# ON ACCURACY OF NUMERICAL EMC/EMI MODELING OVER A WIDE FREQUENCY RANGE

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**Abstract -** Numerical EMC/EMI modeling over a wide frequency range requires for an appropriate sampling scheme. Limited computer resources give rise to the strong limitation on the number of samples, especially during analysis of a complex system. A simple method for estimation of the sampling accuracy and the required number of samples is given. A technique for improving of the modeling computational efficiency is proposed.

## I. INTRODUCTION

Almost all the EMC problems are wide frequency range ones in their nature. Numerical modeling by computers requires for an appropriate sampling scheme. A typical decomposition of an EMC problem is shown in Fig.1 [1]. In accordance with this Figure, emitter spectra,

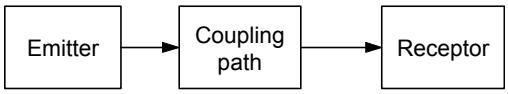


Fig.1. Typical decomposition of an EMC problem

receptor susceptibilities and coupling path responses must be sampled at the computer-aided modeling process. The number of samples is limited because of limited computer resources. The limitation of the samples number manifests itself in the case of narrow-band spectrum/susceptibility analysis over a wide frequency range and, especially, when the system under analysis is a complex one and a lot of objects (or emitter-receptor pairs) must be analyzed. This limitation may result in errors due to the sampling process. In general, the exponential sampling scheme has many advantages for wide-range modeling [2,5] because the relative frequency accuracy $Df_i/f_i$ , where $f_i$ is i-th sample frequency, $Df_i$ is i-th sample interval width ( $Df_i = (f_{i+1} - f_{i-1})/2$ ), is constant in the whole frequency range. The number of samples must be properly determined in order to avoid large sampling error. There is obviously the optimum number of samples since (1) samples number increase causes computational time growth as well as the sampling error reduction, (2) samples number increase exceeding the certain quantity does not result in noticeable sampling error decrease. When the integrated interference margin is used as a figure-of-merit (we consider just this case in the present paper), there is an analogy between the error of numerical integration and the sampling error.

The purpose of this paper is an estimation of the spectrum and susceptibility sampling accuracy in the case of complex systems EMC analysis in the frequency domain, when there is a large number of emitter-receptor pairs and, consequently, the number of samples is strictly limited because of limited computer resources, especially of PCs. We shall not consider adaptive sampling schemes [6] since they require much computational time being unacceptable in this case. We shall also present a method for increasing analysis efficiency.

## II. THE SAMPLING SCHEMES

Let us study the sampling error on the basis of the sampling schemes given in [2]. The broad-band spectrum sampling scheme is illustrated in Fig.2 (it's assumed that an envelope spectrum model is used as a sampling starting point). For each sampling interval $Df_{ei}$ with the central sample frequency $f_{ei}$ , the maximum value of the spectrum is found and assigned

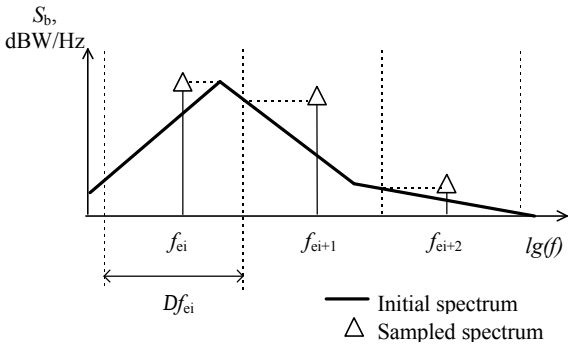


Fig.2 The broad-band spectrum sampling scheme. $S_b$ is broad-band spectrum (envelope model), $f_{ei}$ and $Df_{ei}$ are emitter sample frequencies and sampling intervals respectively.

as a sampled spectrum value for frequency $f_{ei}$ . Fig.2 shows that this sampling scheme leads to the spectrum overestimation but all peak values are taken into account

during the sampling process. The total power of the spectrum,

$$P_{total} = \int_{f_{\min}}^{f_{\max}} S_b(f)df \ , \qquad (1)$$

where $f_{min}$ and $f_{max}$ are the minimum and the maximum frequencies in the spectrum, will be overestimated too. For the sampled spectrum, integral in (1) is reduced to the sum

$$P_{total} = \sum_{i=1}^{N_e} S_{b,i}\Delta f_{ei} \ , \qquad (2)$$

where $S_{b,i}$ are the sampled spectrum values (note that $S_{b,i} \neq S_b(f_{ei})$ ), $N_e$ is the number of emitter sample frequencies. We should note that the estimations of the numerical integration error [7] can not be directly applied to the present case since we use the worst case analysis technique. Besides, the use of spectrum/susceptibility specific features allows one to obtain more accurate estimations.

If the number of samples is properly determined, the overestimation will be small enough. The susceptibility sampling scheme is similar to the spectrum one. But the minimum level of susceptibility is found at the sampling interval rather than the maximum one (see Fig. 3).

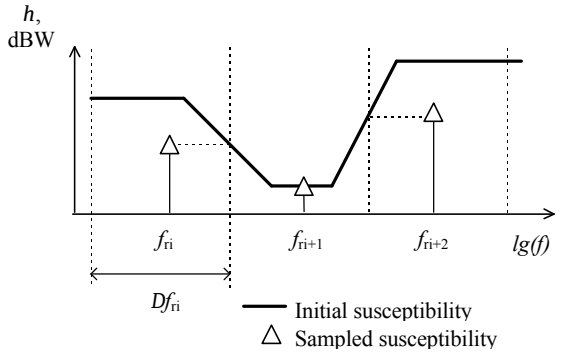


Fig.3 The susceptibility sampling scheme. $h$ is receptor's susceptibility, $f_{ri}$ and $Df_{ri}$ are receptor sample frequencies and sampling intervals respectively.

The integrated EMI (electromagnetic interference) margin is calculated for each receptor as a criterion of distortion [2-4]

$$IM = \int_{f_{\min}}^{f_{\max}} \frac{S_r(f)}{h(f)}df \ , \qquad (3)$$

where $IM$ is the integrated EMI margin, $S_r(f)$ is a non-required broad-band spectrum (interference) at the receptor input, $h(f)$ is the receptor susceptibility. If $IM>0$ $dB$, then there is interference, if $IM<0$ $dB$, then there is no interference. Since IM is the final figure-of-merit for the analysis under consideration, we shall estimate all errors in terms of IM. For sampled spectrum and susceptibility the last equation is reduced to the sum,

$$IM = \sum_{i=1}^{N_r} \frac{S_{r,i}}{h_i}\Delta f_{ri} \ , \qquad (4)$$

where $N_r$ is the number of receptor sample frequencies, $Df_{ri}$ are receptor sampling intervals, $h_i$ is the receptor sampled susceptibility. Note that $h_i = h(f_{ri})$ and that the emitter sample frequencies and the receptor ones need not be coincident ($f_{ei} = f_{ri}$) so one must transform the emitter sampled spectrum from the emitter sample frequencies to the receptor ones.

The sampled spectrum value $S_{r,i}$ at the receptor input on receptor sample frequency $f_{ri}$ is determined as follows (see Fig. 4) [2,8]. First of all, the emitter sample frequencies within the receptor sampling interval $Df_{ri}$ are identified ($f_{en}$ - $f_{en+2}$ in our case). Then the corresponding sampled spectrum values are calculated and the spectrum value at the frequency $f_{ri}$ is calculated by the log-linear interpolation of sampled spectrum values on the two successive emitter sample frequencies which enclose $f_{ri}$ ($f_{en}$ and $f_{en+1}$ in our case). Then $S_{r,i}$ is chosen to be the maximum of all these values.

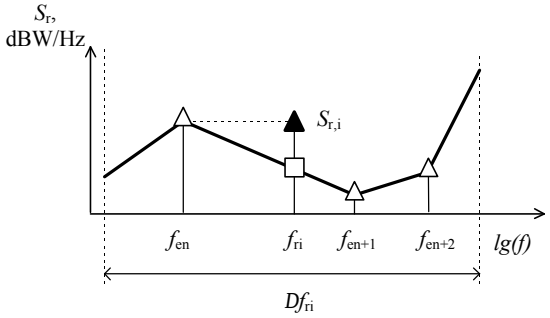


Fig.4 The sampled spectrum transformation from the emitter sample frequencies to the receptor ones.

For the exponential sampling scheme, the sample frequencies are given by

$$f_i = q^i \cdot f_{\min} \ , \quad q = \sqrt[N]{\frac{f_{\max}}{f_{\min}}} \ , \quad i = \overline{0,N} \ , \qquad (5)$$

where $N$ is the number of frequency samples. The parameter $q$ can be determined by the following approximate formula (for $N >> 1$)

$$q \approx 1 + \frac{1}{N} \ln\left(\frac{f_{\max}}{f_{\min}}\right) \quad . \tag{6}$$

The width of the frequency interval which corresponds to the frequency $f_i$ is

$$\Delta f_i \approx \frac{f_i}{N} \ln\left(\frac{f_{\max}}{f_{\min}}\right) \quad . \tag{7}$$

This equation shows that for exponentially-spaced frequencies a large increase in the analysis range width $[f_{min}, f_{max}]$ doesn't produce essential increase of the sample interval width $Df_i$ (or the number of samples for constant $Df_i$ ) because of a logarithm in (7).

## III. ESTIMATION OF THE SAMPLING ERROR

By a sampling error is meant an error due to the sampling process only, i.e. an error due to the substitution of continuous functions and equations by their discrete analogues (for instance, an integral by the sum).

Equations (1)-(7) can be used for an estimation of the sampling accuracy in the following manner [8]. First of all we should note that there are four components of the sampling error:

- an increase in the power of a broadband spectrum due to the sampling;
- an expansion of a receptor (susceptibility) bandwidth;
- an increase in the power of a broadband spectrum due to the transformation from emitter sample frequencies to receptor ones;
- a change in relative position of spectra and susceptibilities on the frequency axis.

The spectrum/susceptibility truncation at the frequencies $f_{min}$ and $f_{max}$ also gives a contribution to the error, but we assume that these frequencies are appropriately chosen so that this contribution is negligible.

The power of a broadband spectrum concentrated within the bandwidth $Df_0$ increases proportionally to the expansion of the frequency interval $Df_{ei}$ (starting from some specific value of $Df_{ei}$ ) due to the sampling process. This increase can be estimated by the following formula

$$P^* \approx P \frac{\Delta f_{ei}}{\Delta f_0}, \quad \Delta f_{ei} > \Delta f_0 \quad , \tag{8}$$

where $P» S_{max}Df_0$ is the power of the broadband spectrum, $P^*» S_{max}Df_{ei}$ is the power of the sampled broadband spectrum, $Df_{ei}$ is the width of the frequency interval the broadband spectrum belongs to (see Fig.5). For example, if the broadband signal of 1 W power is concentrated in the bandwidth 1 kHz (the spectral power density is $10^{-3}$ W/Hz) and the frequency interval width is 1 MHz, then the spectrum after the sampling process will obtain the total power 1 kW instead of 1 W.

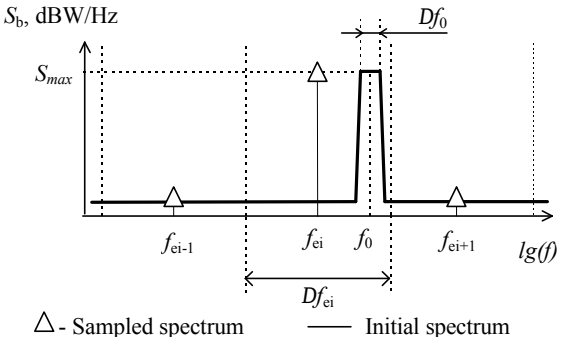


Fig.5 Increase in the power of a broadband spectrum due to the sampling process.

The expansion of $Df_{ri}$ also expands the receptor bandwidth raising the signal level at the receptor output. This level can be estimated by the following formula analogous to (8) (provided the broadband spectrum variations being inessential within the interval $Df_{ri}$),

$$\frac{P_{out}^*}{P_{out}} \approx \frac{\Delta f_{ri}}{\Delta f_0}, \quad \Delta f_{ri} > \Delta f_0 \quad , \tag{9}$$

where $P_{out}$ is the broadband signal power at the output of the receptor having the bandwidth $Df_0$ , $P_{out}^*$ is the power of the same signal at the output of the receptor with sampled susceptibility. Thus if the receptor bandwidth is 10 kHz and the frequency interval is 1MHz then the sampling process will cause 100 times increase of the signal power at the receptor output.

The error due to variations in spectra and susceptibilities relative position depends on their type and in the general case cannot be estimated by such a simple method. It is worth noting that the relative change in position of spectra and susceptibilities cannot exceed $Df_i$.

The sampled spectrum transformation from emitter frequencies to receptor ones gives a contribution to the sampling error. In general, it is not useful to determine the sampling error for an emitter and a receptor separately, it's necessary to consider the "emitter-receptor" pair. If the sampling error is small enough for the emitter and for the receptor, the error in IM due to the sampling does not need to be small. This is the case when the relative interval widths $Df_i/f_i$ are quite different for the emitter and the receptor. Let's illustrate

this statement by an example shown on Fig. 6 (we don't take into account the coupling path response).

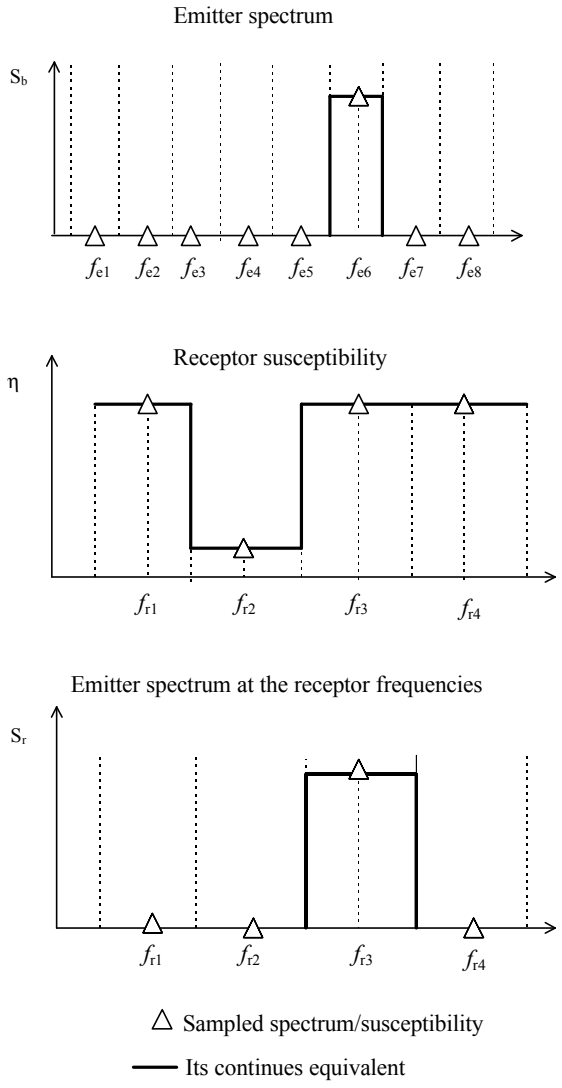


Fig.6. Error in IM due to the spectrum transformation from emitter frequencies to receptor ones.

The sampled spectrum and susceptibility as well as their continuous equivalents from the viewpoint of IM calculated by Eq.(3) and (4) are shown on this figure. As it can be seen from the figure, the spectrum transformation from the emitter frequencies to the receptor ones causes the spectrum bandwidth to expand (as much as two times in our case) and, correspondingly, IM to increase (by two times) in comparison to the rigorous equation (3). The transformation resembles the second sampling of the emitter spectrum in this respect. Accordingly, an increase in spectrum power (and in IM) can be estimated by an expression similar to (8)

$$P^* \approx P\frac{\Delta f_r}{\Delta f_e}, \quad \Delta f_r > \Delta f_e \quad . \tag{10}$$

There are, of course, such spectrum forms which don't give rise to a spectrum expansion even if the receptor sampling interval is much wider than the emitter one (for instance, when the emitter spectrum is nearly constant within the receptor sampling interval), but we now consider the worst case.

The last equation shows that there is no any spectrum expansion when the width of the emitter sampling interval and the receptor one are nearly equal: $Df_r \gg Df_e$ Using (7), we obtain the following "matching condition":

$$\frac{1}{N_e}\ln\left(\frac{f_{e,\max}}{f_{e,\min}}\right) \approx \frac{1}{N_r}\ln\left(\frac{f_{r,\max}}{f_{r,\min}}\right) \quad , \tag{11}$$

where $f_{e,min}$ , $f_{e,max}$ , $f_{r,min}$ and $f_{r,max}$ are minimum and maximum frequencies for the emitter and the receptor correspondingly. This condition quarantines that there will be no any spectrum expansion during the transformation.

Turn now our attention to the determination of the required number of samples (it means such a number for which the sampling error is small).

The required number of samples for an emitter (for sampling the emitter spectrum) can be estimated on the basis of the considerations given above and Eq. (7)

$$N_e \approx k\frac{f_0}{\Delta f_0}\ln\left(\frac{f_{e,\max}}{f_{e,\min}}\right) , \tag{12}$$

where $Df_0$ is the spectrum bandwidth (see Fig. 5), $f_0$ is the central frequency for this bandwidth, $k$ is a safety factor (in practice, $k \gg 1 \div 10$). In fact, $k$ is the number of samples per bandwidth $Df_0$ . For a receptor, the required number of samples (for sampling the receptor susceptibility) can also be estimated by this formula (in this case, $Df_0$ is the receptor bandwidth, $f_0$ is the central frequency for this bandwidth, and the receptor minimum and maximum frequencies are used). The emitter spectrum transformation from emitter frequencies to receptor ones requires for a small modification of (12). The ratio $f_0/Df_0$ must be calculated for the emitter coupling to the receptor as well as for the receptor itself and the maximum value of $f_0/Df_0$ is used in (12) in accordance with the "matching condition" (11). This modification prevents an overestimation of the broadband signal power at the receptor output from an emitter having bandwidth smaller than that of the receptor.

## IV. IMPROVING THE MODELING COMPUTATIONAL EFFICIENCY

For a complex system, the total number of samples (for all emitters and receptors) can be very large. In this case, the computation time is enormous too, and some methods of making the computer modeling more efficient are necessary.

The feature of the sampling scheme discussed above is the following: if the number of samples is decreased (either for a receptor or for an emitter coupling to the receptor), then the integrated EMI margin at the receptor output increases. So, for a small number of samples, IM value is always overestimated due to the sampling process. This feature allows one to carry out the analysis by the two-step method:

(1) In the first step, the analysis is carried out with a small number of samples ( $k \approx 0.1 \div 1$ ). All emitter-receptor pairs with no interference ($IM<0$ *dB*) is excluded from farther consideration since the sampling only gives rise to an IM overestimation.

(2) In the second step, the emitter-receptor pairs which possessed interference (IM>0 dB) in the first step are analyzed more accurately (with a large number of samples, $k \approx 1 \div 10$ ).

This method allows one to exclude all non-interference pairs in the first step and to analyze more accurately all "suspicious" pairs in the second step. Since the number of non-interference pairs is, as a rule, much greater than that of interference ones, the use of the method proposed will result in a large decrease of the computation time.

## V. CONCLUSION

The technique presented in this paper can be used for an order-of-magnitude estimation of the sampling error and for the estimation of the required number of samples. The two-step analysis method enables one to improve the computational efficiency. Farther improvement in the efficiency can be achieved by use of optimum integration techniques [9] since IM which is the analysis final result is expressed in terms of integral (3).

For an optimum analysis method, the errors on each sampling interval should be approximately the same [9]. In general, the more optimum method (shorter computation time for constant accuracy, higher accuracy for constant computation time etc.) we want to obtain, the more information on spectra, susceptibilities and coupling paths should be used, i.e. the more specific should be a problem statement.

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