttt{abs}(a) \otimes \texttt{abs}(b)) \otimes \downarrow u.\ \texttt{unchanged}(V \setminus \{a,b\},u)).\end{aligned}$$

- *Inhibition*:

$$\begin{aligned}\texttt{inhib}(V,a,b) \stackrel{\text{def}}{=} & (\texttt{pres}(a) \oplus (\texttt{pres}(a) \otimes \texttt{pres}(b)) \oplus (\texttt{pres}(a) \otimes \texttt{abs}(b))) \\ & \to \delta_1 \ \ (\texttt{pres}(a) \otimes \texttt{abs}(b)) \otimes \downarrow u.\ \texttt{unchanged}(V \setminus \{a,b\},u)).\end{aligned}$$

- *Inhibition with consumption*:

$$\begin{aligned}\texttt{inhib}_c(V,a,b) \stackrel{\text{def}}{=} & (\texttt{pres}(a) \oplus (\texttt{pres}(a) \otimes \texttt{pres}(b)) \oplus (\texttt{pres}(a) \otimes \texttt{abs}(b))) \\ & \to \delta_1 \ \ (\texttt{abs}(a) \otimes \texttt{abs}(b)) \otimes \downarrow u.\ \texttt{unchanged}(V \setminus \{a,b\},u)).\end{aligned}$$

- *Strong inhibition*:

$$\begin{aligned}\texttt{inhib}_s(V,a,b) \stackrel{\text{def}}{=} & (\texttt{abs}(a) \oplus (\texttt{abs}(a) \otimes \texttt{pres}(b)) \oplus (\texttt{abs}(a) \otimes \texttt{abs}(b))) \\ & \to \delta_1 \ \ (\texttt{abs}(a) \otimes \texttt{pres}(b)) \otimes \downarrow u.\ \texttt{unchanged}(V \setminus \{a,b\},u)).\end{aligned}$$

- *Well definedness*:

$$\texttt{well\_defined}_0(V) \stackrel{\text{def}}{=} \forall a \in V.\ [\texttt{pres}(a) \otimes \texttt{abs}(a) \to 0].$$
$$\texttt{well\_defined}_1(V) \stackrel{\text{def}}{=} \forall a \in V.\ [\texttt{pres}(a) \oplus \texttt{abs}(a)].$$
$$\texttt{well\_defined}(V) \ \stackrel{\text{def}}{=} \texttt{well\_defined}_0(V), \texttt{well\_defined}_1(V).$$

- *The system*

$$\begin{aligned}\texttt{vars} \ \ & \stackrel{\text{def}}{=} \{\texttt{p53}, \texttt{Mdm2}, \texttt{DNAdam}\} \\ \texttt{rule}(1) & \stackrel{\text{def}}{=} \texttt{inhib}(\texttt{vars}, \texttt{DNAdam}, \texttt{Mdm2}) \\ & \stackrel{\text{def}}{=} (\texttt{pres}(\texttt{DNAdam}) \oplus (\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2}))) \\ & \quad \to \delta_1(\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{p53}, u) \\ \texttt{rule}(2) & \stackrel{\text{def}}{=} \texttt{inhib}_\texttt{s}(\texttt{vars}, \texttt{Mdm2}, \texttt{p53}) \\ & \stackrel{\text{def}}{=} (\texttt{abs}(\texttt{Mdm2}) \oplus (\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \oplus (\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53}))) \\ & \quad \to \delta_1(\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{DNAdam}, u))\end{aligned}$$

$$\texttt{rule}(3) \stackrel{\text{def}}{=} \texttt{active}(\texttt{vars}, \texttt{p53}, \texttt{Mdm2})$$
$$\stackrel{\text{def}}{=} (\texttt{pres}(\texttt{p53}) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{Mdm2})))$$
$$\rightarrow \delta_1(\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{DNAdam}, u))$$
$$\texttt{rule}(4) \stackrel{\text{def}}{=} \texttt{inhib}(\texttt{vars}, \texttt{Mdm2}, \texttt{p53})$$
$$\stackrel{\text{def}}{=} (\texttt{pres}(\texttt{Mdm2}) \oplus (\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \oplus (\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53})))$$
$$\rightarrow \delta_1(\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{DNAdam}, u))$$
$$\texttt{rule}(5) \stackrel{\text{def}}{=} \texttt{inhib}_\texttt{c}(\texttt{vars}, \texttt{p53}, \texttt{DNAdam})$$
$$\stackrel{\text{def}}{=} (\texttt{pres}(\texttt{p53}) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{DNAdam})) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{DNAdam})))$$
$$\rightarrow \delta_1(\texttt{abs}(\texttt{p53}) \otimes \texttt{abs}(\texttt{DNAdam})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{Mdm2}, u))$$
$$\texttt{rule}(6) \stackrel{\text{def}}{=} \texttt{inhib}_\texttt{s}(\texttt{vars}, \texttt{DNAdam}, \texttt{Mdm2})$$
$$\stackrel{\text{def}}{=} (\texttt{abs}(\texttt{DNAdam}) \oplus (\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2})))$$
$$\rightarrow \delta_1(\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \otimes \downarrow u.\ \texttt{unchanged}(\texttt{p53}, u))$$

$$\texttt{system} \stackrel{\text{def}}{=} \texttt{vars}, \texttt{rule}(1), \texttt{rule}(2), \texttt{rule}(3), \texttt{rule}(4), \texttt{rule}(5), \texttt{rule}(6), \texttt{well\_defined}(\texttt{vars}).$$

– *Initial state*:

$$\texttt{initial\_state} \stackrel{\text{def}}{=} \texttt{abs}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2}),$$
$$\texttt{initial\_state at } 0.$$

– *Hypothesis*:

$$\texttt{dont\_care}(x) \stackrel{\text{def}}{=} \texttt{pres}(x) \oplus \texttt{abs}(x)$$
$$\texttt{dont\_care}(V) \stackrel{\text{def}}{=} \otimes_{x \in V} \texttt{dont\_care}(x)$$
$$\texttt{fireable}(1) \stackrel{\text{def}}{=} (\texttt{pres}(\texttt{DNAdam}) \oplus (\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2}))) \otimes \texttt{dont\_care}(\texttt{p53})$$
$$\texttt{fireable}(2) \stackrel{\text{def}}{=} (\texttt{abs}(\texttt{Mdm2}) \oplus (\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \oplus (\texttt{abs}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53}))) \otimes \texttt{dont\_care}(\texttt{DNAdam})$$
$$\texttt{fireable}(3) \stackrel{\text{def}}{=} (\texttt{pres}(\texttt{p53}) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{Mdm2}))) \otimes \texttt{dont\_care}(\texttt{DNAdam})$$
$$\texttt{fireable}(4) \stackrel{\text{def}}{=} (\texttt{pres}(\texttt{Mdm2}) \oplus (\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{pres}(\texttt{p53})) \oplus (\texttt{pres}(\texttt{Mdm2}) \otimes \texttt{abs}(\texttt{p53}))) \otimes \texttt{dont\_care}(\texttt{DNAdam})$$
$$\texttt{fireable}(5) \stackrel{\text{def}}{=} (\texttt{pres}(\texttt{p53}) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{pres}(\texttt{DNAdam})) \oplus (\texttt{pres}(\texttt{p53}) \otimes \texttt{abs}(\texttt{DNAdam}))) \otimes \texttt{dont\_care}(\texttt{Mdm2})$$
$$\texttt{fireable}(6) \stackrel{\text{def}}{=} (\texttt{abs}(\texttt{DNAdam}) \oplus (\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{pres}(\texttt{Mdm2})) \oplus (\texttt{abs}(\texttt{DNAdam}) \otimes \texttt{abs}(\texttt{Mdm2}))) \otimes \texttt{dont\_care}(\texttt{p53})$$
$$\texttt{not\_fireable}(1) \stackrel{\text{def}}{=} \texttt{abs}(\texttt{DNAdam}) \otimes \texttt{dont\_care}(\{\texttt{Mdm2}, \texttt{p53}\})$$
$$\texttt{not\_fireable}(2) \stackrel{\text{def}}{=} \texttt{pres}(\texttt{Mdm2}) \otimes \texttt{dont\_care}(\{\texttt{p53}, \texttt{DNAdam}\})$$
$$\texttt{not\_fireable}(3) \stackrel{\text{def}}{=} \texttt{abs}(\texttt{p53}) \otimes \texttt{dont\_care}(\{\texttt{Mdm2}, \texttt{DNAdam}\})$$
$$\texttt{not\_fireable}(4) \stackrel{\text{def}}{=} \texttt{abs}(\texttt{Mdm2}) \otimes \texttt{dont\_care}(\{\texttt{p53}, \texttt{DNAdam}\})$$
$$\texttt{not\_fireable}(5) \stackrel{\text{def}}{=} \texttt{abs}(\texttt{p53}) \otimes \texttt{dont\_care}(\{\texttt{DNAdam}, \texttt{Mdm2}\})$$
$$\texttt{not\_fireable}(6) \stackrel{\text{def}}{=} \texttt{pres}(\texttt{DNAdam}) \otimes \texttt{dont\_care}(\{\texttt{Mdm2}, \texttt{p53}\})$$