On The Capacity of IRS-Assisted Gaussian SIMO Channels

Milad Dabiri, Sergey Loyka

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***Abstract*—Intelligent reflective surface (IRS) has recently emerged as a valuable addition to other key technologies for 5/6G to improve their energy efficiency and achievable rate at low cost. An IRS-assisted single-input multiple-output (SIMO) channel is studied here from an information-theoretic perspective. Its channel capacity includes an optimization over IRS phase shifts, which is not a convex problem and for which no closed-form solutions are known either. A number of closed-form bounds are obtained for the general case, which are tight in some special cases and thus provide a globally-optimal solution to the original problem. Based on a closed-form globally-optimal solution for the single reflector case, a computationally-efficient iterative algorithm is proposed for the general case. Its convergence to a local optimum is rigorously proved and a number of cases are identified where its convergence point is also globally optimal. Numerical experiments show that the algorithm converges fast in practice and its convergence point is close to a global optimum.**

I. Introduction

Massive multi-antenna (MIMO) systems are a key technology for 5/6G as they offer high spectral efficiency and simplified processing in multi-user environments [1]. However, their hardware complexity and energy consumption are high for many applications so that various techniques are explored to reduce them. A promising new approach is an intelligent reflective surface (IRS), which assists the regular MIMO channel and improves signal quality at the receiver via constructive reflective paths [2]. Unlike the standard approach, whereby the channel is considered to be out of control, this new approach aims at controlling the channel to improve transmission rate and energy efficiency. Furthermore, since IRS makes use of passive elements (except for simple control circuits) and due to its overall simplicity (as opposed to massive MIMO), this can be achieved in a cost-efficient way.

One of the key problems is to optimize IRS to increase the rate, spectral or energy efficiency or to decrease transmit power. This problem has a difficult analytical structure and it is non-convex, even in its most simple (but non-trivial) form, i.e. for a MISO or SIMO channel. Therefore, various iterative algorithms have been proposed [3]-[6].

In particular, the MISO channel with IRS assistance was considered in [3] and a joint optimization problem of maximizing the receive (Rx) SNR over the transmit (Tx) beamforming vector and IRS phase shifts was formulated. Since this problem is not convex, the semidefinite relaxation technique was used to obtain numerically an upper bound and an alternating optimization method was used to minimize the Tx power. A similar MISO setup was considered in [4], where an iterative algorithm was proposed based on a fixed-point iteration for Tx beamforming vector and manifold optimization for IRS phase shifts. This approach provides higher rates and lower computational complexity compared to [3]. In [7], an IRS-assisted cognitive communication system was considered and the rates of secondary users were optimized while satisfying interference constraint for a primary user.

A common feature of the above studies is that no analytical solutions were obtained due to a difficult analytical structure of the considered problems and that the proposed algorithms were shown to converge only to a local rather than global optimum; it is not clear how far away is an achieved local optimum from a global one or whether the algorithms can converge to a global optimum under certain conditions. Since the considered problems are not convex, their numerical complexity is in general exponential (i.e. prohibitively large for moderate to large systems, especially for real-time optimization).

Some practical aspects of IRS design and implementation, such as implementing phase shifts over the whole continuous interval $[0, 2\pi]$, or the availability and accuracy of channel state information (CSI), have been also considered [7]-[9]. An experimental study to validate the benefits of IRS in practice has been reported in [12].

In this paper, we consider an IRS-assisted SIMO channel from an information-theoretic perspective via its channel capacity, where IRS-induced phase shifts are also optimization variables. Since an analytical solution to this problem is not known in the general case and since the known numerical algorithms exhibit local convergence at best, we obtain a number of closed-form global bounds that hold in the general case and are tight in some special cases. This yields a number of closed-form solutions to optimal IRS phase shifts and also serves as a benchmark for evaluating numerical algorithms (i.e. how close they are to a global optimum): if an algorithm convergence point is close to an upper bound, then automatically it is also close to a global optimum (since global optimum is sandwiched between the upper bound and convergence point) and this can serve as a stopping criterion in some cases.

Based on an analytical closed-form and globally-optimal solution for the single reflector case, a semi-analytical alternating optimization algorithm is proposed for the general case. At each alternation, it makes use of the closed-form solution and hence does not need any numerical procedure involving gradients and/or Hessians (as in e.g. gradient or Newton method) and thus it is computationally-efficient, even for a large number of antennas and reflectors (as in 5/6G). Its convergence to a local maximum point is rigorously proved and a number of cases are identified whereby it converges to a global optimum or close to it. Numerical experiments show its computational efficiency and fast convergence. A comparison to the fixed point iteration (FPI) method in [4] shows that the proposed algorithm achieves higher IRS gain for all tested channels.

M. Dabiri and S. Loyka are with the School of Electrical Engineering and Computer Science, University of Ottawa, Canada, e-mail: sergey.loyka@uottawa.ca

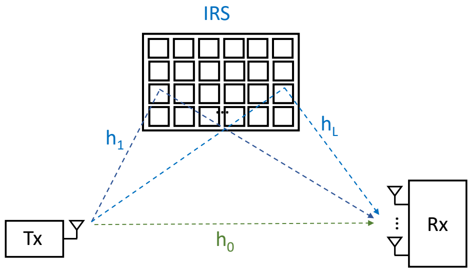

Fig. 1. An illustration of IRS-assisted SIMO channel.

Notations: bold capitals and bold lower-case letters denote matrices and vectors, respectively; $|\mathbf{h}|$, $\mathbf{h}'$ and $\mathbf{h}^+$ denote Euclidean norm (length), transposition and Hermitian conjugation of column vector $\mathbf{h}$ and h_i is its i-th entry; $\Re\{z\}$ and $\Im\{z\}$ denote real and imaginary parts of complex number z while $\arg\{z\}$ is its argument (phase).

II. Channel Model

Let us consider an IRS-assisted SIMO channel as in Fig. 1, including a single-antenna transmitter (Tx), an IRS equipped with L reflectors, and a receiver (Rx) equipped with N antennas. This may represent an uplink of a cellular system where a single-antenna user communicates with a multi-antenna base station. The Rx signal can be expressed as

$$\mathbf{y} = (\mathbf{h}_0 + \sum_{l=1}^{L} e^{j\phi_l}\mathbf{h}_l)x + \mathbf{z} \tag{1}$$

where x is the scalar transmitted (Tx) signal satisfying the power constraint $\mathbb{E}[|x|^2] = P$, $\mathbf{y} \in \mathbb{C}^{N\times 1}$ is the received (Rx) signal, $\mathbf{z} \in \mathbb{C}^{N\times 1}$ denotes the AWGN noise vector; $\mathbf{h}_0, \mathbf{h}_l \in \mathbb{C}^{N\times 1}$ are the channel vectors representing the direct link and the reflected link via l-th IRS reflector, which introduces phase shift ϕ_l (the reflection loss is absorbed into $\mathbf{h}_l$). The noise is circularly symmetric complex Gaussian with zero mean and variance of σ_0^2 per Rx antenna. We further assume that the channel is static or quasi-static (i.e. stays fixed for a sufficiently long time) and that full channel state information is available.

For a standard SIMO channel without IRS assistance, the Rx SNR and thus the rate are maximized by matched filtering (or maximal-ratio combining) at the Rx [10], so that the maximum SNR γ_0 is

$$\gamma_0 = |\mathbf{h}_0|^2 P/\sigma_0^2. \tag{2}$$

Without loss of generality, we further assume that $|\mathbf{h}_0| = 1$. With IRS assistance, matched filtering at the Rx is still optimal (maximizes the SNR and rate) so that

$$\gamma(\boldsymbol{\phi}) = |\mathbf{h}_0 + \sum_{l=1}^{L} e^{j\phi_i}\mathbf{h}_i|^2\gamma_0 = g(\boldsymbol{\phi})\gamma_0 \tag{3}$$

where $\boldsymbol{\phi} = [\phi_1, \cdots, \phi_L]'$ is the vector of IRS phase shifts, and we emphasize that the SNR $\gamma(\boldsymbol{\phi})$ depends on $\boldsymbol{\phi}$ (to be optimized later on); $g(\boldsymbol{\phi}) = |\mathbf{h}_0 + \sum_{l=1}^{L} e^{j\phi_i}\mathbf{h}_i|^2$ is the IRS gain.

III. IRS-Assisted Channel Capacity

Unlike the standard SIMO channel, whereby the channel vector $\mathbf{h}_0$ is constant and the channel capacity involves maximization of mutual information over the input distribution only [10], the IRS-assisted channel provides extra degrees of freedom for maximizing mutual information, i.e. IRS-induced phase shifts ϕ_l. Since the equivalent IRS-assisted channel vector $\mathbf{h}_0 + \sum_{l=1}^{L} e^{j\phi_l}\mathbf{h}_l$ is independent of the input x, Gaussian input is still optimal [10] and the maximum rate (spectral efficiency) supported by the IRS-assisted SIMO channel for a given $\boldsymbol{\phi}$ is

$$R(\boldsymbol{\phi}) = \log(1 + \gamma(\boldsymbol{\phi})) \tag{4}$$

so that its capacity is

$$C_{IRS} = \max_{\boldsymbol{\phi}} R(\boldsymbol{\phi}) = \log(1 + \max_{\boldsymbol{\phi}} \gamma(\boldsymbol{\phi})) \tag{5}$$

We note that this problem is not convex and hence is difficult to solve, either analytically or numerically, and, in the general case, it remains open. Here, we address it by obtaining explicit closed-form solutions for some special cases, establish bounds and conditions for their achievability, propose an iterative algorithm for the general case and prove its (local) convergence.

A. IRS-assisted capacity via bounds

In this section, we establish some bounds on the IRS-assisted SNR and conditions for their achievability, which can be further used to solve (5) in some cases in closed-form.

Proposition 1. *The IRS-assisted SNR $\gamma(\boldsymbol{\phi})$ is upper bounded as follows:*

$$\gamma(\boldsymbol{\phi}) \le G\gamma_0,\ G = \left(1 + \sum_{l=1}^{L} |\mathbf{h}_l|\right)^2. \tag{6}$$

and the upper bound is attained if

$$\mathbf{h}_l = a_l\mathbf{h}_0,\ \phi_l = -\arg\{a_l\},\ l = 1..L. \tag{7}$$

for some complex a_l.

Proof. Using the triangle inequality,

$$\begin{aligned}\gamma(\boldsymbol{\phi}) &= |\mathbf{h}_0 + \sum_{l=1}^{L} e^{j\phi_l}\mathbf{h}_l|^2\gamma_0 \\ &\le \gamma_0\left(1 + \left|\sum_{l=1}^{L} e^{j\phi_l}\mathbf{h}_l\right|\right)^2 \\ &\le G\gamma_0\end{aligned} \tag{8}$$

It can be verified that the equality is attained under (7). □

Since the upper bound in (6) is independent of ϕ, a solution to (5) follows.

Corollary 1. *The maximum IRS-assisted SNR γ_{IRS} is bounded as follows*

$$\gamma_{IRS} = \max_{\boldsymbol{\phi}} \gamma(\boldsymbol{\phi}) \le G\gamma_0 \tag{9}$$

and the IRS-assisted capacity is bounded as

$$C_{IRS} \le \log(1 + G\gamma_0) \tag{10}$$

Both bounds are attained if $\mathbf{h}_l = a_l \mathbf{h}_0,\ l = 1..L$, so that $\phi_l^* = -\arg\{a_l\}$ solve the problem in (5).

Next, we consider a more general case, where $\mathbf{h}_l$ and $\mathbf{h}_0$ are not required to be parallel. To this end, let us define the channel matrix $\mathbf{H}$ and the phase shift vector $\mathbf{w}$ as follows:

$$\mathbf{H} = [\mathbf{h}_0, \cdots, \mathbf{h}_L],\ \mathbf{w} = [1, e^{j\phi_1}, \cdots, e^{j\phi_L}]' \tag{11}$$

Proposition 2. *In the general case, the IRS-assisted SNR can be bounded as follows:*

$$\sigma_{min}^2(\mathbf{H})(L+1)\gamma_0 \leq \gamma(\boldsymbol{\phi}) \leq \sigma_1^2(\mathbf{H})(L+1)\gamma_0 \tag{12}$$

where $\sigma_1(\mathbf{H})$ and $\sigma_{min}(\mathbf{H})$ are the largest and smallest singular values of $\mathbf{H}$. The upper bound is attained if $\mathbf{w} = \alpha_1 \mathbf{v}_1$ for some α, where $\mathbf{v}_1$ is the right singular vector of $\mathbf{H}$ corresponding to its largest singular value. The lower abound is attained if $\mathbf{w} = \alpha_2 \mathbf{v}_{min}$ for some α_2, where $\mathbf{v}_{min}$ is the right singular vector of $\mathbf{H}$ corresponding to its smallest singular value.

Proof. Using (11),

$$\gamma(\boldsymbol{\phi}) = |\mathbf{H}\mathbf{w}|^2\gamma_0 \leq \sigma_1^2(\mathbf{H})|\mathbf{w}|^2\gamma_0 = \sigma_1^2(\mathbf{H})(L+1)\gamma_0 \tag{13}$$

where the inequality is due to the singular value inequality $|\mathbf{H}\mathbf{w}| \leq \sigma_1(\mathbf{H})|\mathbf{w}|$ [11, pp. 267-268.] and the equality is from $|\mathbf{w}|^2 = L+1$. The achievability follows from $|\mathbf{H}\mathbf{v}_1| = \sigma_1(\mathbf{H})$. The lower bound is proved in the same way. □

Since the upper bound in (12) is independent of $\boldsymbol{\phi}$, it can be used to solve (5).

Corollary 2. *The maximum IRS-assisted SNR is bounded as follows*

$$\gamma_{IRS} \leq \sigma_1^2(\mathbf{H})(L+1)\gamma_0 \tag{14}$$

and the IRS-assisted capacity is bounded as

$$C_{IRS} \leq \log(1 + \sigma_1^2(\mathbf{H})(L+1)\gamma_0) \tag{15}$$

Both bounds are attained if $\mathbf{v}_1$ has equal-magnitude entries, i.e. $|v_{1l}| = 1/\sqrt{L+1}$, then $\phi_l^* = -\arg\{v_{1l}\}$ solve the problem in (5).

Since these bounds are global (i.e., apply to a globally-optimal IRS and any channel), they can be used as a benchmark to evaluate numerical algorithm's performance: if its convergence point is close to the upper bound, then it is also close to a global optimum, since the latter is sandwiched between the upper bound and a convergence point. This can also be used as algorithm's stopping criterion in some cases.

B. Single-reflector case

Next, we consider the case of $L = 1$, which is the least complex IRS implementation, and solve (5) in full generality for a global optimum, i.e. not imposing any conditions on $\mathbf{h}_0$ and $\mathbf{h}_1$. This analytical solution will be used later on for efficient iterative algorithm to solve (5) for general L and general $\mathbf{H}$. To this end, let

$$\begin{aligned} a_{kn} &= |h_{kn}|,\ \theta_{kn} = \arg\{h_{kn}\},\ k = 0,1,\ n = 1...N \\ c_n &= a_{0n}a_{1n},\ \Delta\theta_n = \theta_{0n} - \theta_{1n}, \\ C_I &= \sum_{n=1}^N c_n \cos(\Delta\theta_n),\ C_Q = \sum_{n=1}^N c_n \sin(\Delta\theta_n) \end{aligned} \tag{16}$$

so that a_{kn} and θ_{kn} represent magnitude and phase of each link, and $\Delta\theta_n$ are the phase differences between LOS and reflected links.

Proposition 3. *For a single-reflector IRS, the following ϕ^* is globally-optimal for (5):*

$$\phi^* = \arg\{C_I + jC_Q\} \tag{17}$$

so that the IRS-assisted capacity is $C_{IRS} = \log(1 + \gamma_{IRS})$, where

$$\gamma_{IRS} = \Big(1 + |\mathbf{h}_1|^2 + 2|C_I + jC_Q|\Big)\gamma_0 \tag{18}$$

Proof. The SNR $\gamma(\phi)$ can be bounded as follows:

$$\begin{aligned} \gamma(\phi) &= |\mathbf{h}_0 + e^{j\phi}\mathbf{h}_1|^2\gamma_0 \\ &= \Big(1 + |\mathbf{h}_1|^2 + 2\sum_{n=1}^N c_n \cos(\Delta\theta_n - \phi)\Big)\gamma_0 \\ &= \Big(1 + |\mathbf{h}_1|^2 + 2(C_I\cos(\phi) + C_Q\sin(\phi))\Big)\gamma_0 \\ &= \Big(1 + |\mathbf{h}_1|^2 + 2\Re\left\{(C_I + jC_Q)e^{-j\phi}\right\}\Big)\gamma_0 \\ &\leq \Big(1 + |\mathbf{h}_1|^2 + 2|C_I + jC_Q|\Big)\gamma_0. \end{aligned} \tag{19}$$

where the inequality is from $\Re\{z\} \leq |z|$ for any complex z and the equality is attained if $\Im\{z\} = 0$ and $\Re\{z\} \geq 0$, i.e. under (17). □

It follows from (17) and (16) that weak links, i.e. those with small c_n, contribute little to ϕ^* while strong links contribute most. When all links are equally strong (all c_n are equal), (17) simplifies to

$$\phi^* = \arg\left\{\sum_{n=1}^N e^{j\Delta\theta_n}\right\} \tag{20}$$

If all $\Delta\theta_n$ are the same, $\Delta\theta_n = \Delta\theta$, then $\phi^* = \Delta\theta$ so that LOS and reflected path are added constructively in each antenna, as it should be.

IV. ALTERNATING OPTIMIZATION ALGORITHM

While Proposition 3 provides a globally-optimal closed-form solution to the non-convex problem in (5), it applies to the single-reflector IRS only. However, it can be used as a building block to construct a semi-analytical iterative algorithm for any number of reflectors. The key idea is to optimize a single reflector phase at each iteration using the closed-form solution in (17) while keeping all other phases fixed. This can be done sequentially for all reflectors. In the optimization literature, this is known as alternating optimization (optimizing only single variable at a time). Let us illustrate this idea for $L = 2$.

- Step 1: optimize ϕ_1 using (17) and (16) with $\mathbf{h}_0 + e^{j\phi_2}\mathbf{h}_2$ in place of $\mathbf{h}_0$.
- Step 2: optimize ϕ_2 using (17) and (16) with $\mathbf{h}_0 + e^{j\phi_1}\mathbf{h}_1$ in place of $\mathbf{h}_0$ and $\mathbf{h}_2$ in place of $\mathbf{h}_1$.

These iterations can be repeated as many times as necessary, using some initial value of ϕ_2 at step 1 of very first iteration. Since this algorithm generates non-decreasing sequences of SNRs and since this sequence is bounded, it will converge, which is a welcome property.

In the general case (arbitrary L), ϕ_l is optimized at step l while all other phases are kept constant:

- Step l: optimize ϕ_l using (17) and (16) with $\mathbf{h}_0 + \sum_{k\neq l} e^{j\phi_k}\mathbf{h}_k$ in place of $\mathbf{h}_0$ and $\mathbf{h}_l$ in place of $\mathbf{h}_1$.

This is summarized in Algorithm 1. Phase optimization is performed alternatingly in the inner loop, one phase at a time, using the closed-form solution in (17) (which is globally-optimal for that particular step), as in Step l above. Since one-by-one optimization is not necessarily globally-optimal (even though each step is), multiple iterations are needed, which are performed by the outer loop. Step 3 of Algorithm 1 is needed to account for a rare (but possible) case $\mathbf{h}_0^{(l)} = 0$, for which ϕ_l is arbitrary so it is randomly generated. $\boldsymbol{\phi}_0$ is an initial phase vector (to be optimized), $\Delta\gamma$ is a convergence tolerance, i_0 is the number of outer iterations over which the increase in the SNR is evaluated in the termination criterion; $\gamma^{(i)} = 0$ for $i < 0$.

Since a closed-form solution is used in each iteration of the inner loop, no gradients or Hessians are necessary and no numerical optimization is used. Hence, Algorithm 1 is computationally-efficient. As we show below, the SNR sequence $\gamma^{(i)}$ generated by this Algorithm is non-decreasing and bounded and therefore converges, and so is the Algorithm.

Algorithm 1 Alternating optimization of $\gamma(\boldsymbol{\phi})$

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Required: H,  φ_0, Δγ > 0, i_0 ≥ 1
Initialization: i = 0, γ^(0) = γ(φ_0), φ = φ_0
repeat  (outer loop)
    1. Update i → i + 1.
    for l = 1 to L do (inner loop)
        2. Set h_0^(l) = h_0 + Σ_{k≠l} e^{jφ_k} h_k
        if h_0^(l) = 0
            3. Set φ_l ~ uni(0, 2π).
        else
            4. Compute C_I, C_Q using (16) with
                 h_0 → h_0^(l),  h_1 → h_l.
            5. Set φ_l = arg{C_I + jC_Q}
        end if
        6. Set γ^(i,l) = γ(φ)
    end for
    7. γ^(i) = γ^(i,L).
until γ^(i) − γ^(i−i_0) ≤ Δγ
Output: φ
```

Proposition 4. *Algorithm 1 generates a non-decreasing and bounded sequence of SNRs $\gamma^{(i)}$ and therefore converges. Its convergence point is a local maximum for the problem in* (5).

Proof. To see that Algorithm 1 generates non-decreasing sequence $\gamma^{(i)}$, let $\boldsymbol{\phi}^{(i,l)}$ be the phase vector after step l of the inner loop has been completed at step i of the outer loop, and observe the following:

$$\gamma^{(i)} = \gamma^{(i,L)} \tag{21}$$

$$\leq \max_{\phi_1} \gamma(\boldsymbol{\phi}^{(i,L)}) = \gamma(\boldsymbol{\phi}^{(i+1,1)}) = \gamma^{(i+1,1)} \tag{22}$$

$$\leq \max_{\phi_2} \gamma(\boldsymbol{\phi}^{(i+1,1)}) = \gamma^{(i+1,2)} \tag{23}$$

$$\leq \cdots \leq \max_{\phi_L} g(\boldsymbol{\phi}^{(i+1,L-1)})) = \gamma^{(i+1)} \tag{24}$$

where (21) is the SNR after step i of the outer loop has been completed; (22), (23) and (24) represent steps 1, 2 and L of the inner loop at step $i+1$ of the outer loop. Thus, $\gamma^{(i)} \leq \gamma^{(i+1)}$, i.e. $\gamma^{(i)}$ is a non-decreasing sequence. Intuitively, this is so because, at each step of the inner loop, the SNR cannot decrease since the respective phase is optimal, i.e. maximizes the SNR, at that step. This sequence is bounded, as has been established in Corollaries 1 and 2. Therefore, it converges and hence the termination criterion in Algorithm 1, i.e. $\gamma^{(i)} - \gamma^{(i-i_0)} \leq \Delta\gamma$, will be eventually satisfied, for any $\Delta\gamma > 0$ and any initial point $\boldsymbol{\phi}_0$. To see that this convergence point is a local maximum, observe that it cannot be a local minimum or an inflection point since the latter would mean that at least one ϕ_l is not (locally) optimal and hence it can be improved at step l of the inner loop (since Step 5 of Algorithm 1 sets optimal ϕ_l while all other phases are fixed). □

Since the problem in (5) is not convex, its local maximum obtained by Algorithm 1 is not necessarily a global one. However, numerical experiments show that in many cases Algorithm 1 does achieve an SNR close to the upper bounds established above and hence close to a global optimum. In fact, it can be shown that, under some conditions, Algorithm 1 does converge to a global optimum provided that its initial point $\boldsymbol{\phi}_0$ is properly selected.

V. Numerical Experiments

In this section, we illustrate the analytical results and performance of Algorithm 1 via numerical experiments.

For Algorithm 1, due to the non-convex nature of the problem in (5), it is important to select properly an initial point $\boldsymbol{\phi}_0$. While in principle any initial point can be used since the algorithm will converge anyway, its convergence point is a local maximum, not necessarily a global one, and it is affected by $\boldsymbol{\phi}_0$. Extensive simulations suggest that the following point is a good choice,

$$\phi_{0l} = -\frac{1}{N}\sum_n \Delta\theta_{n,l} \tag{25}$$

i.e. the opposite of the average phase difference between LOS and reflector l paths across all antennas, which is indicated by AOA-1 in the figures below. For comparison, we also show the results with all-zero initial point labeled as AOA-2. The IRS gain $g(\boldsymbol{\phi})$ as in (3) is used as a performance measure since it is independent of γ_0.

Fig. 2 shows the performance of Algorithm 1 along with the upper bound in (14) and all $\phi_l = \pi$ IRS (which corresponds to an ideal conductor with no phase adjustments), for $N = 2, L = 2$ scenario (for which no closed-form solution to (5) is known). To have representative results, 10^4 channel realizations were generated randomly (with $|h_{nl}| \sim$ uni$(0,1)$

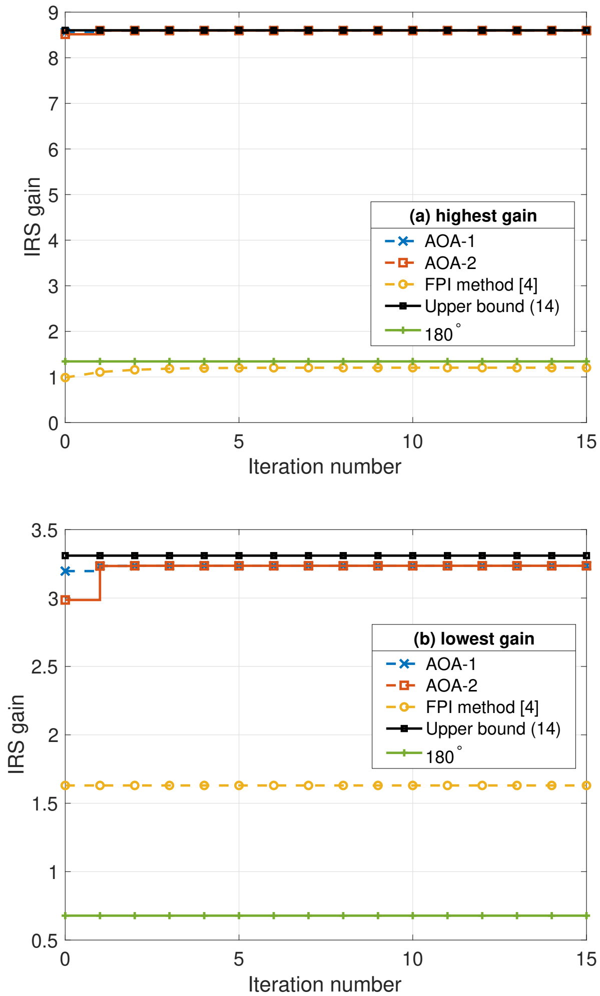

Fig. 2. Highest (a) and lowest (b) IRS gain scenarios among 10^4 randomly-generated channel realizations, with $N = 2, L = 2$.

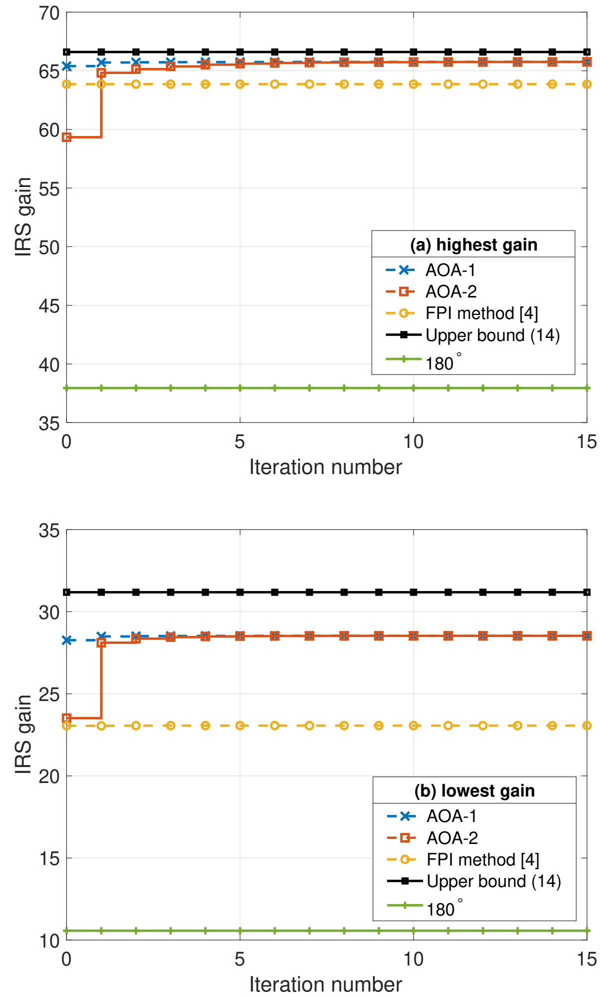

Fig. 3. The same setting as in Fig. 2 but with $N = 5, L = 10$.

and $\Delta\theta_{n,l} \sim \text{uni}(-\pi,\ \pi)$, all i.i.d.) and normalized to $|\mathbf{h}_l| = 1$ (to remove the impact of variable channel gains) and the IRS gain was computed for each one. Fig. 2(a) and 2(b) show the results for the channels with the highest and lowest IRS gains over the entire set. Clearly, Algorithm 1 convergence to the upper bound, thus achieving a global optimum, in case (a) and close to it in case (b), all within a small number of iterations (less than 5). As 2(b) shows, all-zero initial point is not the best one and the one in (25) performs better. The all $\phi_l = \pi$ IRS is far from optimum and much better performance can be achieved via proper phase optimization.

Fig. 3 shows the IRS gain with a larger number of antennas and reflectors, $N = 5, L = 10$. Clearly, this results in larger IRS gain and Algorithm 1 still performs well. Even though it takes more iterations to convergence compared to Fig. 2, this number is still modest (less than 10). Even though a convergence point is not necessarily a global optimum (since the problem is not convex), it is close to the upper bound and hence to the global optimum. Fig. 3(b) also demonstrates the superiority of the initial point in (25). Similar results can also be obtained for larger L.

A comparison to the fixed point iteration (FPI) method in [4] shows that the proposed algorithm achieves higher IRS gain for all tested channels ($2 \cdot 10^4$ in total). A more detailed comparison will be presented in a later publication.

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