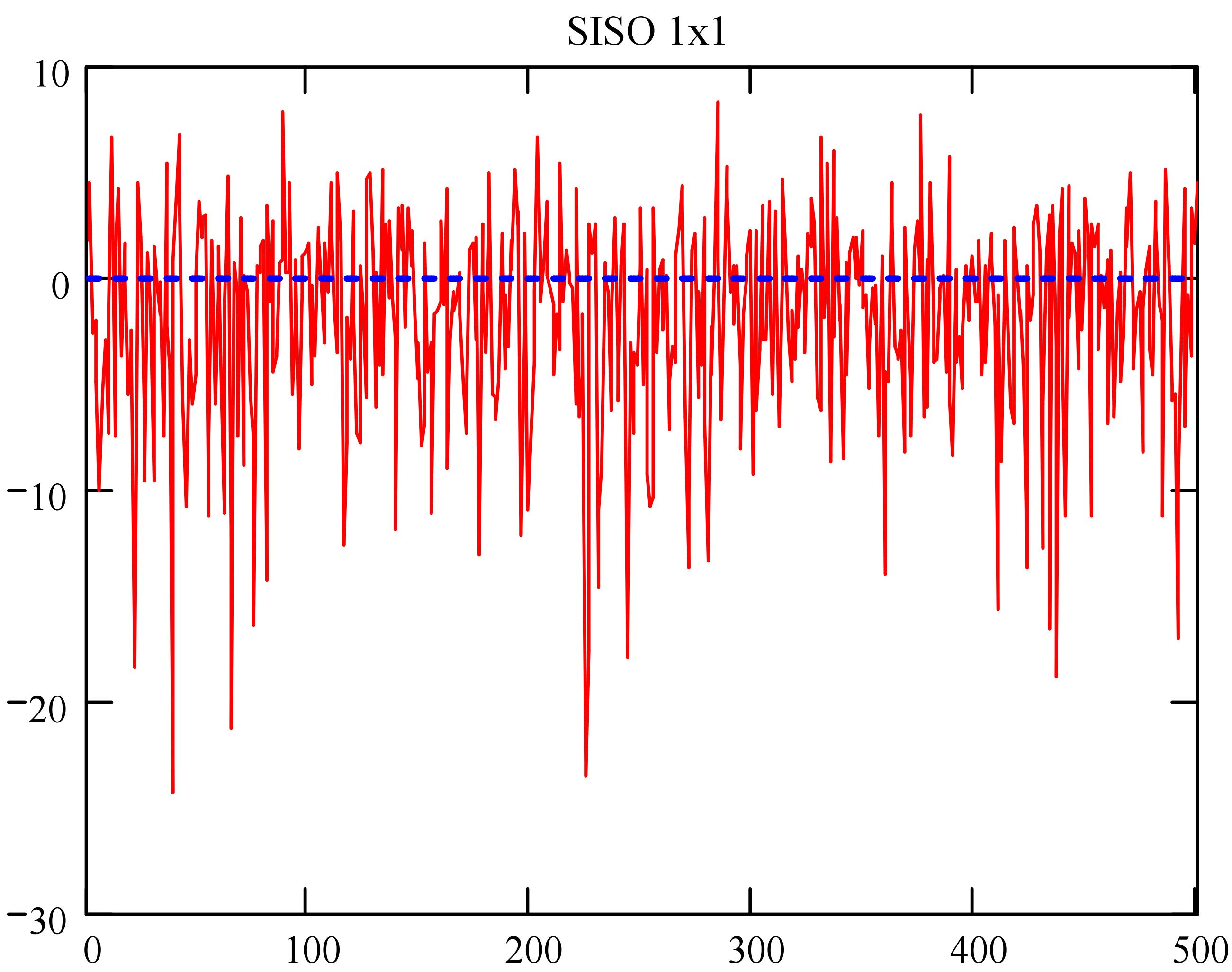
Diversity Combining Techniques

- When the required signal is a combination of several plane waves (multipath), the total signal amplitude may experience deep fades (Rayleigh fading), over time or space.
- The major problem is to combat these deep fades, which result in system outage.
- Most popular and efficient technique for doing so is to use some form of diversity combining.
- **Diversity**: multiple copies of the required signal are available, which experience independent fading or close to that.
- Effective way to combat fading.
- Basic principle: create multiple independent paths for the signal, and combine them in an optimum way.

Rayleigh Fading (multipath)



Types of Diversity

- Space diversity: Antennas are separated in space.
- Angle diversity: Multiple copies of the required signal have different AOA's.
- Frequency diversity: Multiple copies are transmitted at different frequencies.
- Polarization diversity: Multiple copies have different field polarizations.
- Field diversity: Multiple copies are transmitted through different field components (i.e,).
- Time diversity: Multiple copies are transmitted over different time slots.

Types of Diversity

- For all types of diversity, multiple signal copies must be uncorrelated (or weakly correlated, corr. coeff. ≤ 0.5) for diversity combining to operate properly.
- Hence different types of diversity are normally efficient in different scenarios.
- All the forms have their own advantages and disadvantages. In any particular case, some forms are better than the other.
- Example: if the channel is static (or-slow-fading), time diversity is not good.
- If the channel is frequency-flat, frequency diversity is not good.
- A diversity form is efficient when the signals are independent (uncorrelated).

Combining Techniques

Multiple signals are combined in a certain way:

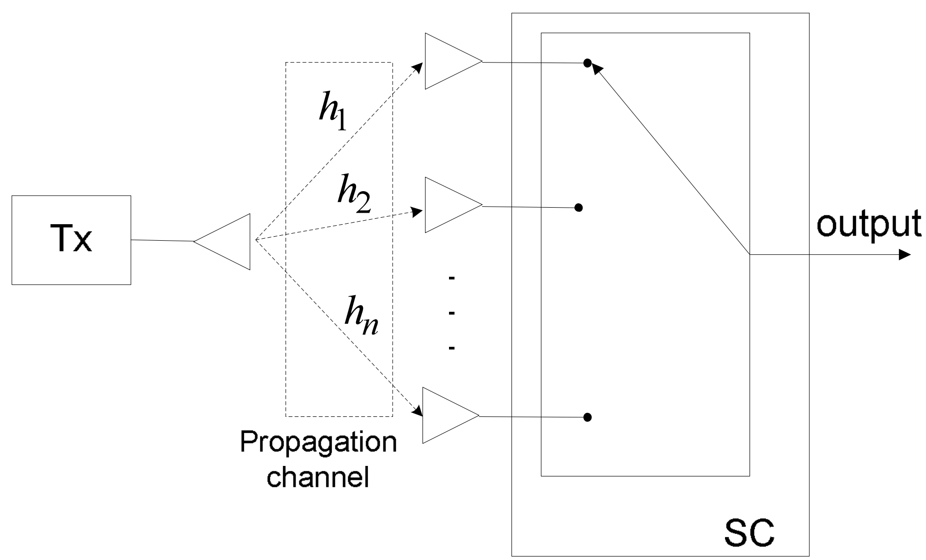
- Selection combining (SC).
- Maximum ratio combining (MRC).
- Equal gain combining (EGC).
- Hybrid combining (different forms are used simltaneously)

<u>Example</u> of hybrid combining: generalized selection combining (select n strongest signals out of N and combine them using MRC).

Micro-diversity and macro-diversity.

Selection combining (SC)

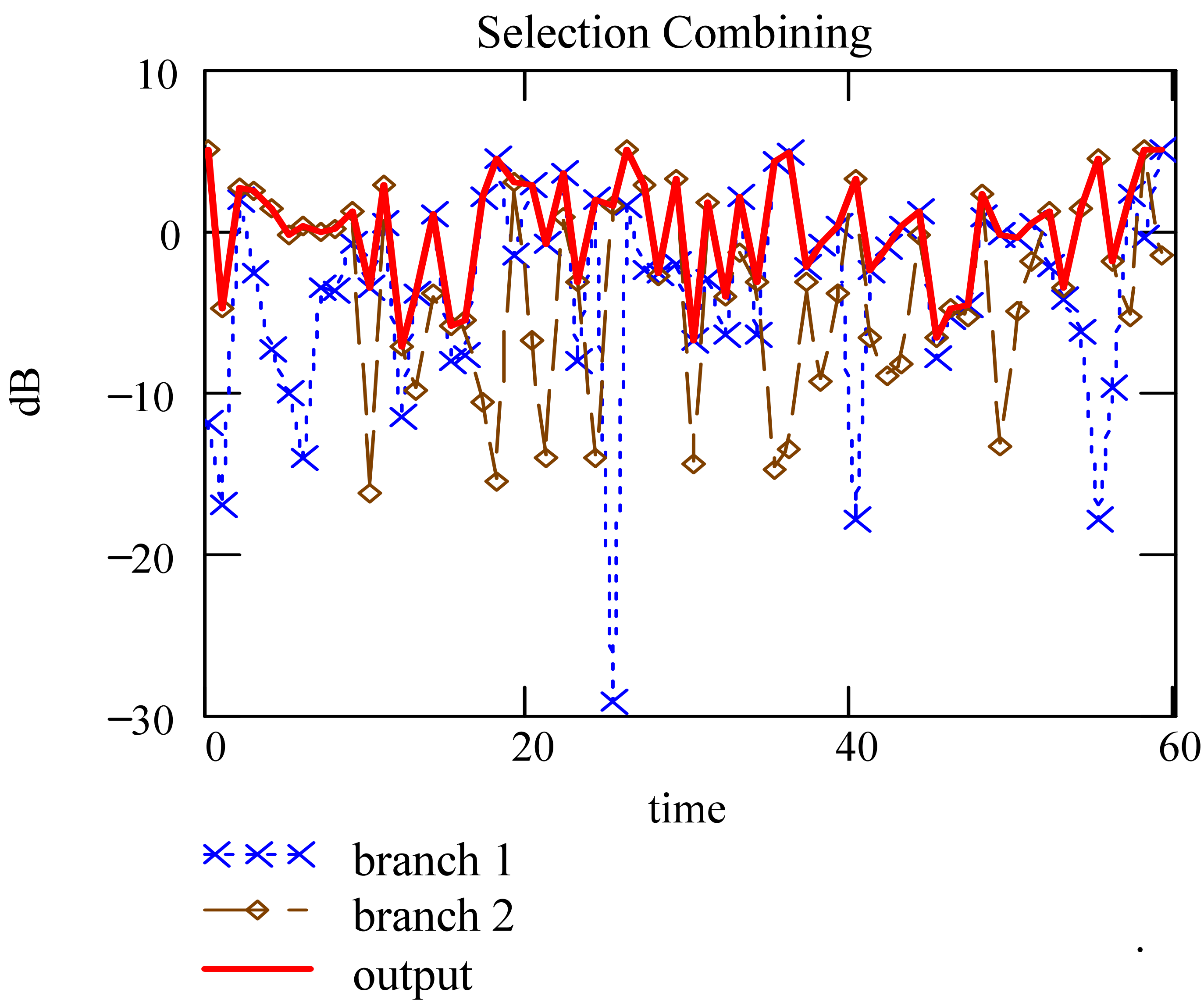
Key idea: at any given moment of time, pick up the branch with the largest SNR.



System model:

$$\mathbf{y} = \mathbf{h}x + \boldsymbol{\xi} \qquad (11.1)$$

Output: select the branch with the largest SNR.


Selection Combining
10
0
−10
−20
−30
dB
0
20
40
60
time
branch 1
branch 2
output

Analysis of a selection combiner

Let's assume that SNR in each branch is Rayleigh distributed , all of them are i.i.d:

$$\rho\left(\gamma_n\right)=\frac{1}{\gamma_0}e^{-\frac{\gamma_n}{\gamma_0}},\quad n=1,\ldots,N \tag{11.2}$$

Let's γ to be a threshold value, then, the probability of outage is

$$P_{out,n}=\Pr\left\{\gamma_n<\gamma\right\}=1-e^{-\frac{\gamma}{\gamma_0}} \tag{11.3}$$

for each branch individually.

The outage probability of the combining system is

$$P_{out}=\prod_{n=1}^{N}P_{out,n}=\left[1-e^{-\gamma/\gamma_0}\right]^N \tag{11.4}$$

Apparently,

$$P_{out}\ll P_{out,n} \tag{11.5}$$

Example:

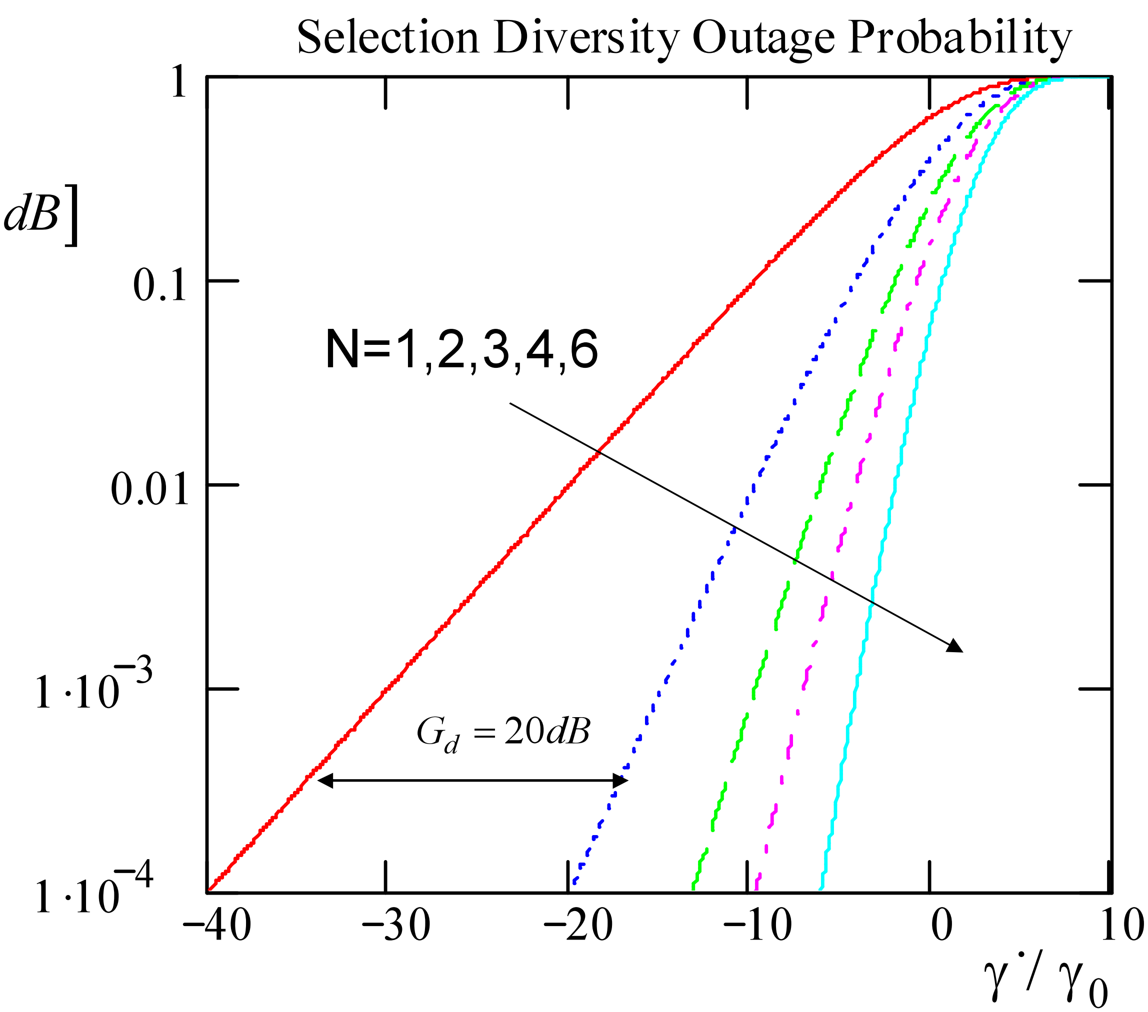
$$P_{out,n} = 10^{-3},\ N = 2:\quad P_{out} = P_{out,n}^2 = 10^{-6} \ll 10^{-3}$$

Selection Combining

Diversity gain: how much less SNR is required to achieve the same P_{out}

$$P_1(\gamma_1) = P_N(\gamma_N) = P_{out}$$

$$G_d = \frac{\gamma_N}{\gamma_1} \text{ or } G_d = (\gamma_N - \gamma_1)[dB]$$



Diversity gain of 10..20 dB: how good is it?

Compare the improvement to that in coding:

Progress toward the Shannon limit

The original turbo codes: about **0.7 dB** from capacity

C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon limit error-correcting coding and decoding: Turbo codes, *IEEE Int. Communications Conference*, 1993.

Irregular LDPC codes: about **0.1 dB** from capacity

T.J. Richardson and R. Urbanke, The capacity of low-density parity-check codes, *IEEE Transactions on Information Theory*, February 2001.

How about 0.01 dB from capacity? And 0.001 dB?

J. Boutros, G. Caire, E. Viterbo, H. Sawaya, and S. Vialle, Turbo code at 0.03 dB from capacity limit, *IEEE Symp. Inform. Theory*, July 2002.

S-Y. Chung, G.D. Forney, Jr., T.J. Richardson, and R. Urbanke, On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit, *IEEE Communications Letters*, February 2001.

Conclusion: For all practical purposes, Shannon's puzzle has been now solved and Shannon's promise has been achieved!

Extracted from A.Vardy, What's New and Exciting in Algebraic and Combinatorial Coding Theory? Plenary Talk at ISIT-06.

Selection Combining

As N increases, the peak of output pdf moves to the right and the curve becomes more and more closer to Gaussian, the Rayleigh channel becomes a Gaussian one as $N \to \infty$.

Q.: prove it!

Diversity order: number of independent branches.

Q.: express a selection combiner as a beamformer.

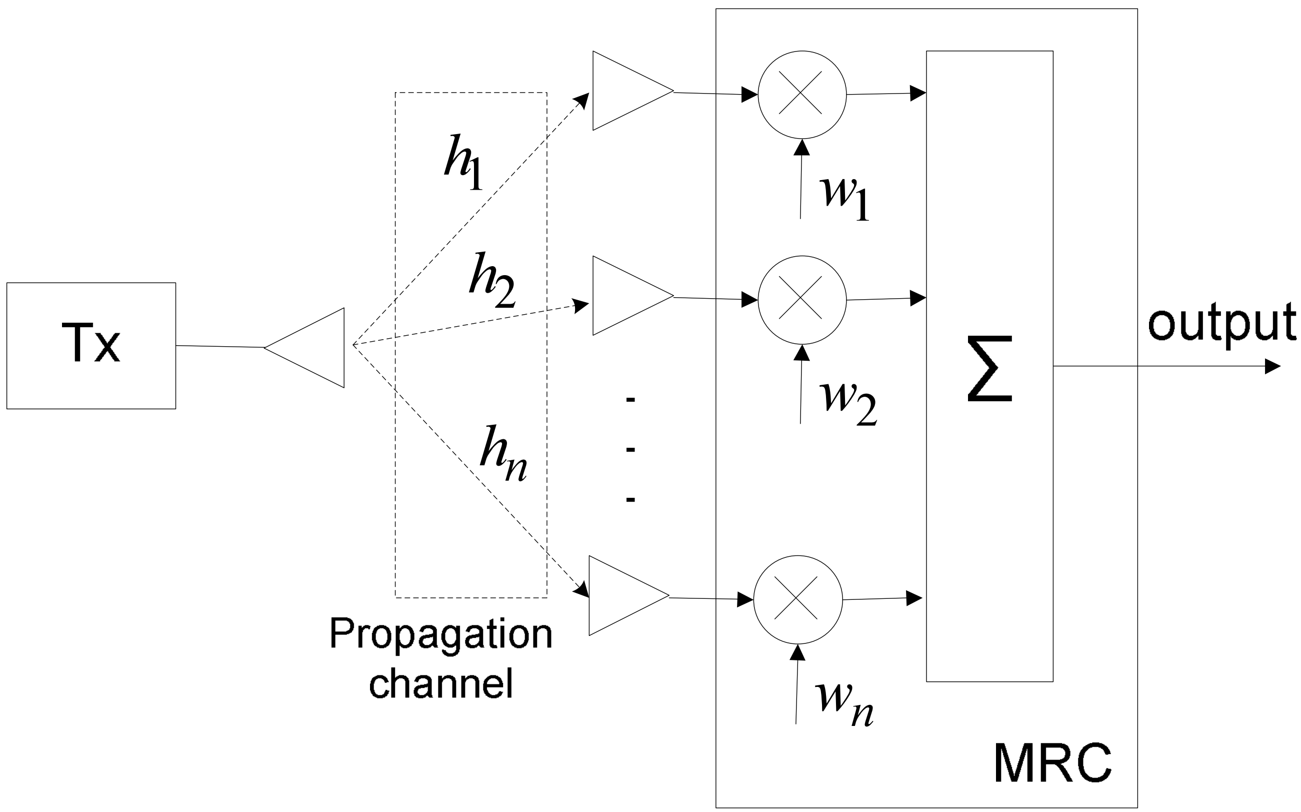
Asymptotic behaviour of P_{out} : $\left(\gamma \ll \gamma_0 \Rightarrow P_{out} \ll 1\right)$

$$P_{out} \approx \left(\frac{\gamma}{\gamma_0}\right)^N, \ \gamma \ll \gamma_0 \qquad (11.3a)$$

Q.: find diversity gain/order in this mode.

Maximal Ratio Combining

Key idea: use linear coherent combining of branch signals so that the output SNR is maximized



Maximal Ratio Combining

The equivalent baseband model is as follows. The individual branch signals are

$$x_n = A \cdot h_n + \xi_n \tag{11.8}$$

where A = complex envelope (amplitude), h_n = channel (complex) gain , $\xi_n \sim CN\left(0, \sigma_0^2\right)$ is AWGN.

Note that h_n acts as multiplicative noise.

The output of the combiner is

$$x_{out} = \sum_{n=1}^{N} w_n^* x_n = \underbrace{A \sum_n w_n^* h_n}_{signal} + \underbrace{\sum_n w_n^* \xi_n}_{noise} \tag{11.9}$$

where w_n are the combining weights.

The signal and noise components are given by 1st and 2nd term correspondingly. The signal and noise power at the output are:

$$P_s = \overline{\left| A\sum_n w_n^* h_n \right|^2} = \frac{1}{2}|A|^2 \left| \sum_n w_n^* h_n \right|^2, \quad P_\xi = \overline{\left| \sum_n w_n^* \xi_n \right|^2} = \sum_n |w_n|^2 \sigma_n^2 \quad (11.10)$$

where $\sigma_n^2 = \overline{|\xi_n|^2}$ is the branch noise power.

Output SNR is

$$SNR_{out} = \frac{P_s}{P_\xi} = \frac{1}{2}|A|^2 \frac{\left| \sum_n w_n^* h_n \right|^2}{\sum_n |w_n|^2 \sigma_n^2} \quad (11.11)$$

We can maximize it using the Lagrange multipliers or better, Cauchy-Schwarz inequality.

Cuachy-Schwartz inequality:

$$\left|\sum_n a_n^* b_n\right|^2 \le \sum_n |a_n|^2 \sum_n |b_n|^2 \qquad (11.12)$$

With the equality achieved when $a_n^* = cb_n$.

Using (11.12), the best combining weights are

$$w_n = h_n / \sigma_n^2 \qquad (11.13)$$

and the output SNR of the best combiner is

$$\gamma_{out} = \sum_n \gamma_n \qquad (11.14)$$

where n-th branch SNR is

$$\gamma_n = \frac{A^2 |h_n|^2}{2\sigma_n^2} \qquad (11.15)$$

Maximal Ratio Combining

The resulting combiner is called MRC. It is the best combiner in terms of the SNR.

Note: $w_n = h_n^* / \sigma_n^2$ is true for any channel, not necessarily Rayleigh.

When all the branch noise powers are equal,

$$\sigma_n = \sigma_0 \rightarrow w_n = \frac{h_n^*}{|\mathbf{h}|}, \ \gamma_{out} = N\gamma_0 \qquad (11.13a)$$

The branch gain is proportional to the channel gain +coherent combining.
Note *N*-fold increase in average SNR (but this is not the main gain!).

Q1: explain it.
Q2: prove (11.14).
Q3: derive this using maximum SNR beamformer discussed earlier. Hint: assume each antenna has different signal amplitude and apply earlier result on max. SNR beamformer.

Outage Probabilities

Consider a Rayleigh channel and assume there is no correlation between branches.

The outage probability can be expressed as

$$P_{out} = 1 - e^{-\frac{\gamma}{\gamma_0}} \sum_{k=1}^{N} \frac{(\gamma/\gamma_0)^{k-1}}{(k-1)!} \qquad (11.16)$$

Asymptotically, $\gamma/\gamma_0 \ll 1$ (small P_{out}):

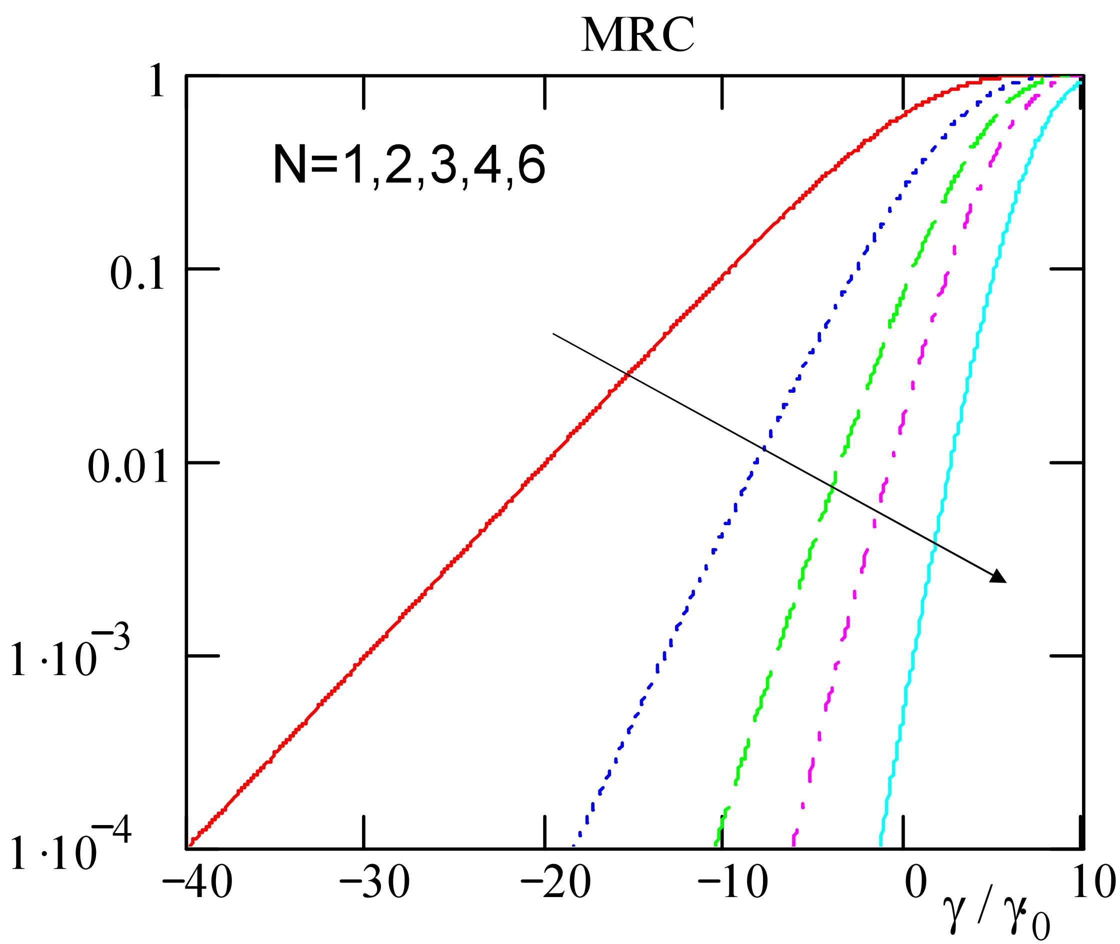
$$P_{out} \approx \frac{(\gamma/\gamma_0)^N}{N!} \qquad (11.17)$$

Diversity order = N.

Q.: Diversity gain?

MRC Outage Probability

Q.: compare to EGC!

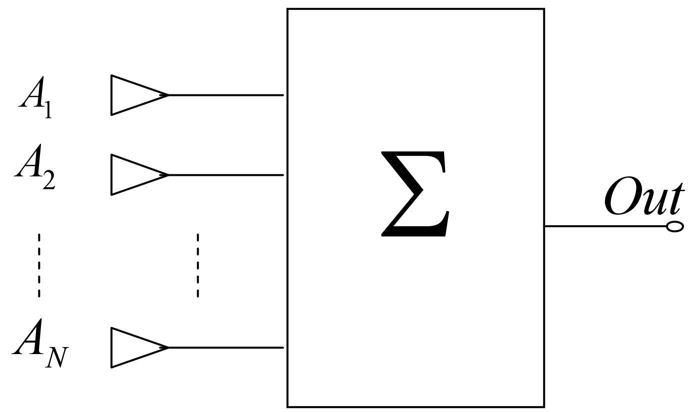


Implementation issues: requires for unequal gains (in general) + coherent combining. May be difficult to implement.
MRC performance is best of all combiners in terms of SNR at the output.

Q: diversity gain =? SNR gain = ?

Equal gain combining

It is MRC with equal gains (but still the combining is coherent).



Coherent combining

$$x_{out} = \sum_{n=1}^{N} |s_n| + \sum_{n=1}^{N} \xi_n \quad (11.19)$$

where s_n is the signal at branch n (without noise).

EGC: performance

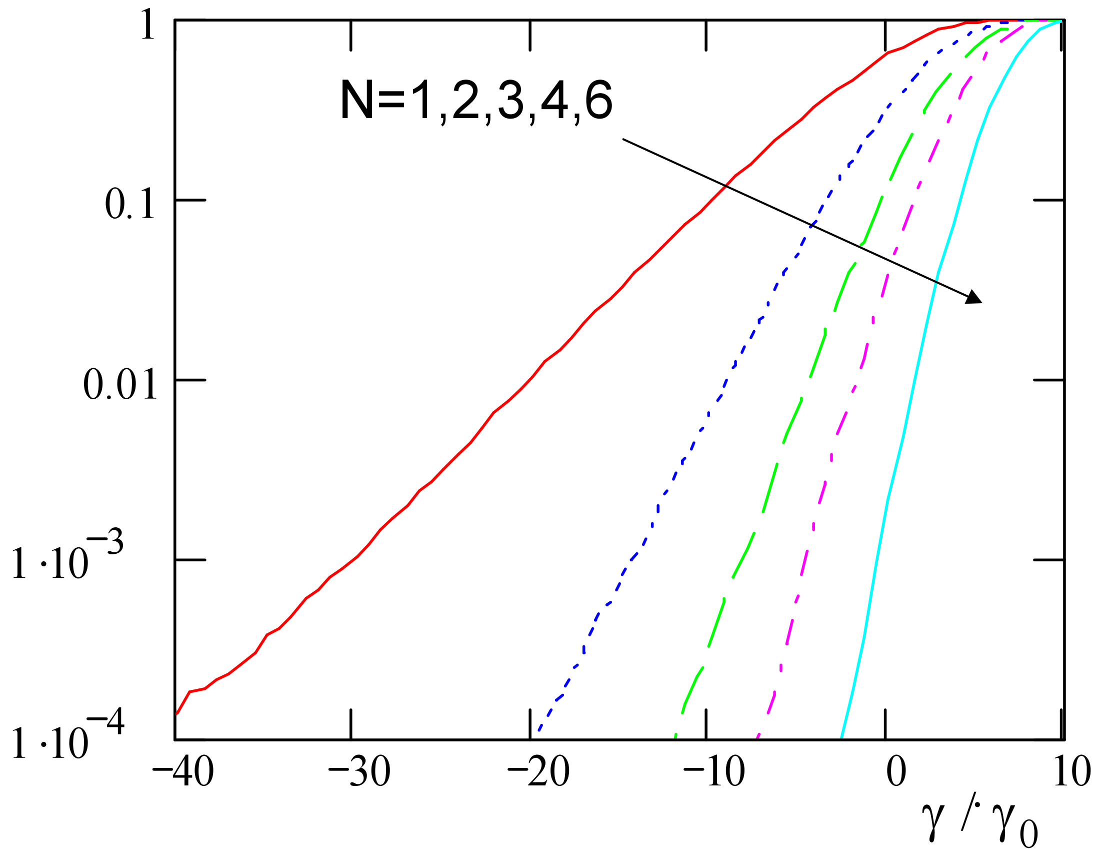
Performance of EGC is between of SC and MRC.

Asymptotic behaviour

$$P_{out} \sim \left(\gamma / \gamma_0\right)^N \qquad (11.20)$$

Implementation issues: easier than MRC. But more difficult than SC (since it is a coherent combiner).

EGC: Outage Probability



Summary of Implementation Issues

- Combiner complexity varies significantly from type to type (as well as performance).
- Overall, there exists performance/complexity trade-off. High-performance combiner (MRC) is difficult to implement; simpler combiner (SC) does not perform that well as MRC, EGC is in between.
- SC is the simplest technique.
- MRC: coherent technique, i.e., signal's phase has to be estimated. All the signals are co-phased and weighted according to their signal voltage to noise power ratios. Requires individual receivers/processing.
- MRC provides the best performance. DSP makes it practical. The output may be acceptable even when all the inputs are not.
- Equal gain combining: compromise. Less complexity than MRC (all the weights are equal), but co-phasing is still required. Can still produce acceptable output from unacceptable inputs.

Summary

- Combating fading channel: diversity combining. Types of combining.
- Selection combining.
- Equal gain combining.
- Maximum ration combining.
- Performance. Outage probability. Diversity gain.

References

1. R.E. Ziemer, R.L. Peterson, Introduction to Digital Communication, Prentice Hall, 2001.
2. J.W. Mark, W. Zhuang, Wireless Communications and Networking, Prentice Hall, Upper Saddle River, NJ, 2003.
3. J. Proakis, Digital Communications, McGraw Hill, Boston, 2001.
4. M. Schwartz, W.R. Bennett, S. Stein, Communication Systems and Techniques, McGraw Hill, New York, 1966.[1]

Homework

Fill in the details in the derivations above. Answer the questions. Do the examples yourself.

[1] This is especially good reference on diversity combining techniques and their performance analysis. Highly recommended.