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A new expression for the ground transient resistance matrix elements of multiconductor overhead transmission lines

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Abstract

The ground impedance matrix elements of a multiconductor overhead transmission line do not have analytical inverse Fourier transforms in the time domain. Thus, in general, the ground transient resistance matrix elements are to be determined numerically. Using the low-frequency approximation analytical expressions are available for the ground transient resistance. These expressions present however a singularity at $t=0$ and require a careful treatment in a direct time domain analysis. In this paper, we show that the singularity in the ground transient resistance is due to the low frequency approximation. Also, we show that at early times, the ground transient resistance elements tend to an asymptotic value, which depend on the line geometrical parameters and ground relative permittivity. Finally, we propose new expressions which are analytical, non-singular, and which describe, within the limits of transmission line theory, both the early-time and late-time behavior of ground transient resistance in a more accurate way.
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1. Introduction

Recent studies dealing with transients in overhead transmission lines due to indirect lightning effects have shown the importance of taking into account the finitely conducting ground in the transmission line equations (e.g. [1–3]). This results in an additional longitudinal impedance term in the transmission line equations which is called the ground impedance matrix [4]. The general expression for the elements of the ground impedance matrix are not suitable for a numerical evaluation since they involve integrals over an infinitely long interval. However, reasonably simple expressions have been proposed in the literature which have been shown to give very accurate results [3] in the frequency range typical of transient surges propagating on transmission lines.

A direct time domain resolution of transmission line equations is sometimes preferred to a frequency domain analysis because of its straightforwardness and its capability in handling nonlinear phenomena such as corona effect and/or presence of protective components such as surge arresters.

On the other hand, one major difficulty of a direct time-domain analysis of transmission line equations is related to the presence of frequency-dependent parameters such as ground impedance, which appear in the equations through a convolution integral. Considering the low frequency approximation, analytical expressions have been derived for the ground transient resistance matrix elements in the time domain [5,6]. However, these expressions present a singularity at $t=0$ and require a careful treatment in the resolution algorithm [3,7].

In this paper, we will show that the presence of this singularity is due to the low frequency approximation, and that such a singularity is not present in the general expression of the ground transient resistance. Also, we will propose a new analytical and non-singular expression that describes, within the limits of transmission line theory, both the early-time and late-time behavior of ground transient resistance in a more accurate way.

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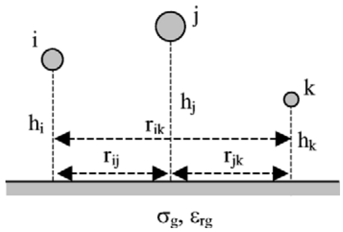

Fig. 1. Definition of the geometry.

The geometry we will refer to is presented in Fig. 1. We consider a uniform overhead multiconductor transmission line above a finitely conducting ground characterized by its conductivity σ_g and its relative permittivity ε_{rg}.

2. Ground impedance expression

Several expressions for the ground impedance have been proposed (e.g. [8–20]). Here, we will use those proposed by Sunde [10] essentially for two reasons:

1) It can be shown that the general more rigorous expressions derived using scattering theory reduce to the Sunde approximation when considering the transmission line approximation [4].
2) The ground impedance calculated using the Sunde expressions are shown to be accurate within the limits of transmission line approximation [4].

The general expression for mutual ground impedance between two conductors i and j derived by Sunde is given by [10]

$$Z'_{g_{ij}} = \frac{j\omega\mu_0}{\pi}\int_0^{\infty} \frac{e^{-(h_i+h_j)x}}{\sqrt{x^2+\gamma_g^2}+x}\cos(r_{ij}x)\mathrm{d}x \tag{1}$$

where h_i, h_j and r_{ij} are geometrical parameters as defined in Fig. 1, and γ_g is the wave propagation constant defined as

$$\gamma_g = \sqrt{j\omega\mu_0(\sigma_g + j\omega\varepsilon_0\varepsilon_{rg})}$$

Adopting a low frequency approximation, the general expression Eq. (1) reduces to the well-known Carson's expression

$$Z'_{g_{ij}}\Big|_{\text{Carson}} \cong \frac{j\omega\mu_0}{\pi}\int_0^{\infty} \frac{e^{-(h_i+h_j)x}}{\sqrt{x^2+j\omega\mu_0\sigma_g}+x}\cos(r_{ij}x)\mathrm{d}x \tag{2}$$

when $\sigma_g \gg \omega\varepsilon_0\varepsilon_{rg}$.

It has been shown in [3] that the validity of the above Carson's approximation extends to frequencies of about a few MHz, for typical overhead power lines and for ground conductivities of about 0.01 s/m. For faster electromagnetic sources, or poorer ground conductivity, the general expression Eq. (1) or their accurate logarithmic approximations (see [3] for a review) should be used to obtain more accurate results.

It is worth mentioning that the presence of a finitely conducting ground results in another additional transverse term in the transmission line equations: the so-called ground admittance $Y'_{g_{ij}}$ (e.g. [4]). However, it can be shown that the contribution of this term for typical overhead power lines remains negligible.

In Fig. 2, we have presented a comparison between $|Z'_g|$ and $\omega L'$ on the one hand, and between $1/|Y'_g|$ and $1/\omega C'$ on the other hand, with L' and C' being the per-unit-length inductance and capacitance of the line, respectively, for a 10-m high wire above ground. It can be seen that while the ground impedance is a non-negligible fraction of $\omega L'$, $1/|Y'_g|$ is about 5 orders of magnitude lower than $1/\omega C'$.

3. Ground transient resistance analytical expressions

The time-domain ground transient resistance matrix $[\xi'_{g_{ij}}]$ elements are defined as (see e.g. [7])

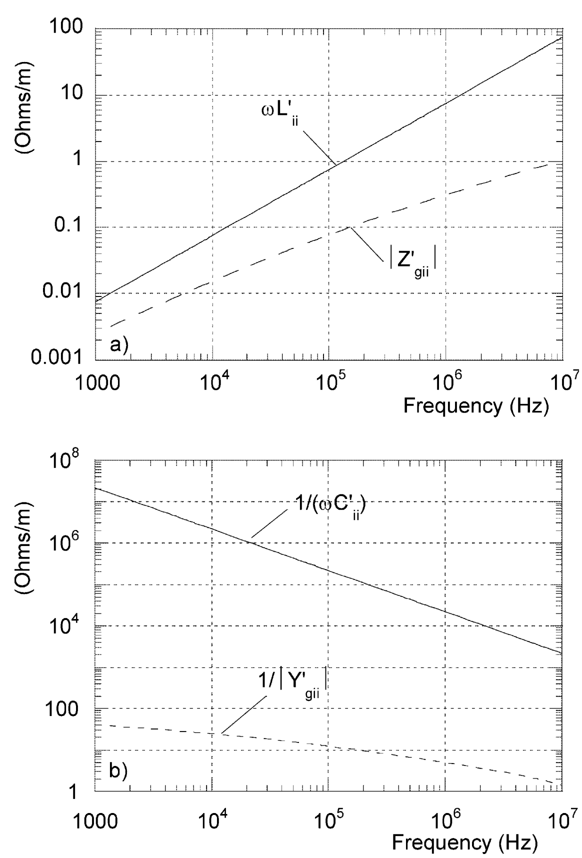

Fig. 2. (a) Comparison between the ground and impedance and $\omega L'_{ii}$. (b) Comparison between the inverse of the ground admittance and $1/\omega C'_{ii}$, for a 10-m high wire above the ground ($\sigma_g = 0.01$ s/m, $\varepsilon_{rg} = 10$).

$$\xi'_{g_{ij}} = F^{-1}\left\{\frac{Z'_{g_{ij}}}{j\omega}\right\} \tag{3}$$

To the best of our knowledge, analytical expressions for the inverse Fourier transform are not available for the ground impedance matrix terms. Thus, the elements of the ground transient resistance matrix in time domain have to be, in general, determined using a numerical inverse Fourier transform algorithm.

In the low-frequency approximation ($\sigma_g \gg \omega\varepsilon_0\varepsilon_{rg}$), however, it is possible to find an analytical inverse Fourier transform for the ground impedance. This was first derived by Timotin [5] for the case of a single-conductor line (diagonal terms of the ground transient resistance matrix). It reads

$$\xi'_{g_{ii}}(t) \cong \frac{\mu_0}{\pi\tau_{g_{ii}}} \times \left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_{g_{ii}}}{t}} + \frac{1}{4}\exp(\tau_{g_{ii}}/t)\mathrm{erfc}\left(\sqrt{\frac{\tau_{g_{ii}}}{t}}\right) - \frac{1}{4}\right] \tag{4}$$

in which $\tau_{g_{ij}} = h_i^2\mu_0\sigma_g$ and erfc is the complementary error function defined as

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-t^2)\mathrm{d}t = 1 - \frac{2\exp(-x^2)}{\sqrt{\pi}}\sum_{n=0}^{\infty} a_n x^{2n+1} \tag{5}$$

where

$$a_n = \frac{2^n}{1\cdot 3\cdots(2n+1)}$$

The Timotin expression has been later extended to the case of a multiconductor line [6]. For convenience, we report hereunder the expression for the general term $\xi'_{g_{ij}}$ of the ground resistance matrix

$$\xi'_{g_{ij}}(t) \cong \frac{\mu_0}{\pi T_{ij}}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{T_{ij}}{t}}\cos(\theta_{ij}/2) + \frac{1}{4}\mathrm{e}^{T_{ij}\cos(\theta_{ij})/t}\cos\left(\frac{T_{ij}}{t}\sin(\theta_{ij}) - \theta_{ij}\right) - \frac{1}{2\sqrt{\pi}}\sum_{n=0}^{\infty} a_n\left(\frac{T_{ij}}{t}\right)^{\frac{2n+1}{2}}\cos\left(\frac{2n-1}{2}\theta_{ij}\right) - \frac{\cos(\theta_{ij})}{4}\right] \tag{6}$$

in which T_{ij} and θ_{ij} are defined as follows

$$\hat{\tau}_{g_{ij}} = \hat{h}_{ij}^2\mu_0\sigma_g = \left(\frac{h_i + h_j}{2} + j\frac{r_{ij}}{2}\right)^2\mu_0\sigma_g = T_{ij}\mathrm{e}^{j\theta_{ij}} \tag{7}$$

4. On the singularity in the ground transient resistance analytical expressions

The expressions Eq. (4) and Eq. (6) are singular for $t = 0$. Indeed, it can be shown that

$$\lim_{t\to 0}\xi'_{g_{ii}} \approx \frac{\mu_0}{2\pi}\frac{1}{\sqrt{\pi\tau_{g_{ii}}t}} \tag{8}$$

$$\lim_{t\to 0}\xi'_{g_{ij}} \approx \frac{\mu_0}{2\pi}\frac{1}{\sqrt{\pi T_{ij} t}}\cos\left(\frac{\theta_{ij}}{2}\right) \tag{9}$$

This singularity is due to the low-frequency approximation used in deriving Eq. (4) and Eq. (6) and it is not present otherwise [21]. Indeed, considering the diagonal terms of the general expression Eq. (1) it can be shown [2] that

$$\lim_{\omega\to\infty} Z'_{g_{ii}} = \frac{1}{2\pi h_i}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}} \tag{10}$$

and applying the initial value theorem, we get

$$\xi'_{g_{ii}}(t=0) = \lim_{\omega\to\infty} j\omega\frac{Z'_{g_{ii}}}{j\omega} = \frac{1}{2\pi h_i}\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_{rg}}} \tag{11}$$

In a similar way, considering the mutual terms of Eq. (1), it can be easily demonstrated that

$$\lim_{\omega\to\infty} Z'_{g_{ij}} = \frac{1}{2\pi\hat{h}}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}} \tag{12}$$

in which

$$\hat{h} = \frac{h_i + h_j}{2} + \frac{r_{ij}^2}{2(h_i + h_j)} \tag{13}$$

and therefore

$$\xi'_{g_{ij}}(t=0) = \lim_{\omega\to\infty} j\omega\frac{Z'_{g_{ij}}}{j\omega} = \frac{1}{2\pi\hat{h}}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}} \tag{14}$$

Eq. (11) and Eq. (14) show that the ground transient resistance tends to an asymptotic value when $t \to 0$. It is interesting to note that this asymptotic value is expressed in terms of the line height and the ground relative permittivity and is independent of the ground conductivity.

Fig. 3 illustrates the singular behavior of the Timotin formula in the early-time region. As it can be seen from this figure, the low frequency analytical expressions Eq. (4) and Eq. (6) are not able to reproduce accurately the early-time behavior of ground transient resistance. In fact, according to a fundamental property of the Fourier transform, the early-time behavior of a time-domain function requires the knowledge of the function spectrum up to the very high-frequency region.

It is worth noting that indeed any low-frequency approximation, and not only the one considered here,

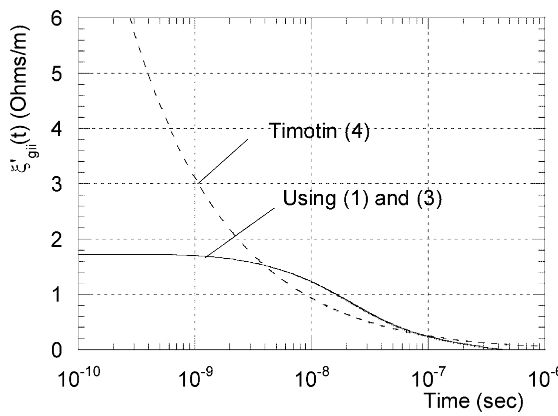

Fig. 3. Comparison between the Timotin analytical formula Eq. (4) and the inverse Fourier transform of $Z'_{g_{ii}}/j\omega$ in the early-time region (10-m high, Single-conductor line above a conducting ground with $\sigma_g = 0.01$ s/m and $\varepsilon_{rg} = 10$).

will give incorrect results for the early-time region [21,22]. Consider the inverse Fourier transform $y(t)$ of a function $Y(\omega)$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}d\omega \tag{15}$$

If $Y(\omega)$ is a constant, then $y(t) = 0$ for all $t \neq 0$ (due to the oscillating factor $e^{j\omega t}$ in the integrand in Eq. (15)). If $Y(\omega)$ is not a constant but it is varied slowly enough in comparison to $e^{j\omega t}$, then the infinite integration limits can be reduced to finite ones since the integration subinterval for which $|\omega|t \gg 1$ does not give a substantial contribution to the total integral due to the fast-oscillating factor $e^{j\omega t}$. The subinterval in which $|\omega|t \gg 1$, gives a substantial contribution since the factor $e^{j\omega t}$ does not oscillate in this subinterval. Taking into account the above considerations, Eq. (15) can be reduced to

$$y(t) \approx \frac{1}{2\pi}\int_{-\omega_{max}}^{\omega_{max}} Y(\omega)e^{j\omega t}d\omega \tag{16}$$

where

$$\omega_{max} \approx \frac{1}{t} \tag{17}$$

Thus, in order to calculate $y(t_0)$, the spectrum of $y(t)$ must be known up to $\omega_{max} \gg 1/t_0$. Similarly, if we know the function $Y(\omega)$ up to ω_{max}, then we can calculate $y(t)$ for $t \geqslant 1/\omega_{max}$. If $y(t)$ must be calculated for smaller values of t, $Y(\omega)$ have to be known for larger values of ω.

Let us now turn our attention to $\xi'_{g_{ij}}(t)$. Since in the low-frequency approximation $Z'_{g_{ij}}(\omega)$ is known up to frequencies for which $\omega \ll \sigma_g/(\varepsilon_0\varepsilon_{rg})$, $\xi'_{g_{ij}}(t)$ can be correctly estimated using Eq. (4) and Eq. (6) only for

$$t > t_{min} = \frac{\varepsilon_0\varepsilon_{rg}}{\sigma_g} \tag{18}$$

For times earlier than t_{min}, the simplified equations Eq. (4) and Eq. (6) give rise to significant inaccuracies. It is however important to remind that the above conclusion applies within the limits of the transmission line approximation[1]. Approximate closed form expression for the ground transient resistance in the early-time region has been proposed in [23].

5. New expressions for the ground transient resistance matrix elements

We have seen that the singularities in the analytical expressions Eq. (4) and Eq. (6) are due to the low-frequency approximation. The ground transient resistance determined by numerical inverse transform tends to an asymptotic value at early times, as described by analytical equations Eq. (11) and Eq. (14). This observation leads us to propose new analytical and non-singular expressions for the ground transient resistance given by

$$\xi'_{g_{ii}}(t) = \min\left\{\frac{1}{2\pi h_i}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_{rg}}},\ \frac{\mu_0}{\pi\tau_{g_{ii}}}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_{g_{ii}}}{t}} + \frac{1}{4}\exp(\tau_{g_{ii}}/t)\mathrm{erfc}\left(\sqrt{\frac{\tau_{g_{ii}}}{t}}\right) - \frac{1}{4}\right]\right\} \tag{19}$$

$$\xi'_{g_{ij}}(t) = \min\left\{\frac{1}{2\pi\hat{h}}\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_{rg}}},\ \frac{\mu_o}{\pi T_{ij}}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{T_{ij}}{t}}\cos(\theta_{ij}/2) + \frac{1}{4}e^{T_{ij}\cos(\theta_{ij})/t}\cos\left(\frac{T_{ij}}{t}\sin(\theta_{ij}) - \theta_{ij}\right) - \frac{1}{2\sqrt{\pi}}\sum_{n=0}^{\infty} a_n\left(\frac{T_{ij}}{t}\right)^{\frac{2n+1}{2}}\cos\left(\frac{2n-1}{2}\theta_{ij}\right) - \frac{\cos(\theta_{ij})}{4}\right]\right\} \tag{20}$$

Note that the above equations combine the late-time response described by the low-frequency expressions Eq. (4) and Eq. (6), and the early-time response given by the asymptotic value of the ground transient resistance (Eqs. (11) and (14)).

Fig. 4 illustrates the proposed expressions which are not singular and also extend, within the limits of

[1] The transmission line theory itself is a low frequency approach, and therefore, it cannot predict accurately the very early-time response of the line, for which radiation from the line should be taken into account. This is the case when the basic TL assumption—transverse dimensions lower than the minimum significant wavelength—is not satisfied. In other words, the transmission line theory applies when $\omega \ll 2\pi c/h$, where h is the line height and c is the speed of light. This means that the transmission line theory can only predict the temporal response of the line for times $t \gg t_{minTL} = h/(2\pi c)$.

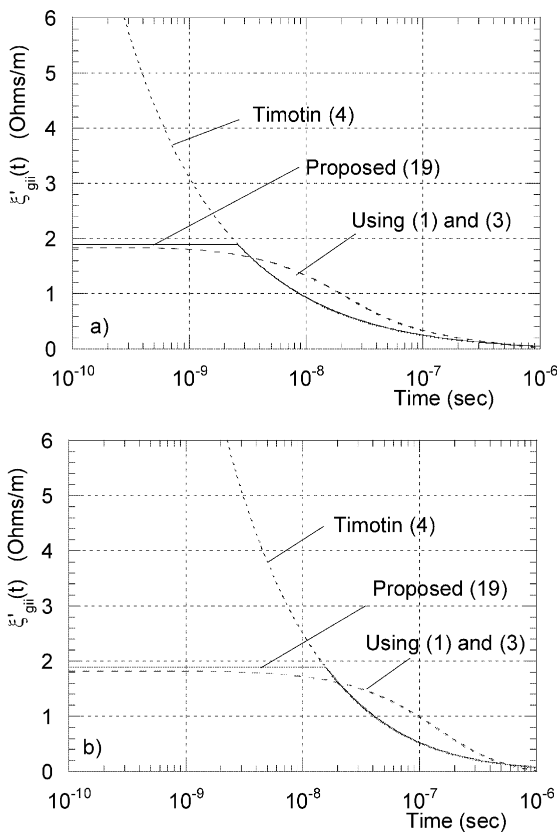

Fig. 4. Comparison between the proposed expression (19), the Timotin analytical formula (4) and the inverse Fourier transform of $Z'_{g_{ii}}/j\omega$ in the early-time region. 10-m high, single-conductor line above a conducting ground (a) $\sigma_g = 0.01$ s/m and $\varepsilon_{rg} = 10$; (b) $\sigma_g = 0.001$ s/m and $\varepsilon_{rg} = 10$.

transmission line theory, the validity of the original analytical expressions to the early-time region.

In order to illustrate the adequateness of the proposed expression, consider an overhead wire of radius $a = 5$ mm located at a height $h = 10$ m above ground. The ground conductivity and relative permittivity are respectively equal to 0.001 s/m and 10. Consider a current propagating waveform consisting of a linear front with a risetime t_r followed by a constant amplitude of 1 A. Consider now the per-unit-length voltage drop due to the ground transient resistance given by

$$v'_g(t) = \int_0^t \xi'_{g_{ii}}(t-\tau)i(\tau)\mathrm{d}\tau \qquad (21)$$

We have computed the above expression for different values of the current risetime ($t_r = 100$ ns, 500 ns, 1 ms), and using for the transient ground resistance (a) the general expression Eq. (3) obtained using an inverse FFT algorithm of the Sunde expression Eq. (1), (b) the low frequency formula by Timotin Eq. (4), and (c) the proposed expression Eq. (19). The results are shown in Fig. 5.

As expected, the late-time response of the two analytical expressions (Timotin's and the proposed

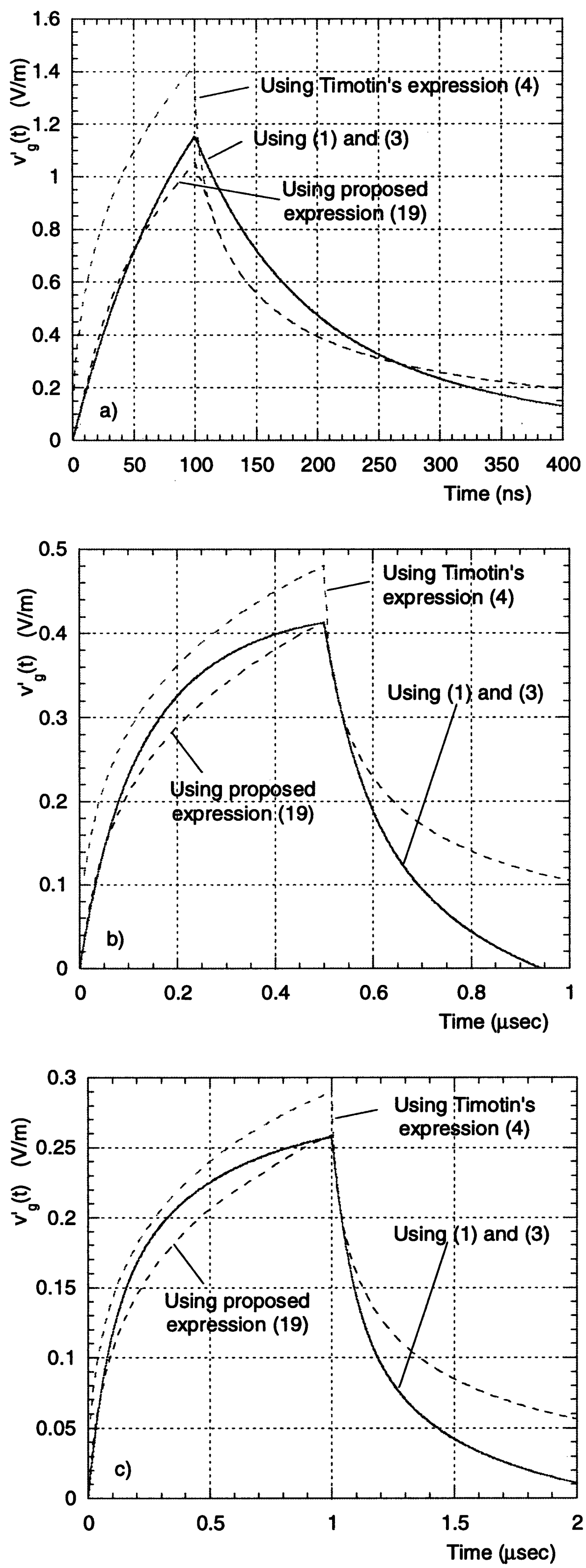

Fig. 5. Per-unit-length voltage drop due to the ground transient resistance computed using the general expression Eq. (3), the formula by Timotin, and the proposed expression. 10-m high conductor above ground ($\sigma_g = 0.001$ s/m and $\varepsilon_{rg} = 10$). (a) 100 ns risetime; (b) 0.5 μs risetime, (c) 1 μs risetime.

formula) are almost identical. However, in the early-time region, the response of the line is better reproduced with the new proposed expression, especially for fast rising signals. Note that for the considered case (10-m high line), the transmission line approximation is still applicable for a pulse with 100 ns risetime. For faster risetimes, however, the minimum wavelength would become comparable to the height of the line and the TL approximation looses its validity. For this reason, we do not present results relevant to risetimes shorter than 100 ns.

6. Summary and conclusion

The ground impedance matrix elements of a multi-conductor overhead transmission line do not have analytical inverse Fourier transforms in the time domain. Thus, in general, the ground transient resistance matrix elements have to be determined numerically.

Using the low-frequency approximation, analytical expressions have been proposed for the ground transient resistance. These expressions present however a singularity at $t = 0$ which requires to be carefully treated in a direct time domain resolution algorithm. We have shown that the singularity in the ground transient resistance is indeed due to the low frequency approximation, and that, at early times, the ground transient resistance elements tend to an asymptotic value, which depend on the line geometrical parameters and the ground relative permittivity.

New expressions for the ground transient resistance are proposed which are analytical, non-singular, and which describe, within the limits of transmission line theory, both the early-time and late-time behavior of ground transient resistance in a more accurate way.

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