ELG6108/CSI5108
Introduction to Convex Optimization

Instructor: Dr. Sergey Loyka (CBY A608)

Lectures: Mon. 16:00 – 18:50 (CRX C410)

Office hours: Wed. 5:30-6pm.

Course web page: http://www.site.uottawa.ca/~sloyka/

Pre-(Co) requisites: solid knowledge of calculus and linear algebra. Matrix theory is a plus. Basic probability theory.

Marking scheme:

Assignments	10%
Course project + presentation	40%
Final exam	50%

Lots of bonus points to everybody who takes active part in the course.

Final exam: 3h, open book.

Pre-requisites:

- Calculus
- Linear algebra/matrices
- Basic probability

Calendar Description: Mathematics of optimization: linear, nonlinear and convex problems. Convex and affine sets. Convex, quasiconvex and log-convex functions. Operations preserving convexity. Recognizing and formulating convex optimization problems. The Lagrange function, optimality conditions, duality, geometric and saddle-point interpretations. Least-norm, regularized and robust approximations. Statistical estimation, detector design. Adaptive antennas. Geometric problems (networks). Algorithms.

Week-by-week Description (approximate):

1. Introduction: Mathematics of optimization. Least-squares and linear programming. Convex and nonlinear optimization.

I Theory

2. Convex sets: Affine and convex sets. Some important examples. Operations that preserve convexity. Generalized inequalities. Separating and supporting hyperplanes. Dual cones and generalized inequalities.

3. Convex functions: Basic properties and examples. Operations that preserve convexity. The conjugate function. Quasiconvex functions. Log-concave and log-convex functions. Convexity with respect to generalized inequalities.
4. Convex optimization problems: Recognizing and formulating convex optimization problems. Linear and quadratic optimization. Geometric programming. Generalized inequality constraints. Vector optimization.
5. Duality: The Lagrange dual function and problem. Geometric and saddle-point interpretations. Optimality conditions. Sub-optimal solutions via duality. Perturbation and sensitivity analysis. Examples. Theorems of alternatives. Generalized inequalities.
6. Application of convex optimization techniques to non-convex problems.

II Applications

6. Approximation and fitting: Norm approximation. Least-norm problems. Regularized approximation. Robust approximation. Function fitting and interpolation.
7. Statistical estimation: Parametric and nonparametric distribution estimation. Optimal detector design and hypothesis testing. Chebyshev and Chernoff bounds. Experiment design.
8. Geometric problems: Projection on a set. Distance between sets. Euclidean distance and angle problems. Extremal volume ellipsoids. Centering. Classification. Placement and location. Floor planning.

9. Adaptive antennas (beamforming): minimum variance, MMSE and max. SNR beamformers. Adaptive equalizers. Diagonal loading and robustness. MIMO systems: optimal precoding and decoding, water-filling algorithm.

III Algorithms

10. Unconstrained minimization (gradient and steepest decent methods). Equality constrained minimization. Interior-point methods (the barrier method).

References: Main Book

1. S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004. - *this is an exceptionally well-written book. It is strongly recommended that you study it carefully to learn not only how to recognize, formulate and solve convex optimization problems, but also how to write well in technical English. Solving end-of-chapter problems is essential for deep understanding of the material.*

Additional Books (available in the library, some in pdf)

2. D. G. Luenberger, Linear and Nonlinear Programming, Springer, 2005
3. J. Brinkhuis, V. Tikhomirov, Optimization: Insights and Applications, Princeton, 2005.
4. R. Fletcher, Practical Methods of Optimization, Wiley, 2000.
5. K. Lange, Optimization, Springer, 2013.
6. C.L. Byrne, A First Course in Optimization, available at http://faculty.uml.edu/cbyrne/opttext.pdf
7. M. Aoki, Introduction to Optimization Techniques. Macmillan, 1971.

Some more references (deep but mathematically demanding)

8. D.P. Bertsekas, Nonlinear Programming, Athena Scientific, 2nd Ed., 2008.
9. A. Ben-Tal, A. Nemirovski, Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications, SIAM, 2001.

10. D. G. Luenberger, Optimization by Vector Space Methods, Wiley-Interscience; 1997.
11. D.P. Bertsekas, A. Nedic, A,E. Ozdaglar, Convex Analysis and Optimization, Athena Scientific, 2003.
12. A.G. Suharev, A.B. Timoxov, B.B. Fedorov, A Course of Optimization Methods, FML, Moscow, 2005 (in Russian).
13. B.M. Alekseev, E.M. Galeev, B.M. Tihomirov, Problems in Optimization Theory, FML, Moscow, 2005 (in Russian).
14. B.T. Polayk, Introduction to Optimization, Nauka, Moscow, 1983 (in Russian).

Matrices/linear algebra

15. A.J. Laub, Matrix Analysis for Scientists and Engineers, SIAM, 2005. - this is a good introductory book with discussion of basic techniques and results in linear algebra and matrix theory.
16. F. Zhang, Matrix Theory, Springer, 1999. – this is a more comprehensive textbook of matrix theory, with many end-of-chapter problems.
17. R.A. Horn, C.R. Johnson, Matrix Analysis, Cambridge University Press. – this and the 2nd volume (next) is a comprehensive book, which treats in detail all important methods and results in matrix theory; it is very well written and end-of-chapter problems are well-selected. Strongly recommended, especially if you use matrix theory in your research.
18. R.A. Horn, C.R. Johnson, Topics in Matrix Analysis, Cambridge University Press, 1991. – see the previous item for comments.

Useful links:

- http://www.stanford.edu/~boyd/cvxbook/
- http://www.stanford.edu/class/ee364a/courseinfo.html
  - there are many useful things at this page, including the videos of lectures by Prof. Boyd.
- http://ocw.mit.edu/OcwWeb/Electrical-Engineering-and-Computer-Science/6-253Spring2004/CourseHome/index.htm

See the course web page for more.

Rationale:

Most problems in engineering can be formulated as optimization problems. Thus, solid knowledge of optimization techniques is essential for both engineering design and research. In the past two decades, a significant amount of new analytical results and techniques have been accumulated in the area of convex optimization. Keeping in mind that convex problems are numerous in many areas of engineering (and many that are not, can be re-formulated or approximated as convex ones), knowledge of these results and techniques can open new horizons in research for our graduate students and faculty. A quotation from the suggested textbook is in order: "*We think that convex optimization is an important enough topic that everyone who uses computational mathematics should know at least a little bit about it. In our opinion, convex optimization is a natural next topic after advanced linear algebra … and linear programming. …Our main goal is to help the reader develop a working knowledge of convex optimization, i.e. to develop the skills and background needed to recognize, formulate and solve convex optimization problems. Developing a working knowledge of convex optimization can be mathematically demanding, especially for the reader interested primarily in applications. In our experience (mostly with graduate students in electrical engineering and computer science), the investment often pays well, and sometimes very well.*" It should be mentioned that the book was written based on courses taught by the authors at Stanford and UCLA for a number of years, and is now used world-wide for similar courses.

This course will concentrate on analytical structure of convex problems and analytical tools to solve them, with detailed discussion of applications in communications, signal processing, statistical estimation, approximation and fitting etc.

At the end, the students will have developed a working knowledge of convex optimization, i.e. an ability to formulate and solve engineering design problems as convex optimization problems.

How to Study: Learning Efficiency Pyramid

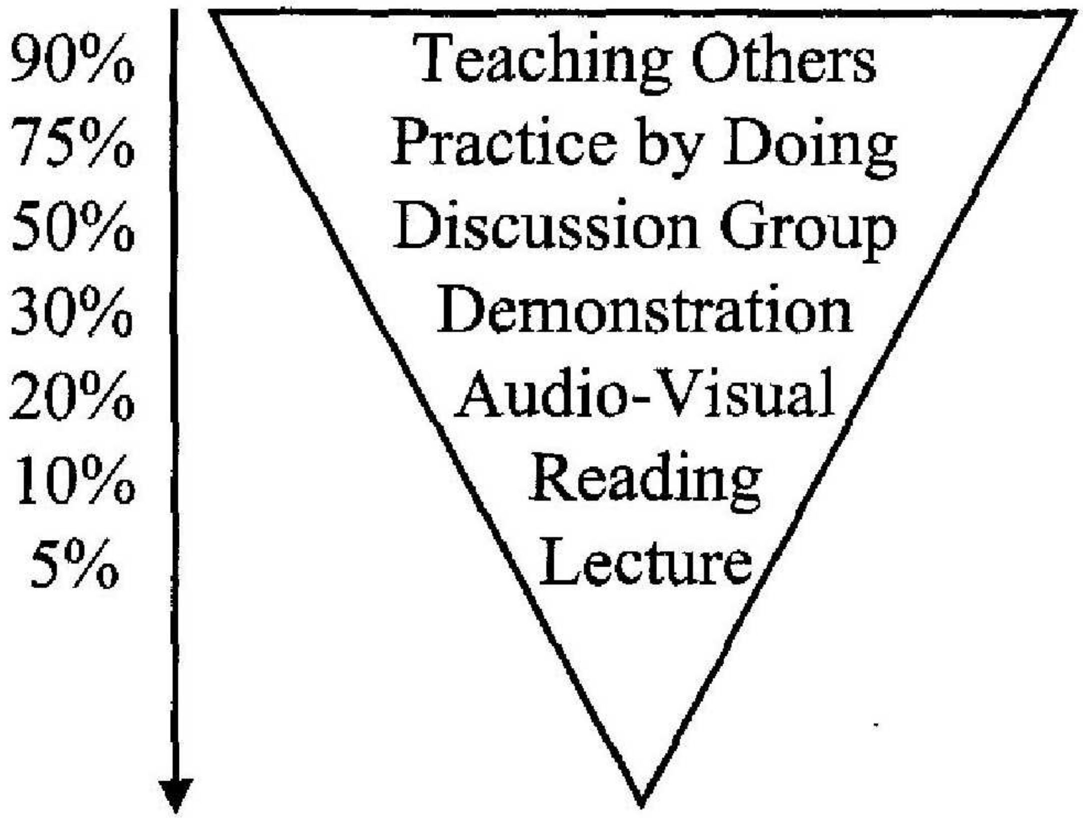

Figure 1. The Learning Pyramid, adapted from David Sousa, *How the Brain Learns*, Reston, VA, The National Association of Secondary School Principals, 1995, ISBN 0-88210-301-6.

"Tell me and I'll forget; show me and I may remember; involve me and I'll understand." – old Chinese proverb.

Another version: "I hear, I forget; I see, I remember; I do, I understand".

"No pain, no gain" – common wisdom.

How to Study

Learning efficiency pyramid is a good guideline

- Reading is necessary, but taken alone is not efficient
- Solving problems (“practice by doing”) is much more efficient
  - examples, assignments, end-of-chapter problems
- Group discussions
  - help provided you contribute something
- Systematic study during the semester
  - is a key to a success.
  - do not leave everything to the last day/night before exams!
- Lectures
  - should be supplemented by the items above
- There is no substitute for active learning! “Seat and watch” approach does not work!