Convex Optimization Tools for Non-Convex Problems

- Apply locally if the problem is locally convex and if an optimum exists, i.e. give local solutions [1][2].
- Inspection of all local solutions reveals a global one.

4-step Method of Brinkhuis and Tikhomirov [2]:

1. Establish an existence of a global solution, e.g. if the objective is a continuous function and the constraint set is closed and bounded (thus compact), the existence of a solution follows from Weierstrass theorem.
2. Find necessary conditions: KKT conditions are necessary for optimality in many cases, see e.g. [3][1] for details, so that a global minimum is a solution of KKT conditions.
3. Find all solutions of KKT conditions.
4. By inspection, find the global minimum.

[1] D. G. Luenberger, Linear and Nonlinear Programming, Springer, 2005.
[2] J. Brinkhuis, V. Tikhomirov, Optimization: Insights and Applications, Princeton, 2005.
[3] D.P. Bertsekas, Nonlinear Programming, Athena Scientific, 2nd Ed., 2008.

KKT conditions are necessary if all functions are continuously differentiable and one of the following holds:

1. Linear equalities, convex inequalities + Slater (see Proposition 3.3.9 in [3]).
2. Linear equalities, concave inequalities (Proposition 3.3.7 in [3]; no Slater is required here).
3. Gradients of all equality and active inequality constraints are linearly independent at a local minimum (no linearity, no convexity/concavity is required here, see Proposition 3.3.1 in [3]).