On Asymptotic Outage Capacity Distribution of Correlated MIMO Channels

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***Abstract* – A general sufficient condition for the asymptotic normality of MIMO channel outage capacity is considered. Some physical aspects of this condition are discussed. Simple alternative conditions, which do not require eigenvalue decomposition, are proposed for the MIMO channels whose correlation matrix has a Toeplitz structure. It is demonstrated that some popular correlation matrix models satisfy this condition. In many cases, the convergence to the asymptotic normality is at least as $1/\sqrt{n_t}$, where n_t is the number of Tx antennas.**

***Index terms* –** MIMO channel, asymptotic capacity, correlation.

I. INTRODUCTION

Outage capacity is one of the major characteristics of fading channels as it gives the ultimate upper limit on the error-free information rate with a given probability of outage [1]. Even though the outage capacity distribution of some MIMO channels is known, its complexity does not allow for significant insight. To overcome this problem, the asymptotic analysis with respect to the number of antennas is used. This approach has already been successfully exploited in [2], [3], [4], [5] and [6], for Rayleigh, keyhole, multi-keyhole and general type of MIMO channels, and has been found to predict reasonably well the outage capacity of channels with a moderate number of antennas. In particular, Martin and Ottersten [3] have proven that when the number of antennas at the transmit end is large, the outage capacity distribution of a spatially correlated Rayleigh MIMO channel is asymptotically normal under some general conditions. In this paper we extend the results of [3] in the following directions:

- The generalized condition for asymptotic normality of the outage capacity distribution is discussed in detail, and some physical implications of this condition are highlighted.
- We define the convergence rate of the outage capacity distribution to the Gaussian one and show that in many cases this rate is bounded from below by $1/2$, i.e. the convergence is not slower than $1/\sqrt{n_t}$, where n_t is the number of transmit antennas.
- While the condition for asymptotic normality in [3] requires eigenvalue decomposition of an asymptotically large correlation matrix at the transmit end, we show that for the channels with Toeplitz correlation structure, this condition is always satisfied if the correlation decays faster than $1/\sqrt{D}$, where D is the distance between the antennas.
- Finally, it is shown that the presented theory applies to a number of popular correlation models.

While the theoretical conditions on asymptotic normality of the outage capacity distribution cannot be verified in practice, the presented study provides theoretical tools to evaluate the accuracy of the Gaussian approximation when the number of antennas is finite.

II. GENERALIZED CONVERGENCE CONDITION

Consider a spatially correlated Rayleigh-fading MIMO channel with n_t transmit (Tx) and n_r receive (Rx) antennas. Let $\mathbf{H}$ be an $n_r \times n_t$ matrix representing complex channel gains from Tx to Rx antennas. [[3], Theorem 1] gives a condition under which the outage capacity distribution of such a channel is asymptotically normal as $n_t \to \infty$, which follows from the Liapounoff Central Limit Theorem [7]. The generality of this condition can be further extended without increase in complexity. Specifically, from a more general formulation of Liapounoff Theorem[1], the generalized condition for [[3], Theorem 1] is that the outage capacity is asymptotically normal if for some $\delta > 0$,

$$\lim_{n_t \to \infty} Z_{n_t}(\delta) = \lim_{n_t \to \infty} \left\|\boldsymbol{\lambda}^t\right\|_{2+\delta} / \left\|\boldsymbol{\lambda}^t\right\|_2 = 0 \qquad (1)$$

where the norm $\left\|\boldsymbol{\lambda}^t\right\|_m = \left(\sum_{i=1}^{n_t} (\lambda_i^t)^m\right)^{1/m}$, and $\boldsymbol{\lambda}^t = \{\lambda_i^t,\ i = 1..n_t\}$ are the eigenvalues of the Tx correlation matrix $\mathbf{R}_t = n_r^{-1} E\{\mathbf{H}^H \mathbf{H}\}$, E denotes expectation, and $\mathbf{H}^H$ is the Hermitian transpose of $\mathbf{H}$. It is important to consider $Z_{n_t}(\delta)$ for a range of δ, rather than for one particular value (such as $\delta = 1$ in [3]) since the convergence rate to asymptotic normality is determined by the supremum over $\delta > 0$, as indicated below. Let $Z_{n_t}(\delta) \to 0$ for some $\delta > 0$. Due to Liapounoff's Inequality [[7], Theorem on p. 228]:

$$Z_{n_t}(\delta) \geq n_t^{-1/2+1/(2+\delta)} \qquad (2)$$

[1] Initially, in 1900, Liapounoff showed that a sum of independent random variables x_n is asymptotically normal if $E(|x_n|^3)$ exists, and $\sqrt[3]{m_3}/\sqrt[2]{m_2} \to 0$ as $n \to \infty$, where $m_k = \sum_{i=1}^{n} E(|x_n|^k)$. Shortly after, in 1901, he found that it is enough to request existence of only some absolute moments $E(|x_n|^{2+\delta})$, $\delta > 0$, and the sum is asymptotically normal if $\sqrt[2+\delta]{m_{2+\delta}}/\sqrt[2]{m_2} \to 0$ as $n \to \infty$ [[7], Ch. 8].

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Proposition 1: Define a convergence rate of $Z_{n_t}(\delta)$ to zero as $n_t \to \infty$ for given δ by

$$R_Z(\delta) = \lim_{n_t \to \infty} -\frac{\ln Z_{n_t}(\delta)}{\ln n_t} \le \frac{1}{2} - \frac{1}{2+\delta} \qquad (3)$$

where the inequality is due to (2). The best convergence rate is determined by the supremum of (3) taken over all $\delta > 0$ such that $Z_{n_t}(\delta) \to 0$. Note that since $\|\lambda^t\|_{2+\delta}$ decreases with δ [8], if $Z_{n_t}(\delta_0) \to 0$, then $Z_{n_t}(\delta) \to 0$ for all $\delta \ge \delta_0$. Therefore,

$$R_Z = \sup_{\delta>0} R_Z(\delta) \le \sup_{\delta>0}[1/2 - 1/(2+\delta)] = 1/2 \qquad (4)$$

Thus, the best possible rate is $R_Z = 1/2$, i.e. $Z_{n_t}(\delta) \to 0$ as $1/\sqrt{n_t}$ in this case. This best rate is achieved, for example, by Toeplitz correlation matrices (see Section IV). Note that using a specific fixed δ to find R_Z may lead to an incorrect result[2], i.e. the supremum in (4) is essential. It should also be pointed out that the generalized Liapounoff Theorem does not require δ to be a constant [9]; it can be a function of n_t, $\delta(n_t) > 0$, which further extends the generality of (1). [9] gives specific examples, which demonstrate greater generality of this formulation.

While the conditions [[3], eq. 10] and (1) are not easy to deal with, some cases when [[3], Theorem 1] does not apply can be characterized in a simple way, which provides simple necessary conditions for that Theorem.

Corollary 1 to [[3], Theorem 1]: Let $\lambda_1^t \ge \lambda_2^t \ge ... \ge \lambda_{n_t}^t$ be the eigenvalues of $\mathbf{R}_t$ sorted in decreasing order. (1) does not hold true, so that [[3], Theorem 1] cannot be applied if there is a finite set of eigenvalues which are not dominated by the rest, i.e. there exists k such that

$$c = \lim_{n_t \to \infty} \frac{\sum_{i=k+1}^{n_t} \left(\lambda_i^t\right)^2}{\sum_{i=1}^{k} \left(\lambda_i^t\right)^2} < \infty, \qquad (5)$$

which physically means that the multipath is not rich enough as $n_t \to \infty$. A proof is straightforward and omitted due to the page limit

From (5), a necessary condition for [[3], Theorem 1] is that $c = \infty$ .While condition (5) is less general than (1), it allows for an insight and is simple to evaluate since it involves only the second-order moments of λ^t. Consider two broad cases where Corollary 1 applies: (i) $\mathbf{R}_t$ has a finite number (k) of non-zero eigenvalues as $n_t \to \infty$, which corresponds to a limited number of multipath in the propagation channel. Then $\sum_{i=k+1}^{n_t} \left(\lambda_i^t\right)^2 = 0$ and consequently $c = 0$. Thus, a necessary physical condition for [[3], Theorem 1] to hold is that the number of multipath components goes to infinity with n_t. (ii) The largest eigenvalue is not dominated by all the other eigenvalues, i.e.

$$c = \lim_{n_t \to \infty} \sum_{i=2}^{n_t} \left(\lambda_i^t\right)^2 \Big/ \left(\lambda_1^t\right)^2 < \infty, \qquad (6)$$

which holds true, for example, when $\lambda_2^t \sqrt{n_t} \big/ \lambda_1^t < \infty$ as $n_t \to \infty$. Thus, a necessary condition for [[3], Theorem 1] is that $\lim_{n_t \to \infty} \lambda_1^t \Big/ \left(\lambda_2^t \sqrt{n_t}\right) = 0$. Consider, as an example, the uniform correlation matrix [10], when all the non-diagonal entries of $\mathbf{R}_t$ are equal to ρ_t. The eigenvalues in this case can be found explicitly in a closed form: $\lambda_1^t = 1 + (n_t - 1)\cdot\rho_t$, $\lambda_2^t = ... = \lambda_{n_t}^t = 1 - \rho_t$, where ρ_t is the parameter indicating the correlation between two adjacent antenna elements. Thus, for $k = 1$ and $\rho_t \ne 0$, $c = 0$, i.e. λ_1^t is not dominated by all the other eigenvalues. Moreover, it is straightforward to show that in this case $\lim_{n_t \to \infty} Z_{n_t}(\delta) = 1$, i.e. [[3], Theorem 1] does not apply.

III. ACCURACY OF GAUSSIAN APPROXIMATION

For finite n_t, the Gaussian distribution serves as an approximation of the true outage capacity distribution. The accuracy of the approximation can be estimated from the following results.

Theorem 1: Let $\Delta_{n_t}(x) = \left|F_{n_t}(x) - \Phi(x)\right|$, where $F_{n_t}(x)$ is the channel outage capacity CDF given n_t, and $\Phi(x)$ is the corresponding Gaussian CDF with the same mean and variance as of the true outage capacity. Then,

$$\Delta_{n_t} = \sup_x \Delta_{n_t}(x) \le c \cdot n_r^{1/4} Z_{n_t}(\delta)^{2+\delta}, \ 0 < \delta \le 1 \qquad (7)$$

where $c \le 4$ is an absolute constant. A proof is based on [[11], Theorem 1.1.][3] and omitted due to the page limit.

In analogy with (3), the rate of convergence $\Delta_{n_t} \to 0$ is defined as

$$R_\Delta = \lim_{n_t \to \infty} -\frac{\ln \Delta_{n_t}}{\ln n_t} \ge (2+\delta) R_Z(\delta), \ 0 < \delta \le 1 \qquad (8)$$

where the inequality is due to (7). From (3), the best convergence rate of $Z_{n_t}(\delta) \to 0$ for $0 < \delta \le 1$ is $R_Z = 1/6$. In this best case,

$$R_\Delta \ge 1/2 \qquad (9)$$

It should be noted that: (i) Even though the lower bound in (9) is achieved at $\delta = 1$, it does not necessarily mean that the upper bound in (7) gives the best estimate of Δ_{n_t} for $\delta = 1$ [9]. (ii) In some cases, the upper bound in (7) significantly overestimates Δ_{n_t}, so that the convergence is better than expected from the bound [9].

IV. CONVERGENCE CONDITION FOR TOEPLITZ MATRICES

While the conditions in [[3], eq. 10] and (1) are important theoretical tools, their usefulness for practical computations is rather limited due to two reasons: (i) The eigenvalues are

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[2] For example, the results in [3] correspond to $\delta = 1$, which implies $R_Z \le 1/6$.

[3] [[11], Theorem 1.1.] is stated for $\delta = 1$, but it can also be extended to $0 < \delta \le 1$ [Private Communications].

known in a closed form only for some simple matrices. Consequently, the above mentioned conditions can be evaluated analytically only in such cases. (ii) Numerical evaluation of these conditions is also difficult, since the numerical complexity (number of operations, inaccuracy, etc.) of the eigenvalue problem increases rapidly with n_t, so that its solution for $n_t \to \infty$ is problematic if possible at all. The following theorem gives a condition that is easier to evaluate.

Theorem 2: Let $\mathbf{R}_t$ be a Toeplitz correlation matrix with elements $\left[\mathbf{R}_t\right]_{k,m} = t_{k-m}$, such that

$$0 < M_t = \lim_{n_t \to \infty} \sum_{k=-n_t+1}^{n_t-1} |t_k|^2 < \infty, \qquad (10)$$

i.e. $\mathbf{R}_t$ is non-degenerate and square-summable[4]. Then for any $\delta > 0$, the following holds:

$$\lim_{n_t \to \infty} Z_{n_t}(\delta) = \left(I_{2+\delta}\right)^{1/(2+\delta)} \left(I_2\right)^{-1/2} \cdot \lim_{n_t \to \infty} n_t^{-\frac{\delta}{2(2+\delta)}} = 0 \quad (11)$$

where

$$I_p = (2\pi)^{-1} \int_0^{2\pi} f^p(x)dx < \infty, \text{ for } \forall p > 0 \qquad (12)$$

and a non-negative real function $f(x) = \sum_{k=-\infty}^{\infty} t_k \cdot e^{jkx}$ is the spectrum of $\mathbf{R}_t$ [12]. A proof is based on Szego Theorem [12] and omitted due to the page limit.

Not only does Theorem 2 give a practical way to evaluate the condition (1) for Toeplitz correlation matrices[5] without using eigenvalue decomposition, it also shows that under condition (10), the channel eigenvalues and so the outage capacity are always asymptotically normal.[6] Moreover, since $Z_{n_t}(\delta) \to 0$ for all $0 < \delta \le 1$, by using (11), (7) and (8), one obtains that $R_\Delta \ge 1/2$ (the best case), i.e. under the conditions of Theorem 2, the convergence is at least as $1/\sqrt{n_t}$. This result is general for a wide class of Toeplitz correlation matrices which satisfy (10), regardless of any other details. As a numerical example, Fig. 1 shows the upper bound in (7) and $\Delta_{n_t}(x_0)$ vs. n_t, where x_0 is the outage capacity such that the corresponding outage probability $\Phi(x_0) = 0.01$. $Z(\delta)$ is calculated at $\delta = 1$ for $\mathbf{R}_t$ given by the exponential correlation model [13] (see also (14)) with correlation parameter $\rho_t = 0.5$. $\Delta_{n_t}(x_0)$ is obtained by Monte-Carlo simulation using 10^5 trials. As predicted, the upper bound (solid line) decreases as $1/\sqrt{n_t}$ ($1/\sqrt{n_t}$ is also shown for comparison). $\Delta_{n_t}(x_0)$ lies well below the upper bound, and decreases with n_t at least as $1/\sqrt{n_t}$.

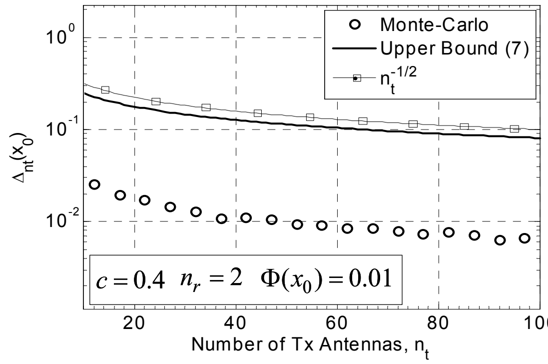

Fig. 1. Distance between the outage capacity distribution of Rayleigh fading channel and Gaussian approximation.

When $\mathbf{R}_t$ is not Toeplitz, Theorem 2 does not apply. However, it is straightforward to show for correlation matrices with an arbitrary structure, that $Z_{n_t}(1)$ is bounded by the norm of $\mathbf{R}_t$ as follows:

$$\left(n_t^{-1} \|\mathbf{R}_t\|\right)^{1/3} \le Z_{n_t}(1) \le 1 \qquad (13)$$

where $\|\mathbf{R}_t\| = \left(\sum_{k,m=1}^{n_t} |[\mathbf{R}_t]_{k,m}|^2\right)^{1/2}$ is Frobenius norm whose calculation does not require eigenvalue decomposition. Note that $n_t^{-1}\|\mathbf{R}_t\|$ is a measure of correlation and power imbalance of a MIMO channel at Tx end introduced in [5]. This measure affects the outage capacity distribution and was motivated by asymptotic analysis of the latter. Thus, a necessary condition for $Z_{n_t}(1)$ to converge to zero and thereby for [[3], Theorem 1] to hold is that the measure of correlation $n_t^{-1}\|\mathbf{R}_t\| \to 0$ as $n_t \to \infty$ [7]. Moreover, from [5], $n_t^{-1/2} \le n_t^{-1}\|\mathbf{R}_t\| \le 1$, where the lower bound corresponds to the case when $\lambda_i^t = 1$, $\forall i$, i.e. there is no correlation at the Tx end, and the upper bound is achieved when there is a single non-zero eigenvalue $\lambda_1^t = n_t$ and $\lambda_i^t = 0$ for $\forall i \ne 1$, i.e. the Tx end is fully correlated. Thus, the overall tendency for $Z_{n_t}(1)$ is to increase with correlation, which results in slower convergence for higher correlated channels (see also [5]).

To validate the general discussion above, consider a number of popular correlation models for $\mathbf{R}_t$.

Exponential Correlation Model: In this model the elements of $\mathbf{R}_t$ are represented through a single complex correlation parameter ρ_t, which indicates the correlation between adjacent antennas [13]:

$$[\mathbf{R}_t]_{k,m} = \begin{cases} \rho_t^{m-k}; & m \ge k \\ \bar{\rho}_t^{k-m}; & m < k \end{cases}, \quad |\rho_t| < 1, \qquad (14)$$

[4] If $\mathbf{R}_t$ is non-degenerate and absolutely summable, it also satisfies (10), since $\sum_{k=-n_t+1}^{n_t-1} |t_k|^2 \le \left(\sum_{k=-n_t+1}^{n_t-1} |t_k|\right)^2$.

[5] Toeplitz correlation matrix physically corresponds to a uniform antenna array geometry, when correlation depends on the spacing between elements only, but not on their positions.

[6] Note that the uniform correlation matrix [10] does not satisfy (10), unless $\rho_t = 0$.

[7] In this sense, the Tx antennas have to be "asymptotically uncorrelated".

where $\overline{\rho}_t$ is the complex conjugate of ρ_t. While the eigenvalues of $\mathbf{R}_t$ are given by a transcendental equation [13], which does not allow easy evaluation of (1), it is straightforward to show using d'Alambert Ratio Test that in this case $M_t < \infty$ for $|\rho_t| < 1$, so that following Theorem 2, condition (1) is indeed satisfied, i.e. the outage capacity of the corresponding channel is asymptotically normal. Moreover, using (11), for $|\rho_t| < 1$

$$\lim_{n_t \to \infty} Z_{n_t}(1) = \frac{\left(1+4|\rho_t|^2+|\rho_t|^4\right)^{1/3}}{\left(1+|\rho_t|^2\right)^{1/2} \cdot \left(1-|\rho_t|^2\right)^{1/6}} \cdot \lim_{n_t \to \infty} n_t^{-1/6} = 0 \quad (15)$$

From the definition of a limit, for any $\varepsilon > 0$, there is n_0 such that for all $n_t > n_0$ $Z_{n_t}(1) \le \varepsilon$. (15) shows that n_0 is an increasing function of $|\rho_t|$, i.e. larger correlation results in slower convergence. This supports the theory presented above, and explains the corresponding observation in [3], which was based on numerical results.

Tri-Diagonal Model: When the correlation is significant only among adjacent antennas, the elements of $\mathbf{R}_t$ are given by the tri-diagonal correlation matrix [13],

$$[\mathbf{R}_t]_{k,m} = \begin{cases} 1, \; k=m \\ \rho_t, \; k=m-1 \\ \overline{\rho}_t, \; k=m+1 \\ 0, \; otherwise \end{cases}, \quad |\rho_t| < \frac{1}{2}\left(\cos\frac{\pi}{n_t+1}\right)^{-1} \quad (16)$$

It is straightforward to show that $M_t < \infty$, i.e. from Theorem 2, $Z_{n_t}(1) \to 0$ as $n_t \to \infty$. Thus, [[3], Theorem 1] applies in this case. Moreover,

$$\lim_{n_t \to \infty} Z_{n_t}(1) = \frac{\left(1+6|\rho_t|^2\right)^{1/3}}{\left(1+2|\rho_t|^2\right)^{1/2}} \cdot \lim_{n_t \to \infty} n_t^{-1/6} = 0, \quad (17)$$

i.e. similarly to (15), n_0 is an increasing function of $|\rho_t|$, i.e. higher correlation results in slower convergence.

Squared-Exponential Correlation Model: This correlation structure has been proposed for the IEEE 802.11n Wireless LANs standard [14] and describes physical propagation channels where the angular spread PDF is truncated Gaussian [15]. The elements of $\mathbf{R}_t$ are given by

$$\left[\mathbf{R}_t\right]_{k,m} = \begin{cases} \rho_t^{(m-k)^2}, \; k \le m \\ \overline{\rho}_t^{(k-m)^2}, \; k > m \end{cases}, \quad |\rho_t| \le 1 \quad (18)$$

While the confirmation of (1) is difficult in this case, it is straightforward to show that $M_t < \infty$, $|\rho_t| < 1$, and thereby from Theorem 2 and [[3], Theorem 1], the channel outage capacity is asymptotically normal.

It can be shown, however, that M_t is unbounded for the correlation models which correspond to the uniform or truncated Laplacian angular distributions of the multipath. Hence, Theorem 2 does not apply in these cases. Clearly, whether M_t is finite or not is determined by asymptotic behavior of $\mathbf{R}_t$'s tails ($k, m \to \infty$). However, the match between the popular correlation models and real correlation structures for $k, m \to \infty$ has not been thoroughly studied, if studied at all, since (i) in practice n_t is always finite, so that the asymptotic behavior of $\mathbf{R}_t$'s tails had little or no importance, and (ii) measuring these tails is difficult from the technical point of view. Thus, the issue of convergence for practical correlation structures seems to remain an open problem. The usefulness of Theorem 2, however, is somewhat more general than just with respect to some particular correlation models. It states that asymptotic normality of outage capacity distribution is a common property of all Rayleigh MIMO channels with Toeplitz correlation structure, where the correlation between antennas decays faster than $1/\sqrt{D}$, D is the distance between the antennas.

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