Differential Phase Shift Keying (DPSK)

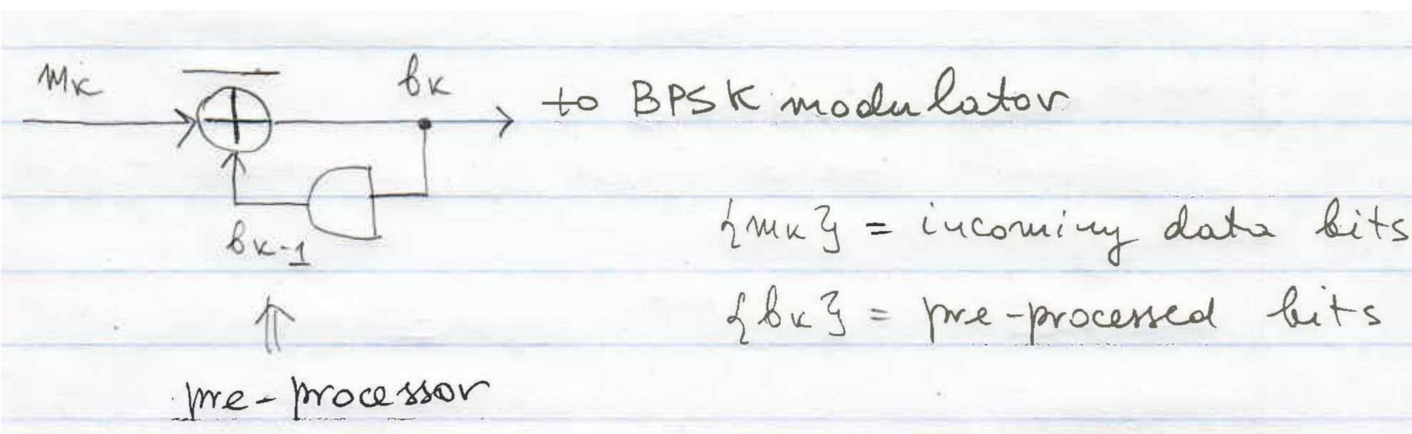
BPSK → need to synchronize the carrier.
DPSK → no such need.

Key idea: transmit the difference between 2 adjacent messages, not messages themselves.

Implementation:

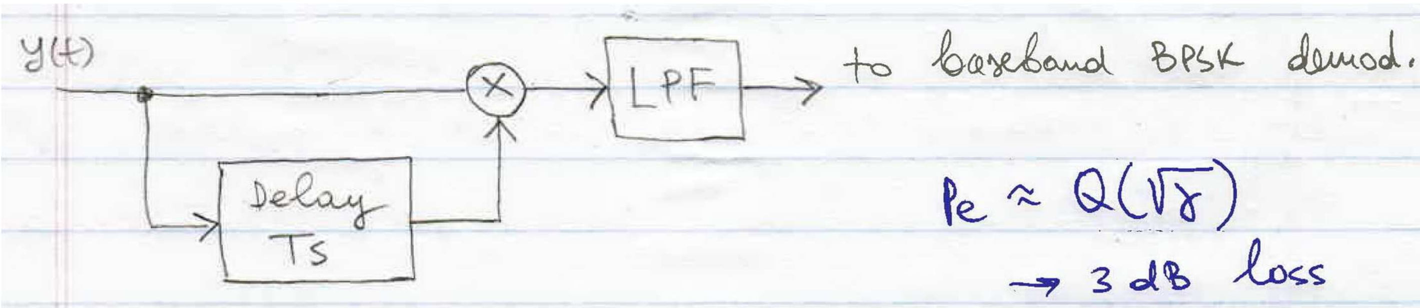
$$b_k = \overline{b_{k-1} \oplus m_k} \Rightarrow \begin{array}{l} m_k = 1 \rightarrow b_k = b_{k-1} \\ m_k = 0 \rightarrow b_k = \overline{b_{k-1}} \end{array} \qquad (7.1)$$

where $\oplus$ is binary (mod-2) addition and $\overline{(\cdot)}$ is binary negation.



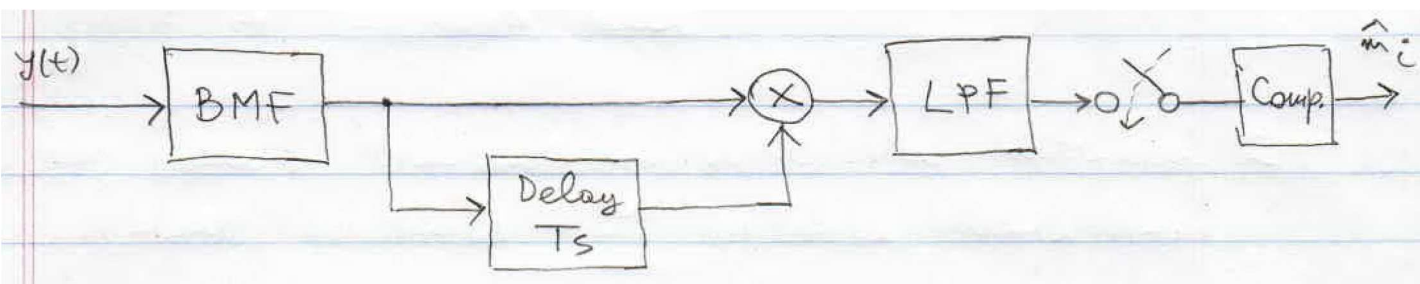
Since BPSK (RF) modulation is used:
- same spectrum (Δf etc.),
- same rate/spectral efficiency.

Demodulator (sub-optimal):



$$P_e \approx Q\left(\sqrt{\gamma}\right) \rightarrow \text{ 3dB loss} \quad (7.2)$$

Demodulator: (optimal)



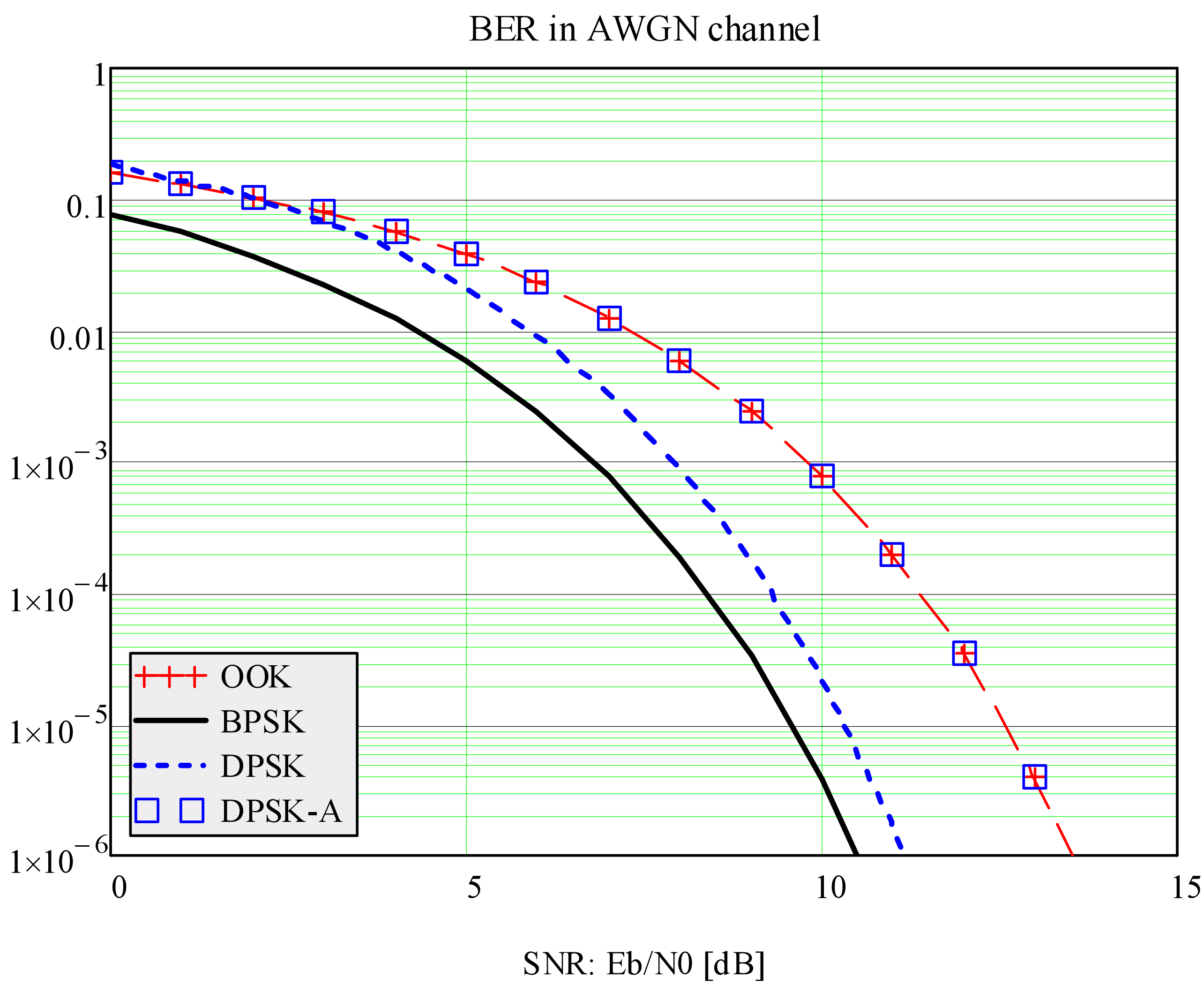
BMF = bandpass matched filter, $h(t) = p(t)\cos\omega t$.

Optimal probability of error (BER):

$$P_e = \frac{1}{2}e^{-\gamma} \quad (7.3)$$

Q.: find expressions for signals at each point of demodulator assuming no noise $\left(y(t)=x(t)\right)$. Compare to the suboptimal demodulator.

BER: $\approx 1\text{dB}$ loss to BPSK.



Signal Constellation

In-phase/quadrature representation[1]:

$$\begin{aligned} x(t) &= A\cos(\omega t + \varphi) \\ &= A\cos\varphi\cos\omega t - A\sin\varphi\sin\omega t \\ &= \underbrace{A_I \cos\omega t}_{I} - \underbrace{A_Q \sin\omega t}_{Q} \end{aligned} \quad (7.4)$$

Complex form:

$$\begin{aligned} x(t) &= A\cos(\omega t + \varphi) = \mathrm{Re}\left\{Ae^{j(\omega t+\varphi)}\right\} \\ &= \mathrm{Re}\left\{A_c e^{j\omega t}\right\} \end{aligned} \quad (7.5)$$

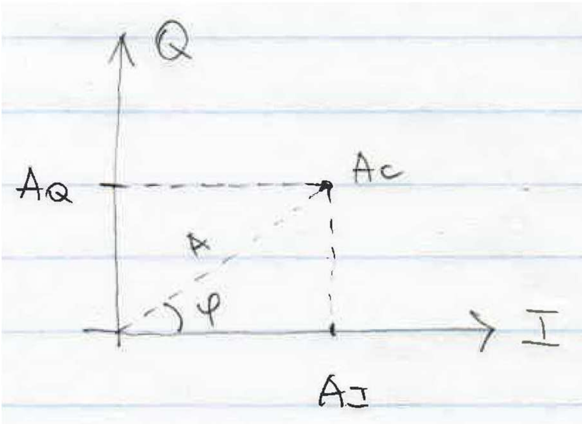
where $A_c = Ae^{j\varphi} =$ complex amplitude, $\omega =$ carrier frequency, $A =$ carrier amplitude (real).

$$A_c = A_I + jA_Q = A\cos\varphi - jA\sin\varphi \quad (7.6)$$

$$A = |A_c| = \sqrt{A_I^2 + A_Q^2} \quad (7.7)$$

$$\varphi = \arg(A_c) = \tan^{-1}(A_Q / A_I) \quad (7.8)$$

[1]For simplicity, assume $p(t) = \Pi(t/T)$ and consider 1 pulse only.

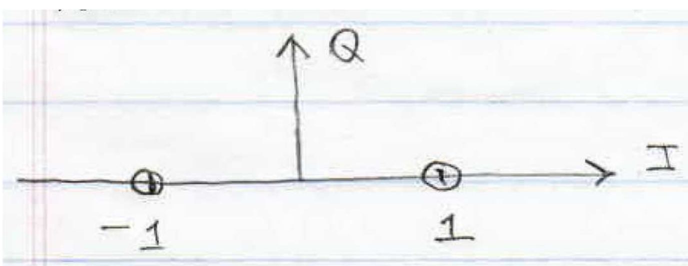


$$\left.\begin{aligned} A &= A(t) \\ \varphi &= \varphi(t) \\ A_c &= A_c(t) \end{aligned}\right\} \leftarrow \text{modulation}$$

BPSK constellation:

$$x(t) = s(t)\cos\omega t;$$

$$s(t) = \pm 1,\ A_I = \pm 1,\ \ A_Q = 0;\ \ A_c = \pm 1,\ \ \varphi = 0, 180^\circ$$



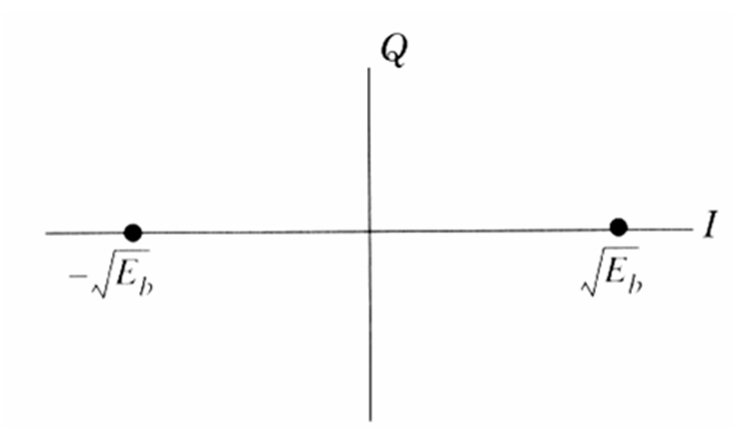


Figure 6.21 BPSK constellation diagram.

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

QPSK

Quadrature phase shift keying.

BPSK: $\varphi_i = 0$ or π

QPSK: $\varphi_i = 0,\ \pi/2,\ 3\pi/2$ or $\pi/4$ (2 bits instead of 1).

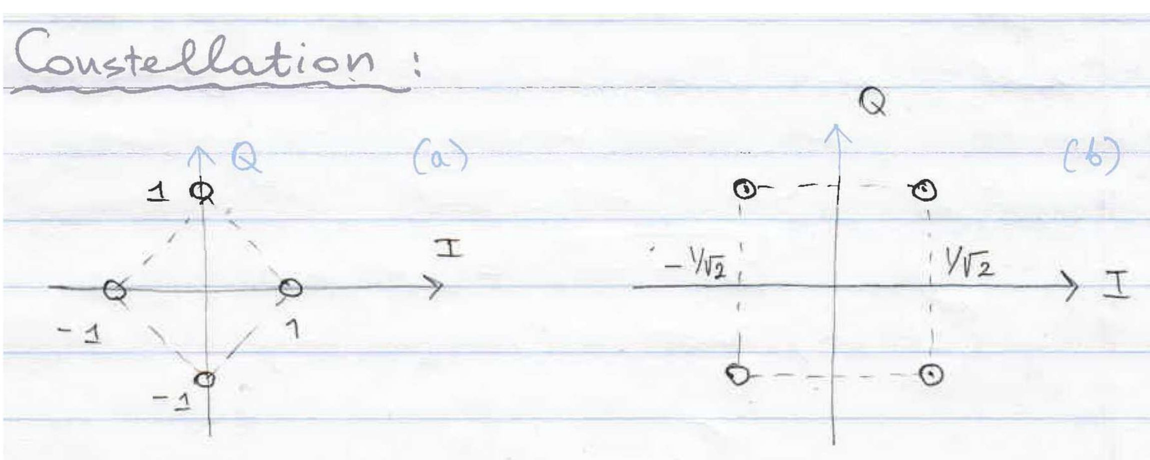
2 forms:

$$x(t) = A\cos(\omega t + \theta_i),\quad \theta_i = i\frac{\pi}{2},\quad i = 0,1,2,3$$
$$\text{or } \theta_i = i\frac{\pi}{2} + \frac{\pi}{4} \tag{7.9}$$

$I - Q$ form:

$$x(t) = A_I \cos\omega t - A_Q \sin\omega t,$$
$$A_I = \pm\frac{A}{\sqrt{2}},\quad A_Q = \pm\frac{A}{\sqrt{2}} \tag{7.10}$$

Constellation:



(b): combination of I and Q BPSK:

$$x(t) = a_i \cos\omega t - b_i \sin\omega t$$
$$a_i, b_i = \pm 1 \quad (I \text{ and } Q \text{ data}) \tag{7.11}$$

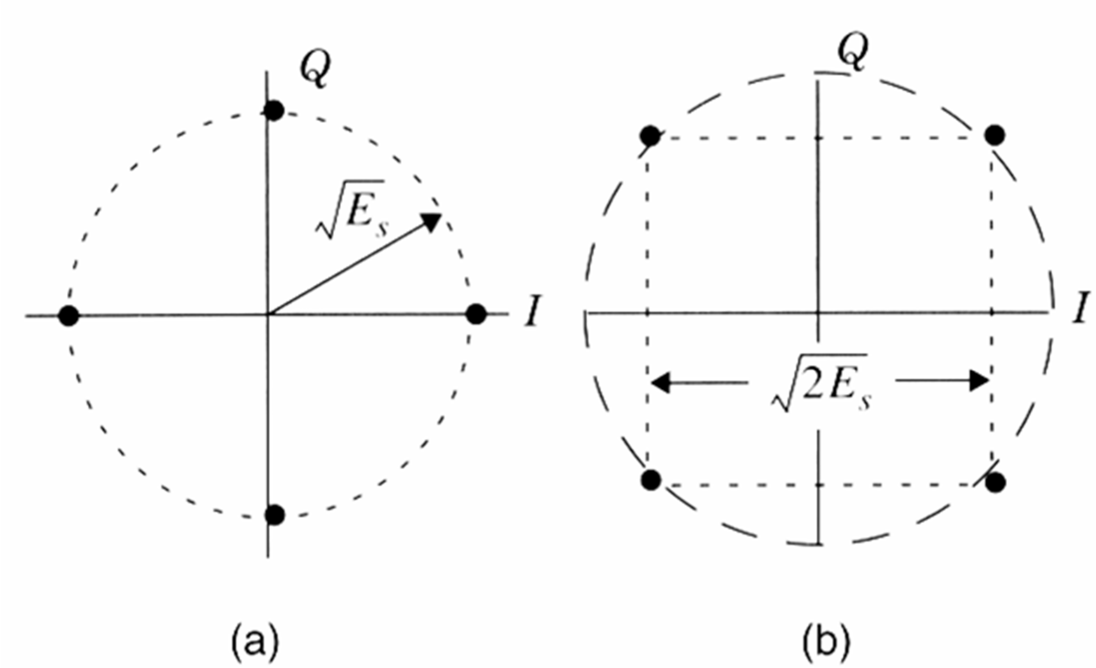


Figure 6.26 (a) QPSK constellation

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

QPSK: Properties

BPSK: 1 bit/symbol (sinc)
QPSK: 2 bit/symbol (sinc)→twice SE! (same Δf)

$$\text{SE} = \eta = \frac{R_b}{\Delta f} = \frac{2\text{bit}/T_s}{1/T_s} = 2 \quad (\text{QPSK}) \tag{7.12}$$

$$\text{BPSK: } \eta = \frac{1\text{bit}/T_s}{1/T_s} = 1 \tag{7.13}$$

I/Q data sequences: constructed in the same way as for BPSK,

$$m_I(t) = \sum_i a_i p(t - iT), \;\; m_Q(t) = \sum_i b_i p(t - iT) \tag{7.14}$$

i.e. separate baseband BPSK over I and Q channels.

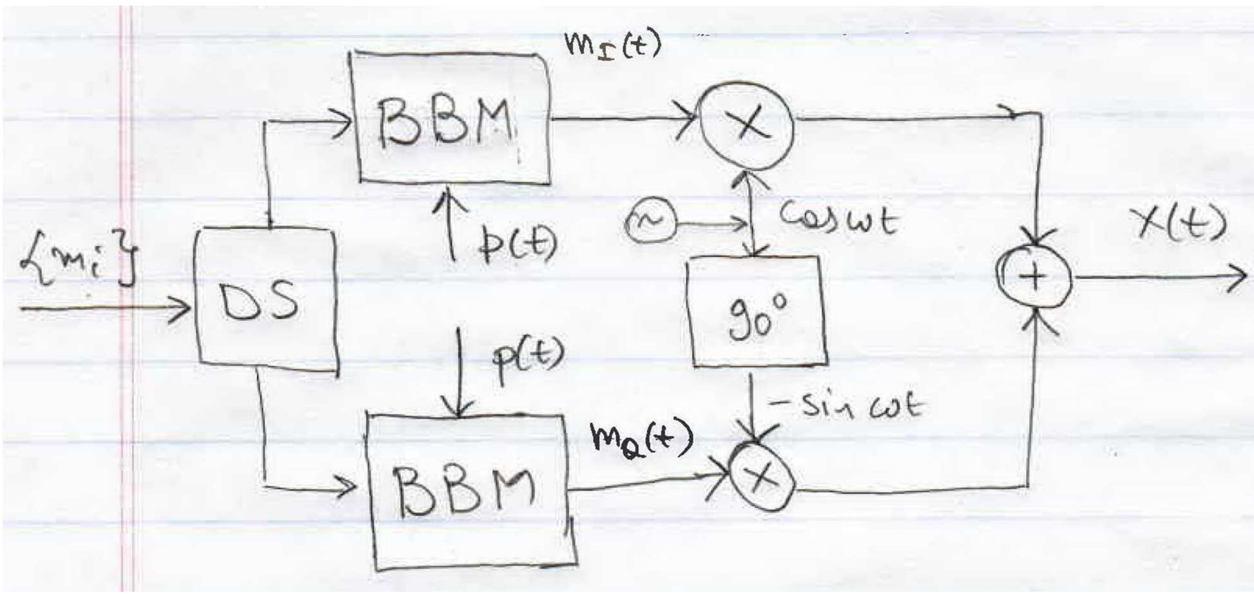
Bandpass modulated signal:

$$x(t) = A\sum_i p(t - iT)\cos(\omega t + \theta_i) \tag{7.15}$$

θ_i = encodes data, e.g. $00 \to \theta_1, \;\; 01 \to \theta_2,$
$10 \to \theta_3, \;\; 11 \to \theta_4$

QPSK Modulator (Tx)

QPSK = 2×BPSK

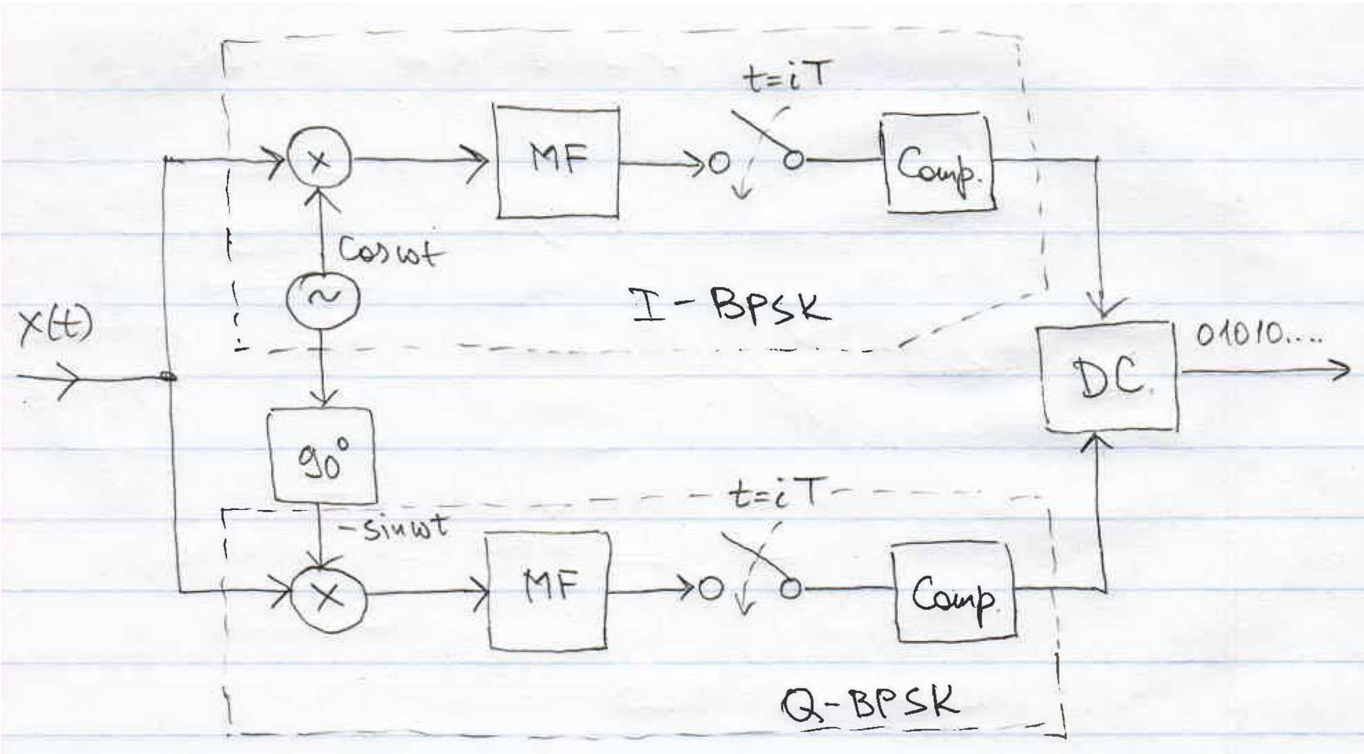


BBM = baseband BPSK modulator,
DS = data splitter.

$$m_I(t) = \sum_i a_i p(t - iT), \;\; m_Q(t) = \sum_i b_i p(t - iT)$$
$$m_i \rightarrow \{a_i, b_i\}, \;\; a_i, b_i = \pm 1 \tag{7.16}$$

$m_I(t), m_Q(t) =$ baseband BPSK-modulated signals.

QPSK Demodulator (Rx)



MF = baseband matched filter $\left(\text{to } p(t)\right)$,
DC = data combiner.

Probability of bit error (BER):

$$P_b = Q\left(\sqrt{\gamma}\right) = Q\left(\sqrt{2\gamma_b}\right) \qquad (7.17)$$

where $\gamma_b = \dfrac{E_b}{N_0} = \dfrac{E_s}{2N_0} = \dfrac{\gamma}{2} =$ SNR/bit; $E_s = 2E_b$.

Bandwidth of QPSK $\left(p(t)=RC\right)$:

$$\Delta f = \frac{1+\alpha}{2}\frac{R_b}{2}; \quad \Delta f_{RF} = \frac{1+\alpha}{2} R_b \tag{7.18}$$

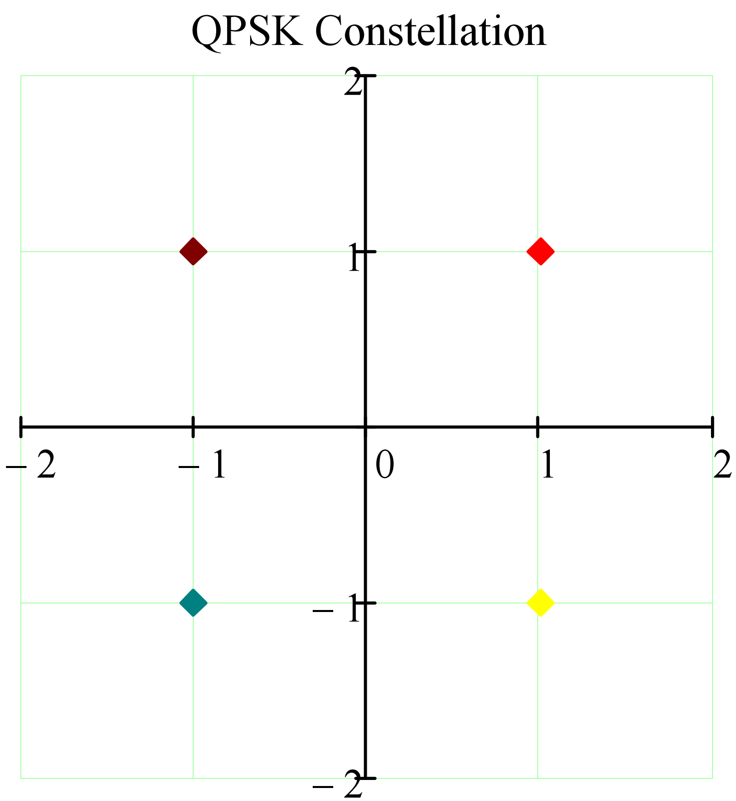
$$R_b = 2R_s = \frac{2}{T_s} = \text{ bit rate } [\text{bits/s}] \tag{7.19}$$

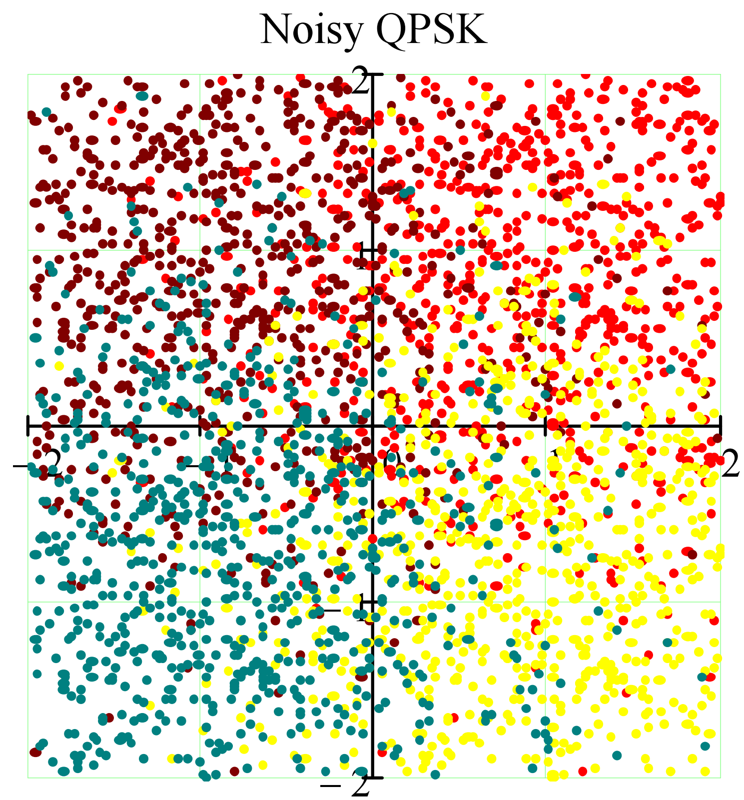
$$\eta = \frac{2}{1+\alpha} \quad [\text{bits/s/Hz}] \Rightarrow \text{QPSK=2xBPSK} \tag{7.20}$$

<u>**Key parameters**</u> (for any modulation):

- data rate R_b
- BER (error probability) P_e
- bandwidth Δf (Δf_{RF}) or spectral efficiency (SE) η

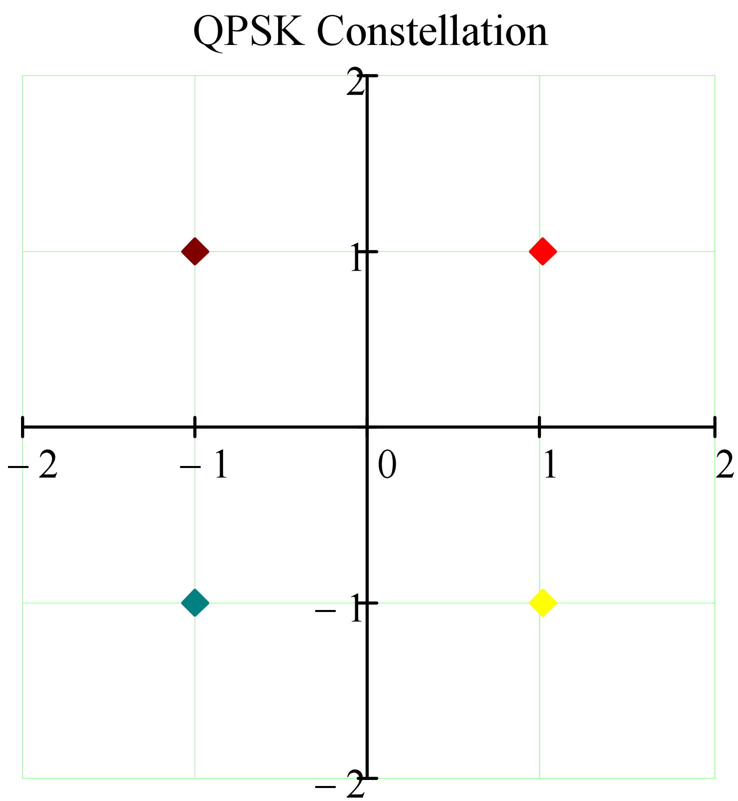
QPSK Constellation in Noise: SNR = 0 dB

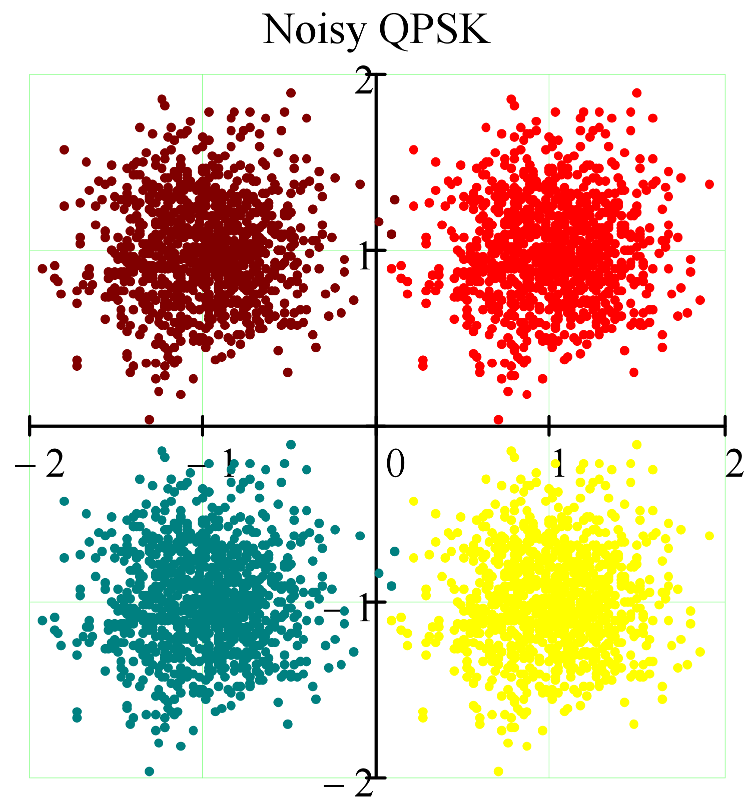




N=1000 symbols transmitted.

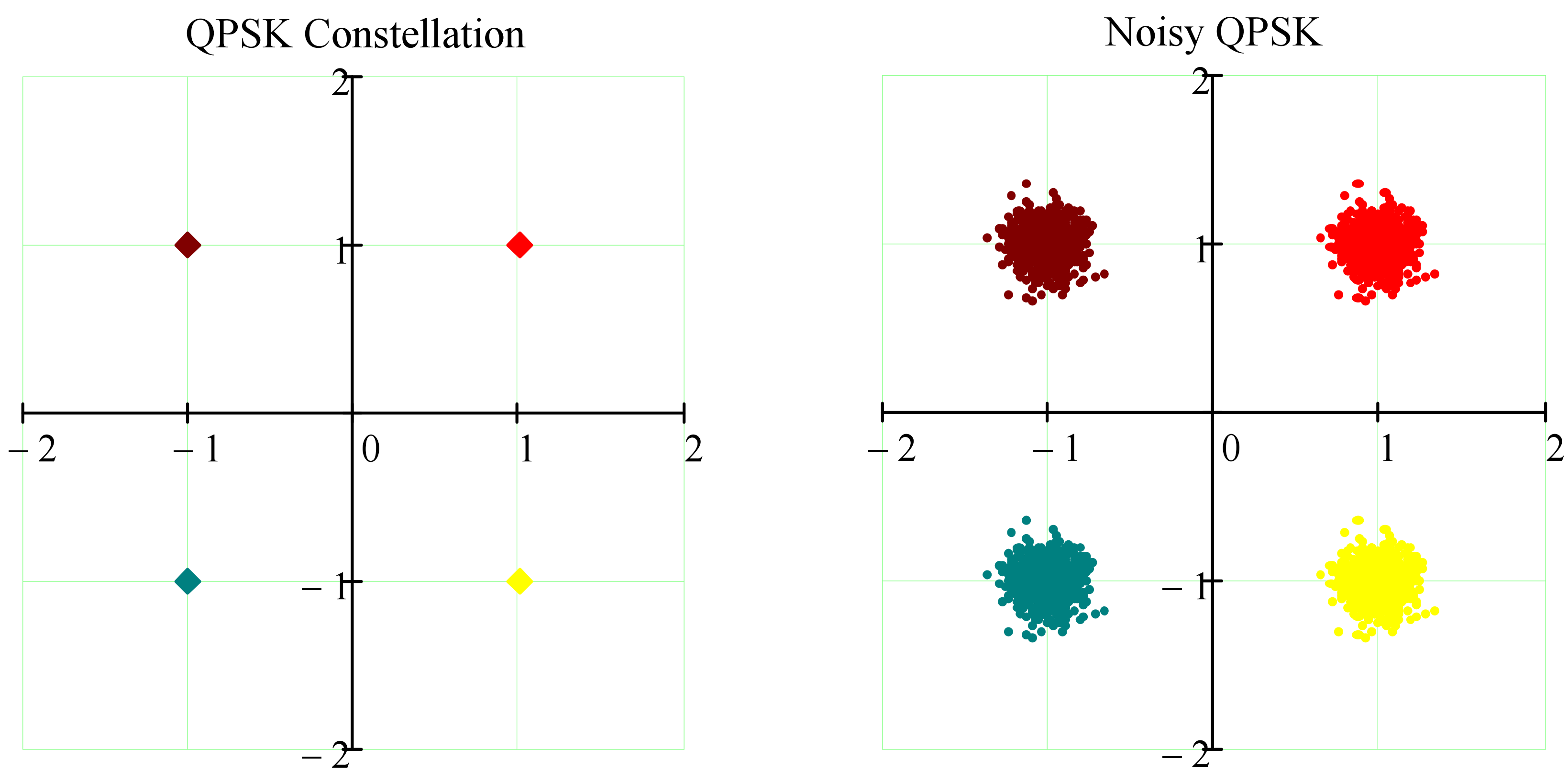
QPSK Constellation in Noise: SNR = 10 dB





N=1000 symbols transmitted.

QPSK Constellation in Noise: SNR = 20 dB



N=1000 symbols transmitted.

Quadrature Amplitude Modulation (QAM)

QAM key idea: use in-phase $(\cos\omega t)$ and quadrature $(\sin\omega t)$ for PAM simultaneously.

- 2 x rate of 1 channel
- independent channels as

$$\int_0^T \cos\omega t \sin\omega t dt = 0 \qquad (7.21)$$

M-PAM:

$$s_i(t) = A_i p(t), \quad i = 1,\ldots,M \qquad (7.22)$$

M-QAM:

$$M\text{-QAM} = \underbrace{\sqrt{M} - \text{PAM}}_{I} \times \underbrace{\sqrt{M} - \text{PAM}}_{Q}$$

RF signal of M-QAM: use $\sqrt{M} - \text{PAM}$ on I and Q:

$$\begin{aligned} & x(t) = Am_I(t)\cos\omega_c t - Am_Q(t)\sin\omega_c t \\ & m_I(t) = a_i p(t), \quad m_Q(t) = b_i p(t), \quad 0 \le t \le T \\ & a_i, b_i : \text{ represent I and Q bits} \\ & \qquad = 2i+1, \ i = -L,\ldots,L-1, \ L = \sqrt{M}/2 \end{aligned} \qquad (7.23)$$

Basis functions of QAM:

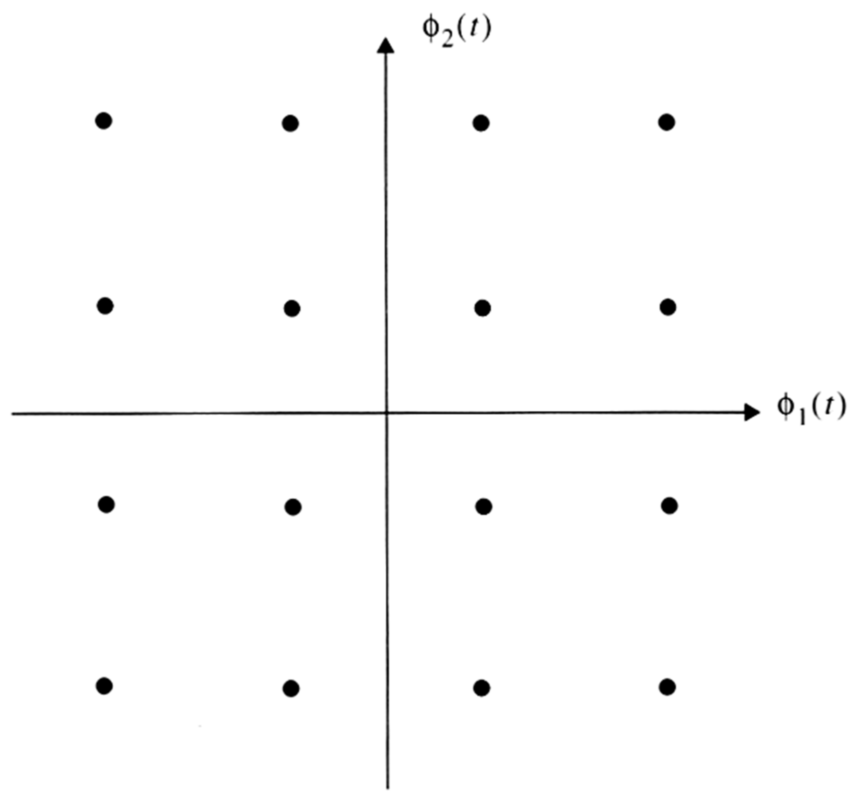
$$\psi_1(t) = p(t)\cos\omega_c t, \quad \psi_2(t) = p(t)\sin\omega_c t \qquad (7.24)$$

Note the orthogonality property:

$$\int_0^T \psi_1(t)\psi_2(t)\,dt = 0 \qquad (7.25)$$

provided that $S_p(f)S_{\cos\omega_c t}(f) = 0$, i.e. the spectrum of $p(t)$ and $\cos\omega_c t$ (or $\sin\omega_c t$) do not overlap.

Q.: Prove this.



T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

Figure 6.47 Constellation diagram of an M-ary QAM ($M = 16$) signal set.

Alternative form of RF QAM signal:

$$x_{(t)} = a_i \psi_1(t) + b_i \psi_2(t) \tag{7.26}$$

Minimum symbol energy $E_{\min}$:

$$E_{\min} = \frac{A^2}{2} E_p$$

$$E_p = \int_0^T p^2(t)\,dt = \text{ energy of } p(t) \tag{7.27}$$

Probability of symbol error (symbol error rate - SER) P_s:

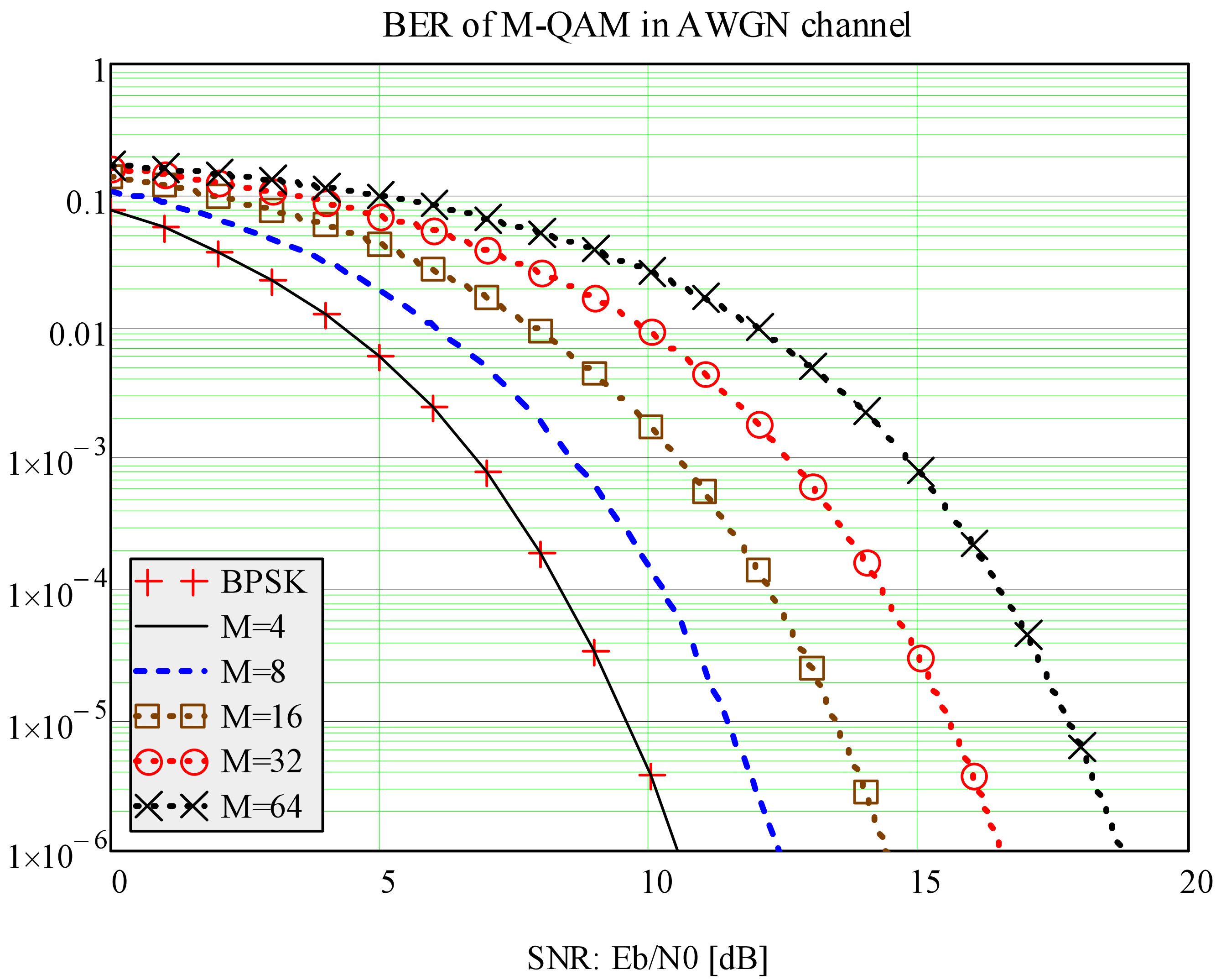
$$P_s \approx 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_{\min}}{N_0}}\right) = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_{av}}{(M-1)N_0}}\right) \tag{7.28}$$

where E_{av} = average symbol energy,

$$E_{av} = \frac{2(M-1)}{3} E_{\min} \tag{7.29}$$

The BER P_b:

$$\frac{1}{\log_2 M} P_s \le P_b \le P_s, \quad P_b \approx \frac{1}{\log_2 M} P_s \tag{7.30}$$



Q.: reproduce the graph.

Q.: how much extra SNR do you need to add 1 extra bit at the same BER?

Adaptive modulation: keep BER (almost) constant.

4G systems:

Optimized for high-speed data service (Internet), VoIP.

Two major standards: LTE (Long Term Evolution) and WiMax (Worldwide Interoperability for Microwave Access).

LTE Standard

Modulation: OFDM + QPSK/16QAM/64QAM, up to 20MHz bandwidth.

Rates: see below.

Table 1. LTE (FDD) downlink and uplink peak data rates from TR 25.912 V7.2.0 Tables 13.1 and 13.1a

FDD downlink peak data rates (64QAM)			
Antenna configuration	SISO	2x2 MIMO	4x4 MIMO
Peak data rate Mbps	100	172.8	326.4
FDD uplink peak data rates (single antenna)			
Modulation depth	QPSK	16QAM	64QAM
Peak data rate Mbps	50	57.6	86.4

3GPP Long Term Evolution: System Overview, Product Development, and Test Challenges. Application Note, Agilent.

Note: MIMO = multiple-input multiple-output, or multi-antenna system.

SISO = single-input single-output, or single-antenna system.

IEEE 802.11n WiFi standard

MCS Index	Type	Coding Rate	Spatial Streams	Data Rate (Mbps) with 20 MHz CH		Data Rate (Mbps) with 40 MHz CH	
				800 ns	400 ns (SGI)	800 ns	400 ns (SGI)
0	BPSK	1 / 2	1	6.50	7.20	13.50	15.00
1	QPSK	1 / 2	1	13.00	14.40	27.00	30.00
2	QPSK	3 / 4	1	19.50	21.70	40.50	45.00
3	16-QAM	1 / 2	1	26.00	28.90	54.00	60.00
4	16-QAM	3 / 4	1	39.00	43.30	81.00	90.00
5	64-QAM	2 / 3	1	52.00	57.80	108.00	120.00
6	64-QAM	3 / 4	1	58.50	65.00	121.50	135.00
7	64-QAM	5 / 6	1	65.00	72.20	135.00	150.00
8	BPSK	1 / 2	2	13.00	14.40	27.00	30.00
9	QPSK	1 / 2	2	26.00	28.90	54.00	60.00
10	QPSK	3 / 4	2	39.00	43.30	81.00	90.00
11	16-QAM	1 / 2	2	52.00	57.80	108.00	120.00
12	16-QAM	3 / 4	2	78.00	86.70	162.00	180.00
13	64-QAM	2 / 3	2	104.00	115.60	216.00	240.00
14	64-QAM	3 / 4	2	117.00	130.00	243.00	270.00
15	64-QAM	5 / 6	2	130.00	144.40	270.00	300.00
16	BPSK	1 / 2	3	19.50	21.70	40.50	45.00
...	...	...	...	...	...	...	...
31	64-QAM	5 / 6	4	260.00	288.90	540.00	600.00

802.11n Primer, Whitepaper, AirMagnet, August 05, 2008.

Baseband/RF bandwidth; spectral efficiency of M-QAM:

$$\Delta f = \frac{1+\alpha}{2} R_s = \frac{1+\alpha}{2} \frac{R_b}{\log_2 M}$$
$$\rightarrow \eta = \frac{R_b}{\Delta f_{RF}} = \frac{\log_2 M}{1+\alpha} \ \left[\text{bit/s/Hz}\right] \quad (7.31)$$

<u>Complex form of QAM signal</u>:

$$x(t) = \text{Re}\left\{ m(t) e^{j\omega_c t} \right\}, \ \ m(t) = m_I(t) + j m_Q(t) \quad (7.32)$$

Signal constellation: via $a_i + jb_i$

$M = 2 \rightarrow$ BPSK, $M = 4 \rightarrow$ QPSK

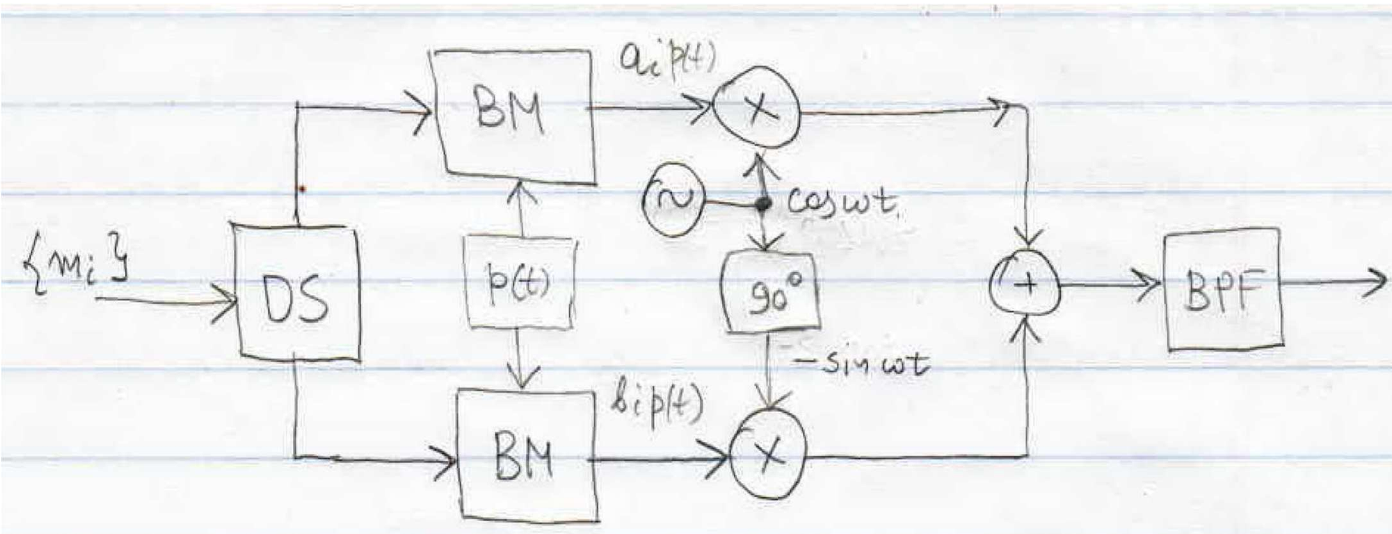
M-QAM $= \left(\sqrt{M} - \text{PAM}\right) \times \left(\sqrt{M} - \text{PAM}\right)$

<u>Demodulation</u>: via

$$m_I(t) = LPF\left\{ x(t) \cos \omega_c t \right\}$$
$$m_Q(t) = -LPF\left\{ x(t) \sin \omega_c t \right\} \quad (7.33)$$

+ baseband demodulation of $m_I(t)$, $m_Q(t)$ (separately, as $\sqrt{M} - \text{PAM}$)

QAM Modulator



BM = baseband modulator

DS = data splitter, $m_i \rightarrow \{a_i, b_i\}$

BPF = bandpass filter

BM = PAM modulator for $a_i p(t)$, or

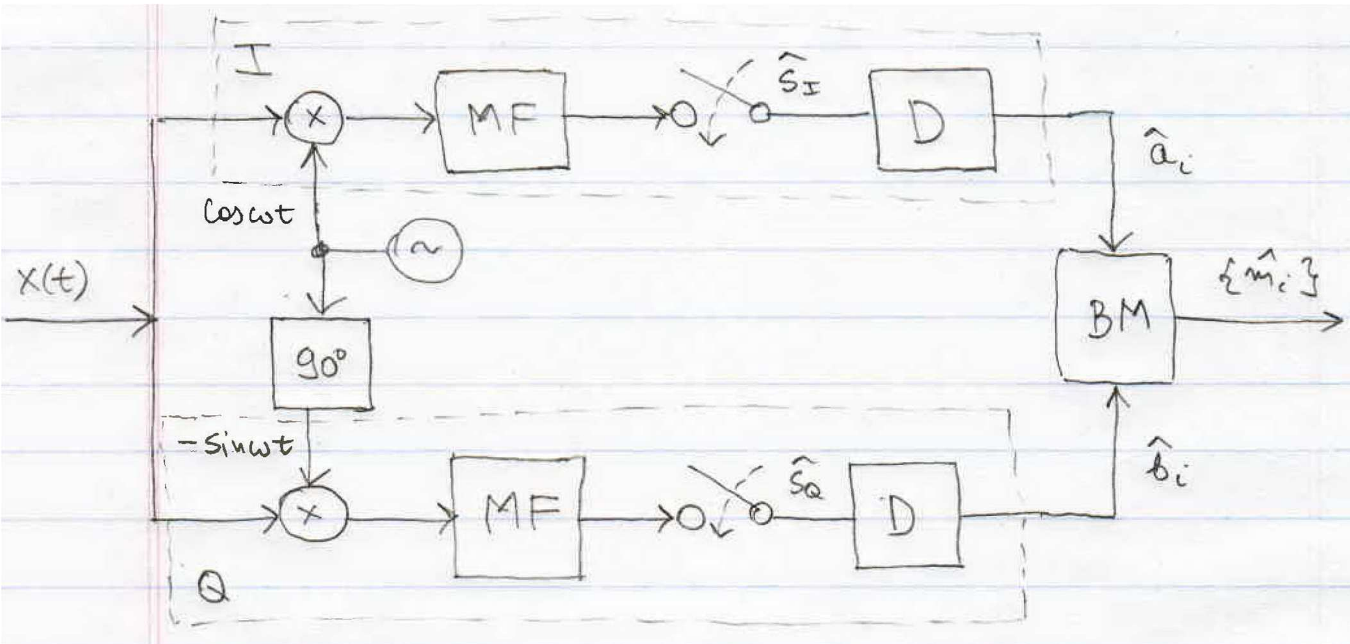
$$s_I(t) = \sum_i a_i p(t - iT); \; s_Q(t) = \sum_i b_i p(t - iT).$$

The RF QAM signal is (single pulse):

$$x(t) = a_i p(t)\cos\omega t - b_i p(t)\sin\omega t, \; 0 \le t \le T \qquad (7.34)$$

QAM Demodulator

Demodulator: down-conversion, MF + detection.



MF = matched filter $(\text{for } p(t))$.

BM = bit mapping, $(\widehat{a_i}, \widehat{b_i}) \rightarrow (0101...)$.

D = detector, $(\hat{s}_I, \hat{s}_Q) \rightarrow (\widehat{a_i}, \widehat{b_i})$.

I, Q = in-phase and quadrature channels.

QAM demodulator = I-PAM + Q-PAM demod.

In practice: $M = 8, 16, 64, ..., 1024$.

Bandwidth and Spectral Efficiency

Baseband (BB):

$$\text{sinc: } \Delta f = \frac{1}{2T_s}; \;\; \text{RC: } \Delta f = \frac{1+\alpha}{2T_s}; \;\; \text{rect: } \Delta f = \frac{1}{T_s}; \quad (7.35)$$

RC = raised-cosine pulse.; rect = rectangular pulse.

Passband or RF:

If DSB is employed:

$$\text{sinc: } \Delta f = \frac{1}{T_s}; \;\; \text{RC: } \Delta f = \frac{1+\alpha}{T_s}; \;\; \text{rect: } \Delta f = \frac{2}{T_s}; \quad (7.36)$$

i.e. $\Delta f_{RF} = 2\Delta f_{BB}$. If SSB, take 1/2 of DSB bandwidth.

Spectral efficiency (SE):

$$\text{SE: } \eta = \frac{R_b}{\Delta f} \quad [\text{bits/s/Hz}] \qquad (7.37)$$

i.e. how many b/s per unit bandwidth (Hz).

Spectral Efficiency

Assume RC pulse everywhere; if not, adjust according to (7.35).
General relationship:

$$R_b = R_s \log_2 M = \frac{\log_2 M}{T_s} \tag{7.38}$$

Baseband (BB):

M-PAM:

$$\eta = \frac{R_b}{\Delta f} = \frac{2\log_2 M}{1+\alpha} \quad [\text{b/s/Hz}], \tag{7.39}$$

Passband or RF:

M-PAM (2-PAM = BPSK):

$$\text{DSB: } \eta = \frac{\log_2 M}{1+\alpha}, \quad \text{SSB: } \eta = \frac{2\log_2 M}{1+\alpha} \tag{7.40}$$

M-QAM (4-QAM = QPSK):

$$\text{DSB: } \eta = \frac{\log_2 M_{QAM}}{1+\alpha} = \frac{2\log_2 M}{1+\alpha}, \tag{7.41}$$

if $M_{QAM} = M^2$, i.e.

$$\text{QAM} = \underbrace{M-\text{PAM}}_{I} \times \underbrace{M-\text{PAM}}_{Q}$$

Q.: which is better?

Summary

- DPSK.
- QPSK.
- QAM.
- Signal constellation.
- Modulators/demodulators.
- Bandwidth and spectral efficiency.
- BER, SER.

Reading:

- Rappaport, Ch. 6 (6.1-6.10).
- L.W. Couch II, Digital and Analog Communication Systems, 7th Edition, Prentice Hall, 2007. (other editions are OK as well)
- Other books (see the reference list).

Note: Do not forget to do end-of-chapter problems. Remember the learning efficiency pyramid!