Digital Communication Systems

Review of ELG3175:

- Fourier Transform/Series
- Spectra
- Modulation/demodulation, baseband/bandpass signals
- Digital signaling
- Intersymbol interference (ISI)

Use your notes/textbook and the Appendix.

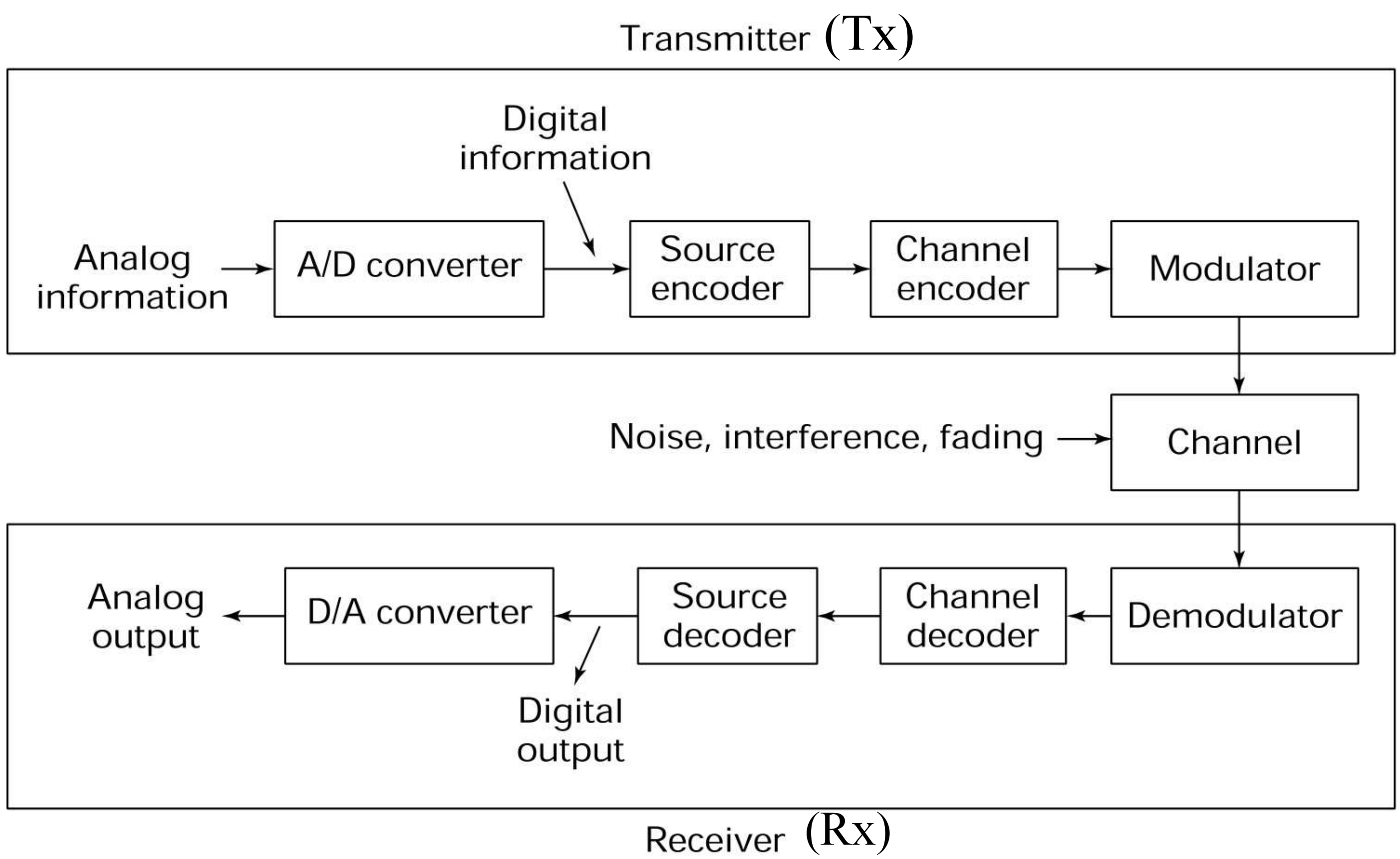
Generic modulation formats:

- AM, PM and FM
- properties

Why digital?

- better spectral efficiency
- better noise immunity (quality)
- allows DSP (not expensive)

Block diagram of a digital communication system



P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002.

Digital Communications

<u>Digital source</u>: select a message m from a set of possible messages $\{m_1, m_2 ....m_N\}$ (alphabet).

<u>Digital modulator</u>: each message m_i is represented via a corresponding (time-limited) symbol $s_i(t),\ 0 \le t \le T_s$.

$$m_i \to s_i(t),\ 0 \le t \le T \tag{7.1}$$

Transmitted sequence:

$$x(t) = \sum_i s_i(t - iT), \tag{7.2}$$

Pulse-amplitude modulation (PAM): $s_i(t) = a_i p(t)$,

$$x(t) = \sum_i a_i p(t - iT) \tag{7.3}$$

i.e. each message is encoded by amplitude a_i: $m_i \to a_i$.

<u>Baseband system</u>: each $s_i(t)$ or $p(t)$ is a baseband pulse.

<u>Bandpass (RF) system</u>: pulses are bandpass (or RF).

Baseband Digital Signaling

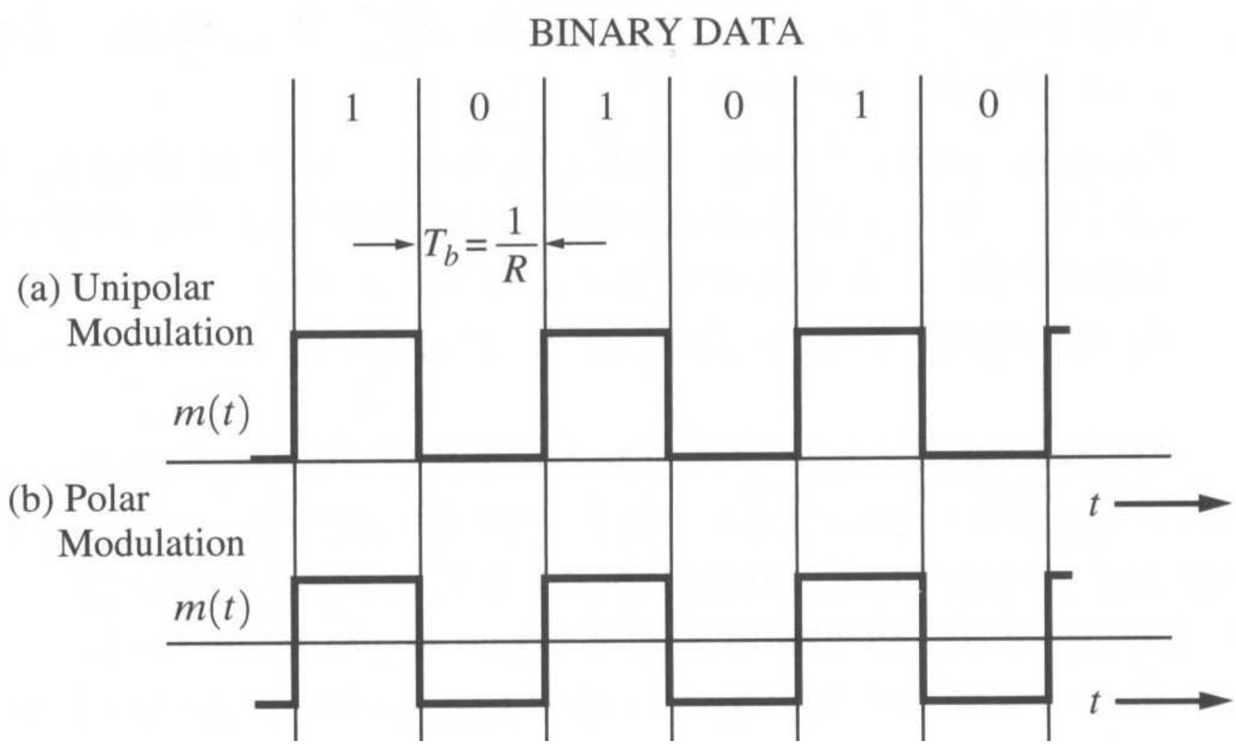
Also known as "line coding": each $s_i(t)$ or $p(t)$ is a baseband pulse.

Examples:

- Binary data representation: 0 or 1.
- Unipolar modulation: high level (e.g., 5V) / zero, or 1/0
- Bipolar modulation: +high level / -high level, or +1/-1

$$R = \frac{1}{T_b} = \text{bit (data) rate [bit/s]}$$

$$x(t) = \sum_i a_i p(t - iT)$$



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Spectral Characteristics

Spectrum is a very limited and expensive resource in wireless communications.

Wireless systems are band-limited.

How to select $p(t)$?

- Rectangular pulse,
- Sinc pulse,
- Raised cosine pulse,
- Others.

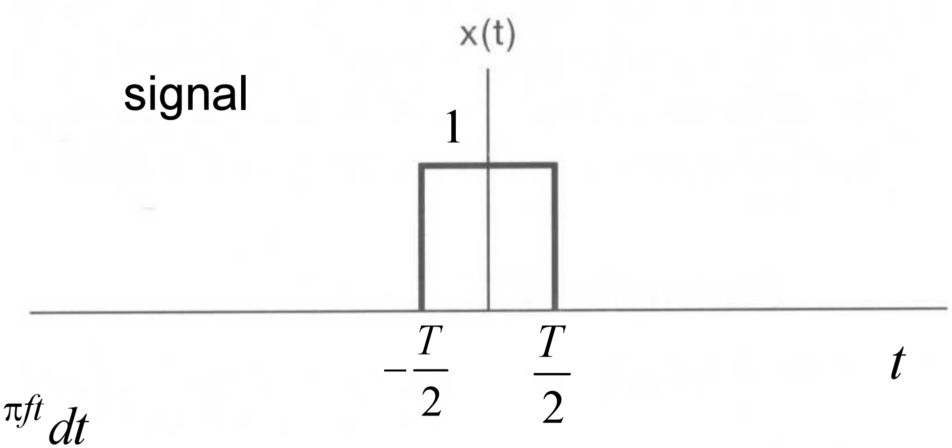
Example: Rectangular Pulse

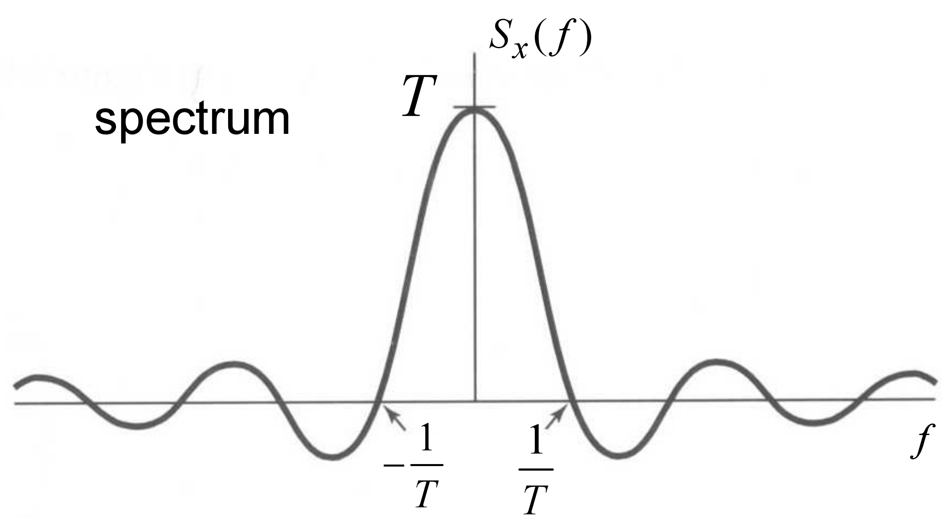
$$x(t) = \Pi(t) = \begin{cases} 1, |t| < T/2 \\ 0, |t| > T/2 \end{cases}$$

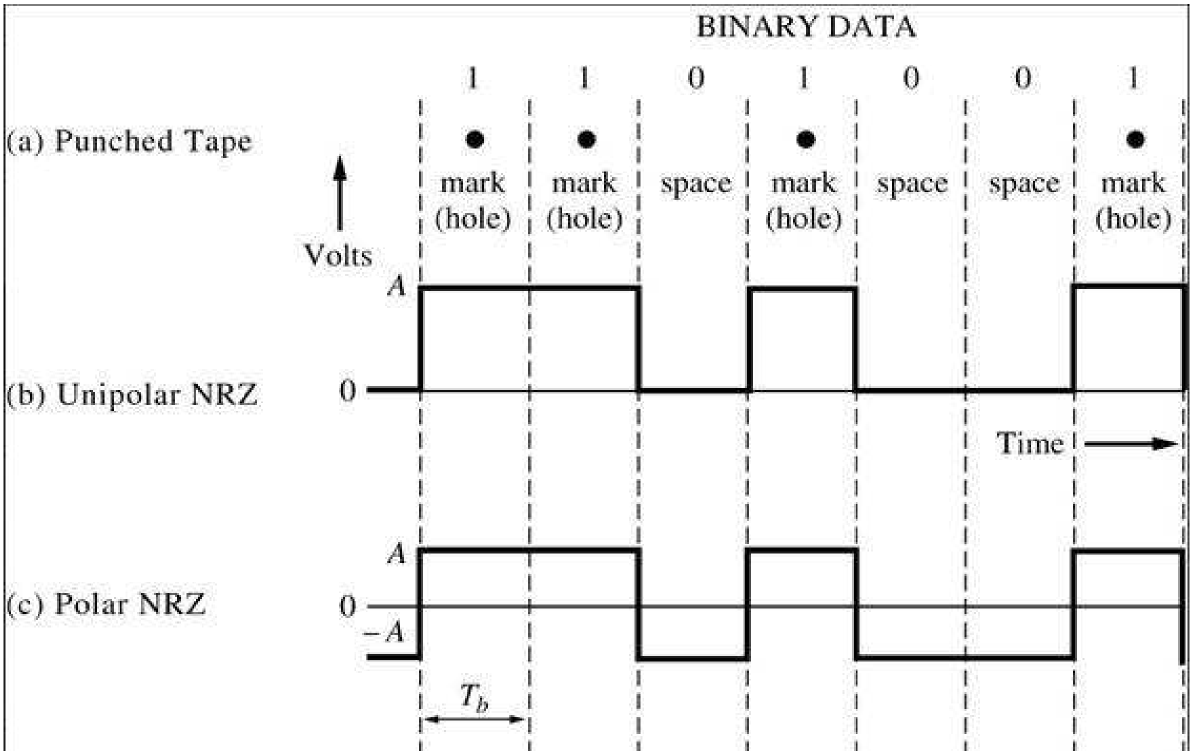
$$S_x(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

$$S_x(f) = T \frac{\sin \pi fT}{\pi fT} = T \mathrm{sinc}(fT)$$

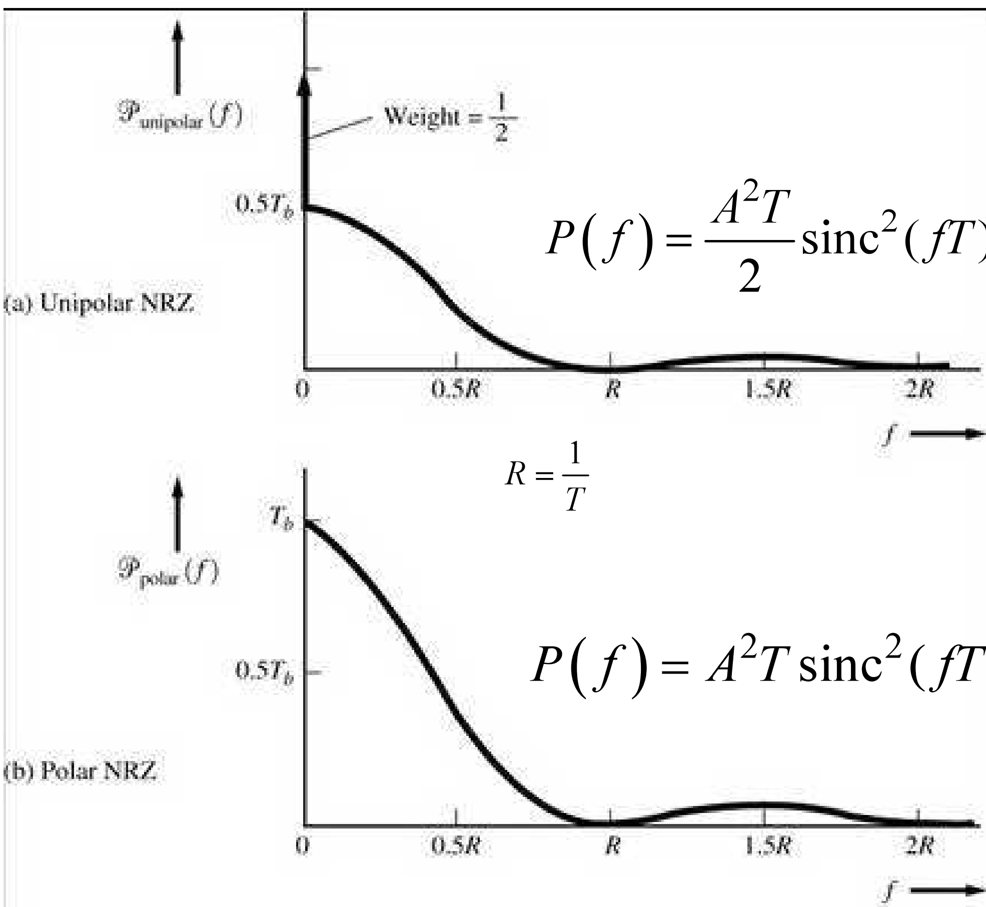
$\Delta f = ?$







L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.



$$P(f) = \frac{A^2 T}{2}\operatorname{sinc}^2(fT) + \frac{A^2}{2}\delta(f)$$

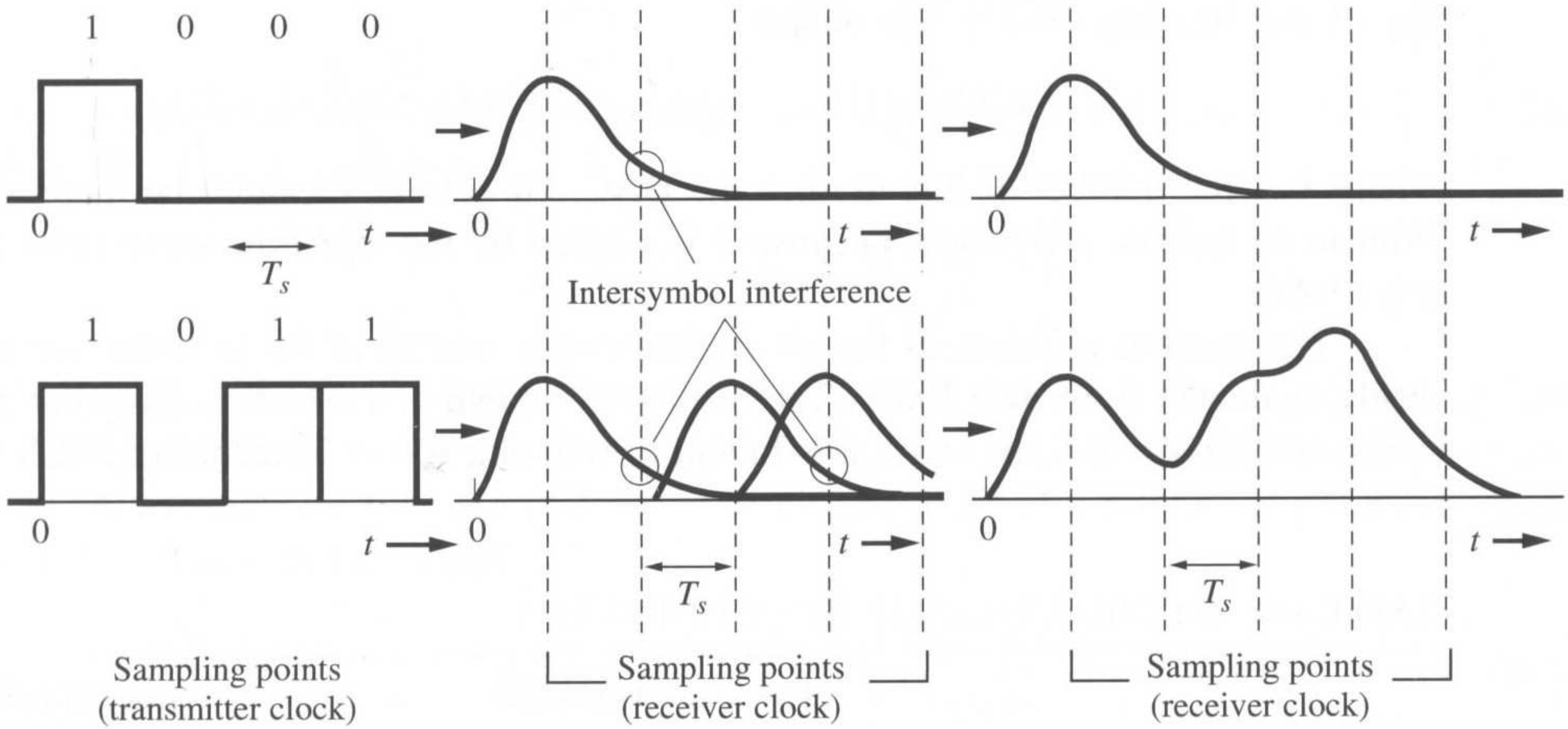
$$R = \frac{1}{T}$$

$$P(f) = A^2 T \operatorname{sinc}^2(fT)$$

$$\Delta f = ?$$

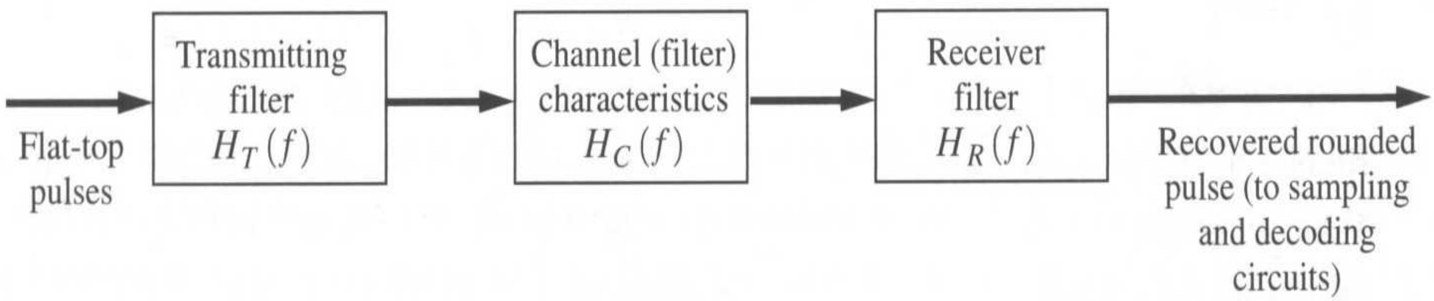
Digital Signaling over Bandlimited Channels: Pulse Shaping and ISI

Practical channels are band-limited -> pulses spread in time and are smeared into adjacent slots. This is intersymbol interference (ISI).



Transmission over a Band-Limited Channel

baseband transmission system



Input PAM signal:

$$x(t) = \sum_n a_n s_T(t - nT) \tag{7.4}$$

Output signal:

$$y(t) = \sum_n a_n s_R(t - nT), \tag{7.5}$$

where $s_R(t) = s_T(t) * h_T(t) * h_C(t) * h_R(t) = s_T(t) * h(t)$ is the Rx symbol.

Sampled (at $t=mT$) output $y_m = y(mT)$:

$$y_m = \sum_n a_n s_{m-n} = a_m s_0 + \sum_{n \neq m} a_n s_{m-n}, \tag{7.6}$$

where T = symbol interval, $s_m = s_R(mT)$.

Pulse Shaping to Eliminate ISI

- Nyquist (1928) discovered 3 methods to eliminate ISI:
  - zero ISI pulse shaping,
  - controlled ISI (eliminated later on by, say, coding),
  - zero average ISI (negative and positive areas under the pulse in an adjacent interval are equal).

- Zero ISI pulse shaping:

$$s(nT) = \begin{cases} s_0, & n = 0 \\ 0, & n \neq 0 \end{cases} \Rightarrow$$
$$y_m = a_m s_0 + \sum_{n \neq m} a_n s_{m-n} \Rightarrow y_m = a_m s_0 \tag{7.7}$$

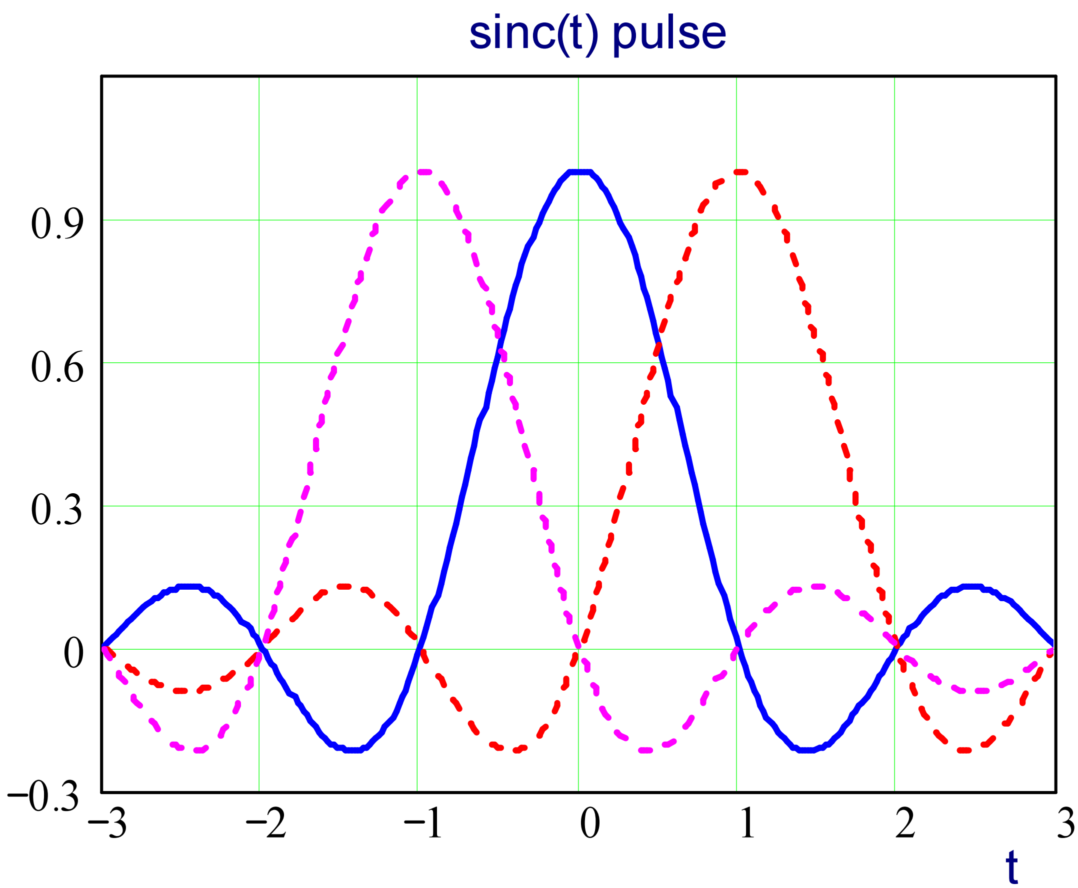
- Pulse shapes:
  - rectangular pulse
  - sinc pulse
  - raised cosine pulse

Example: sinc pulse,

$$s(t) = \mathrm{sinc}(Rt) \Rightarrow s(nT) = \mathrm{sinc}(n) = 0,\ n \neq 0$$

where R=1/T = transmission (symbol) rate [symbols/s].

Zero ISI: sinc Pulse



$$s(t) = \mathrm{sinc}(Rt) \Rightarrow s(nT) = \mathrm{sinc}(n) = 0,\ n \neq 0,$$

$R = 1/T =$ transmission rate [symbols/s].

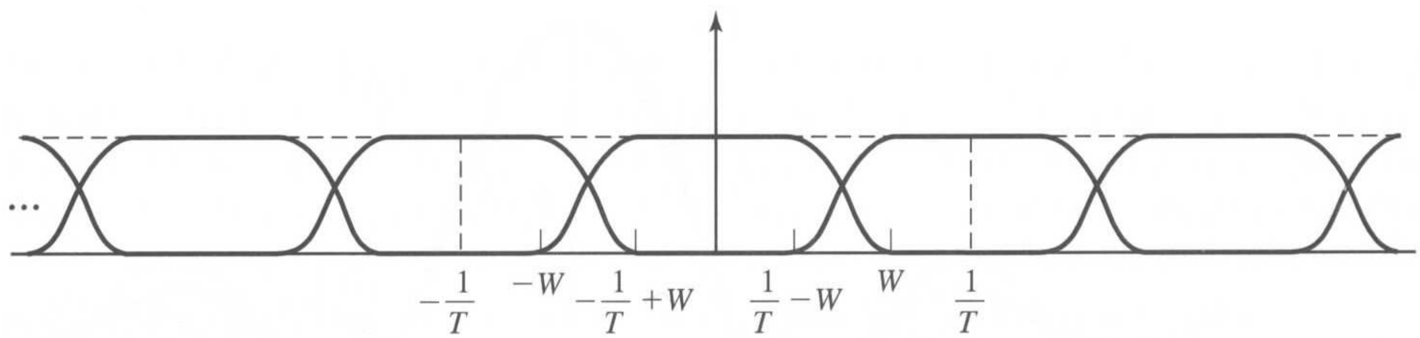
Sinc pulse allows to eliminate ISI at sampling instants.

However, it has some (2) **serious drawbacks**.

Nyquist Criterion for Zero ISI

- Provides a generic solution to the zero ISI problem.
- A necessary and sufficient condition for zero ISI:

$$\sum_{m=-\infty}^{\infty} S_s\left(f+\frac{m}{T}\right)=T \qquad (7.8)$$



$W = F_{\max}$

Consider a channel badndlimited to [0,F_{max}]. There are 3 possible cases:

1) $1/T = R > 2F_{\max}$ → no way to eliminate ISI.

2) $R = 2F_{\max}$ → only $\mathrm{sinc}(Rt)$ eliminates ISI. The highest possible transmission rate f_0 for transmission with zero ISI is $2F_{max}$, not F_{max} (what a surprise!)

3) $R < 2F_{\max}$ → many signals may eliminate ISI in this case.

Sinc pulse: $\Delta f = F_{\max} = \dfrac{R}{2} = \dfrac{1}{2T}$. Q: rectangular pulse?

The Raised Cosine Pulse

When $R < 2F_{\max}$, raised cosine pulse is widely used.

Its spectrum is

$$S_{rc}(f) = \begin{cases} T, & 0 \le |f| \le \frac{1-\alpha}{2}R \\ \frac{T}{2}\left[1+\cos\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2}R\right)\right], & \frac{1-\alpha}{2}R \le |f| \le \frac{1+\alpha}{2}R \\ 0, & |f| > \frac{1+\alpha}{2}R \end{cases} \tag{7.9}$$

where $0 \le \alpha \le 1$ is roll-off factor,

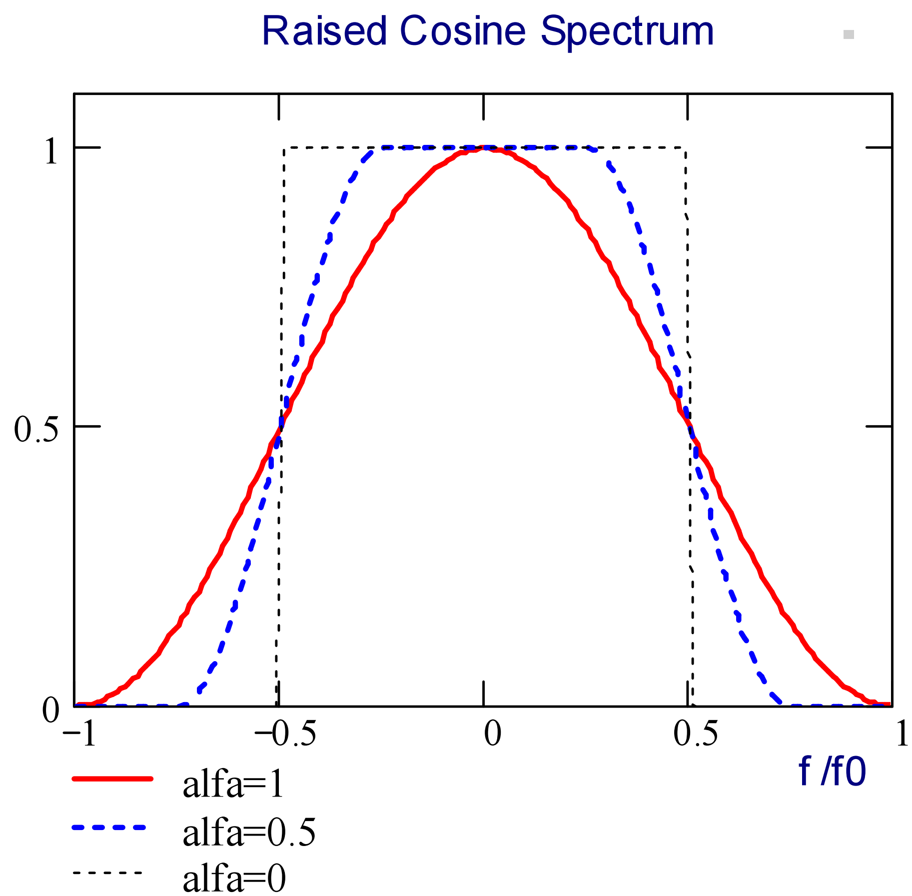
The bandwidth above the Nyquist frequency $f_0/2$ is called excess bandwidth.

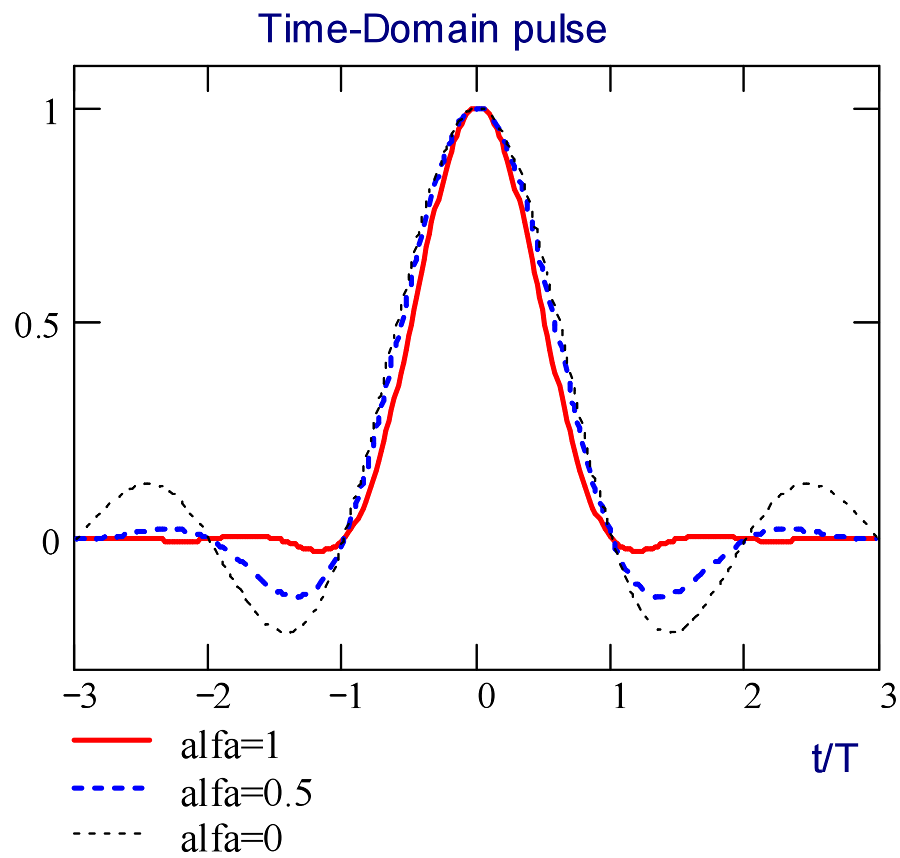
Time-domain waveform (shape) of the pulse is

$$s(t) = \mathrm{sinc}(t/T)\frac{\cos(\pi\alpha t/T)}{1-4\alpha^2 t^2/T^2} \tag{7.10}$$

Baseband bandwidth = ?

RF (bandpass) bandwidth = ?





Note that: (1) $\alpha = 0$, pulse reduces to $\mathrm{sinc}(Rt)$ and $R = f_0 = 2F_{\max}$

(2) $\alpha = 1$, symbol rate $R = F_{\max}$

(3) tails decay as t^3 -> mistiming is not a big problem

(4) smooth shape of the spectrum -> easy to design filter

Bandpass (RF) Digital Signaling

How to transform the baseband signal to the RF frequencies (for efficient wireless transmission)?

Most popular approach: combine the baseband signaling with the DSB-SC modulation:

$$x(t) = s(t) \cdot A_c \cos(\omega_c t) \tag{7.11}$$

i.e. multiply the digitally-modulated message (baseband) $s(t) = \sum_i a_i p(t - iT)$ with the carrier $A_c \cos(\omega_c t)$.

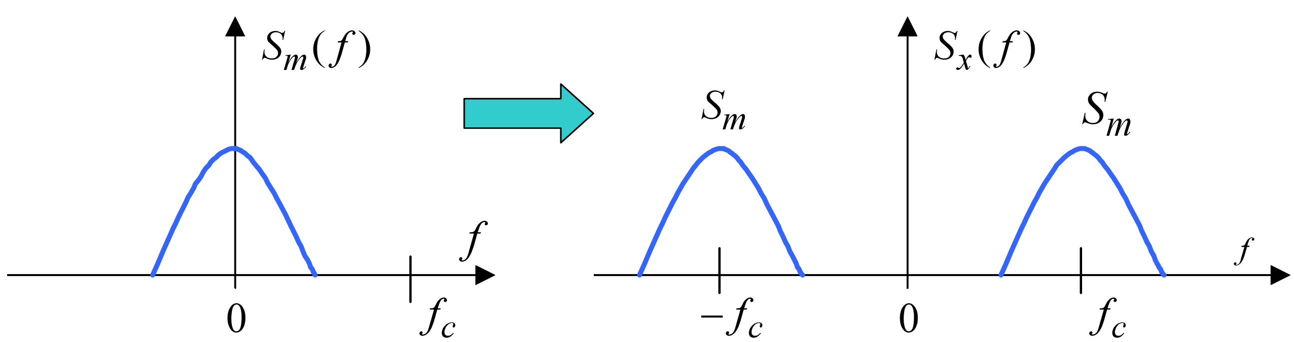
Compare to the analog DSB-SC:

$$x(t) = m(t) \cdot A_c \cos(\omega_c t) \tag{7.12}$$

where $m(t)$ is the message: (7.11) = (7.12) when $m(t) = s(t)$, i.e. $s(t)$ serves as an analog message to DSB-SC.

Spectrum of the modulated bandpass signal:

$$S_x(f) = \frac{A_c}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right] \tag{7.13}$$



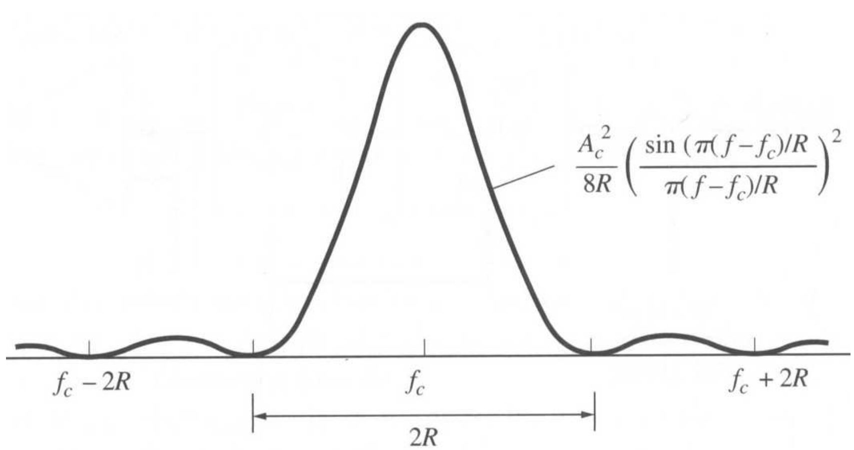
Binary Phase Shift Keying (BPSK)

Assume rectangular pulse shape and random binary message:

$$s(t) = \sum_i a_i p(t - iT),\ a_i = \pm 1,\ p(t) = \Pi(t) \tag{7.14}$$

Then, the PSD of the modulated signal $x(t) = s(t) \cdot A_c \cos(\omega_c t)$ is

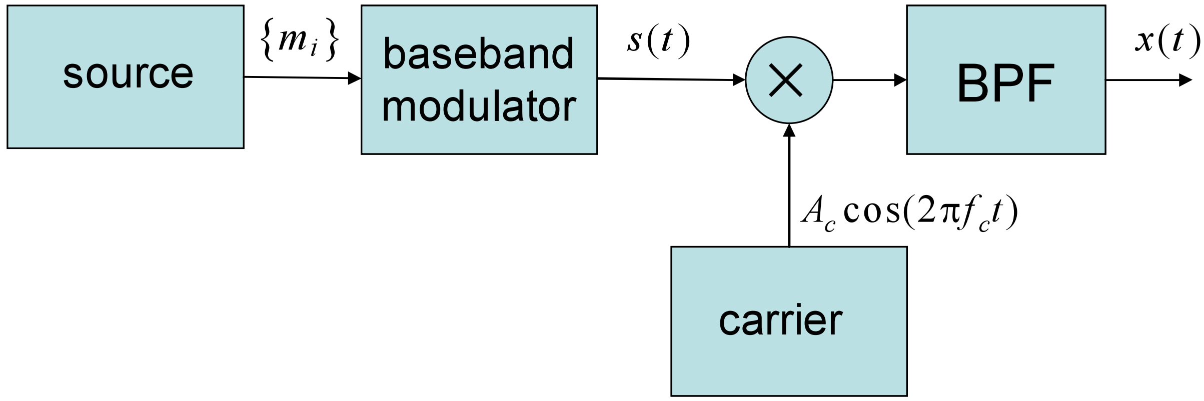
$$P_{BPSK}(f) = \frac{A_c^2 T}{2} \operatorname{sinc}^2((f - f_c)T) \tag{7.15}$$



Bandwidth $\Delta f = ?$

RF vs. baseband badwidth?

Digital DSB-SC Transmitter



Digital baseband modulation: $m_i \rightarrow a_i,\ s(t) = \sum_i a_i p(t - iT)$

Baseband (digitally-modulated) signal: $s(t)$

Bandpass (RF) modulated signal: $x(t) = s(t) \cdot A_c \cos(\omega_c t)$

BPF: eliminates out-of-band emission

* Demodulation (detection) ?

Summary

- Review of digital communications.
- Baseband digital modulation (line coding).
- Band-limited systems and intersymbol interference.
- Bandpass (RF) modulation.
- Spectra.
- Modulators/demodulators (detectors), block diagrams.
- Review of FT & modulation from ELG3175.

Reading:

- Rappaport, Ch. 6 (6.1-6.10).
- L.W. Couch II, Digital and Analog Communication Systems, 7th Edition, Prentice Hall, 2007. (other editions are OK as well)
- Other books (see the reference list).

Note: Do not forget to do end-of-chapter problems. Remember the learning efficiency pyramid!

Appendix 1:

Baseband/Bandpass Signals and Modulation

In what follows is a review of baseband/bandpass signals and modulation (DSB-SC), corresponding modulators and detectors as well as basic digital modulation techniques. This material was covered in ELG3175 and you are advised to consult your notes and the textbook of that course.

A good textbook to follow is this:

L.W. Couch II, Digital and Analog Communication Systems, 7th Edition, Prentice Hall, 2007. (other editions are OK as well)

Lecture 5

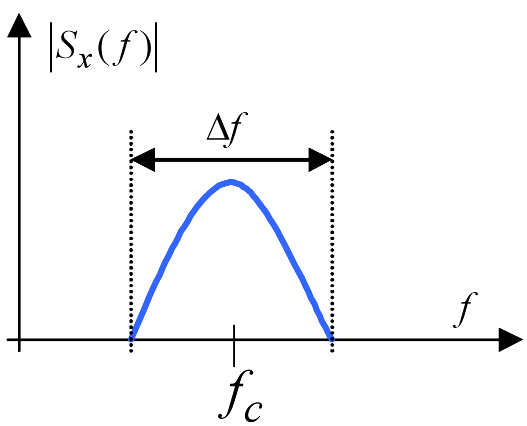
Baseband & Bandpass Signals

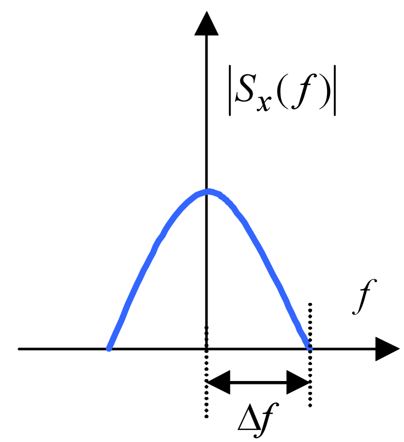
- *Baseband* (lowpass) signal: spectrum is (possibly) nonzero around the origin (f=0) and zero (negligible) elsewhere:

$$\boxed{S_x(f) = 0, \quad |f| > f_{\max}}$$

- *Bandpass* (narrowband) signal: spectrum is (possibly) nonzero around the carrier frequency f_c and zero (negligible) elsewhere:

$$\boxed{S_x(f) = 0, \quad |f - f_c| > B}$$

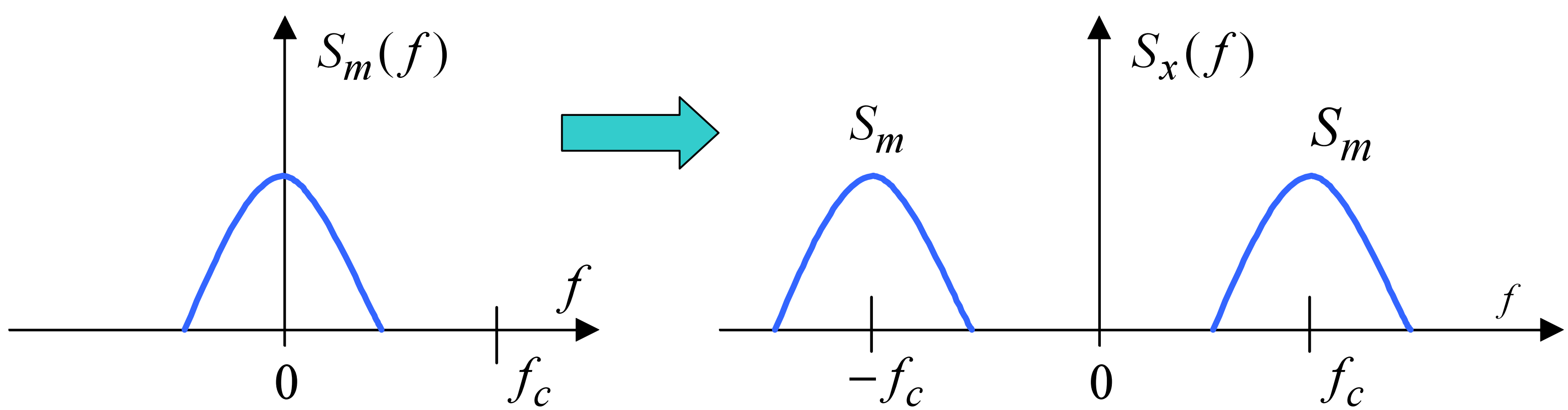




DSB-SC: General Case

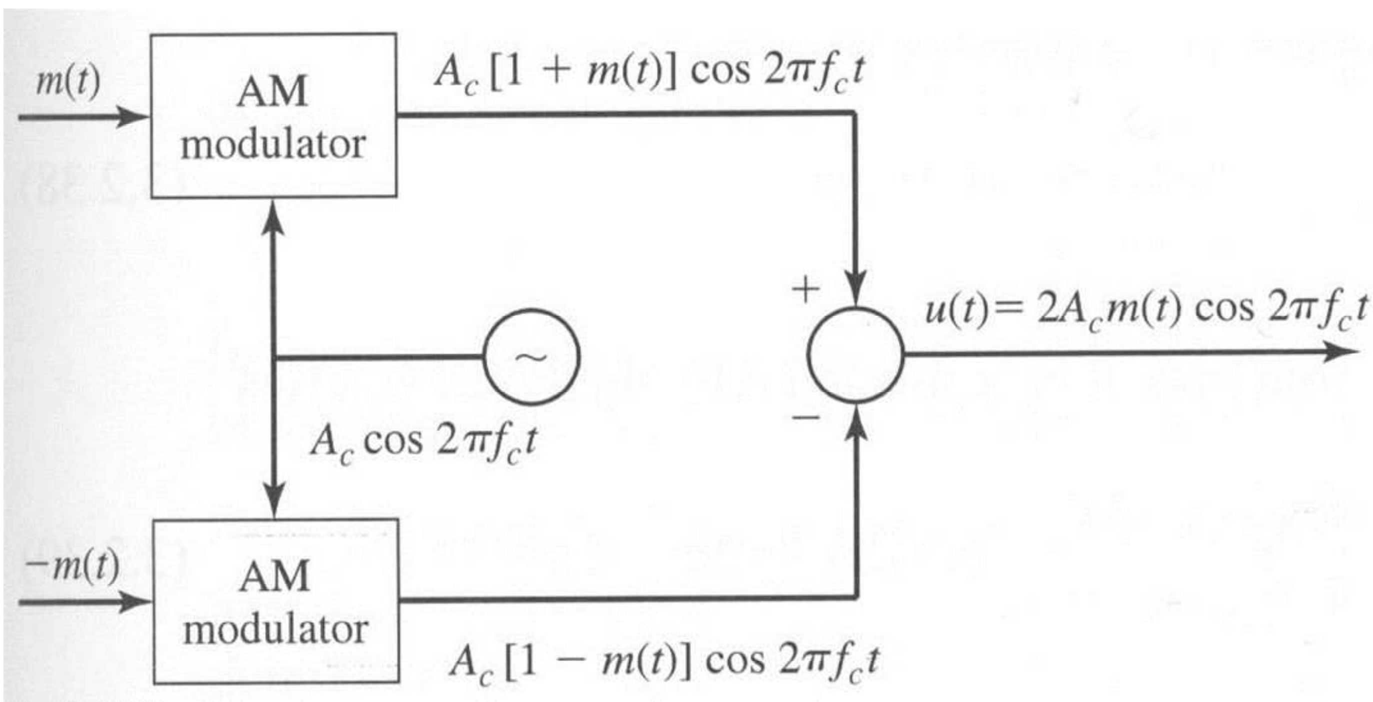
- DSB-SC signal: $\boxed{x(t) = A_c m(t)\cos(2\pi f_c t)}$
- Spectrum: $\boxed{S_x(f) = \frac{A_c}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right]}$

- What do you see on a spectrum analyzer?
- Bandwidth ? Power efficiency? PSD?



Generation of DSB-SC

- Generation:
  - Mixer. Not practical in many cases.
  - Filtered conventional AM. Not practical.
- Balanced modulator:



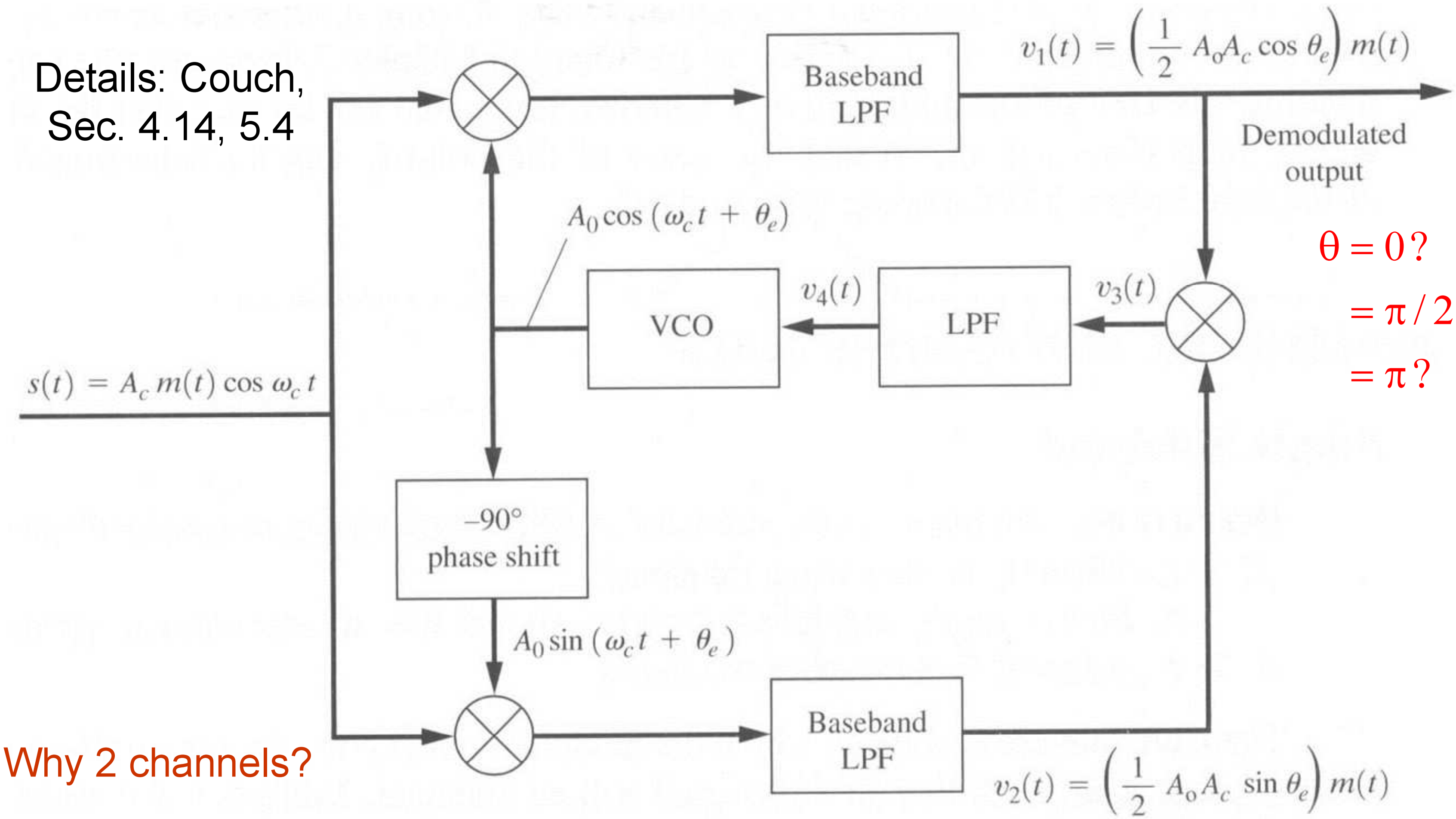
J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Lecture 6

Demodulation of DSB-SC

- Demodulation - Costas loop:

Details: Couch, Sec. 4.14, 5.4



$\theta = 0$?

$= \pi/2$?

$= \pi$?

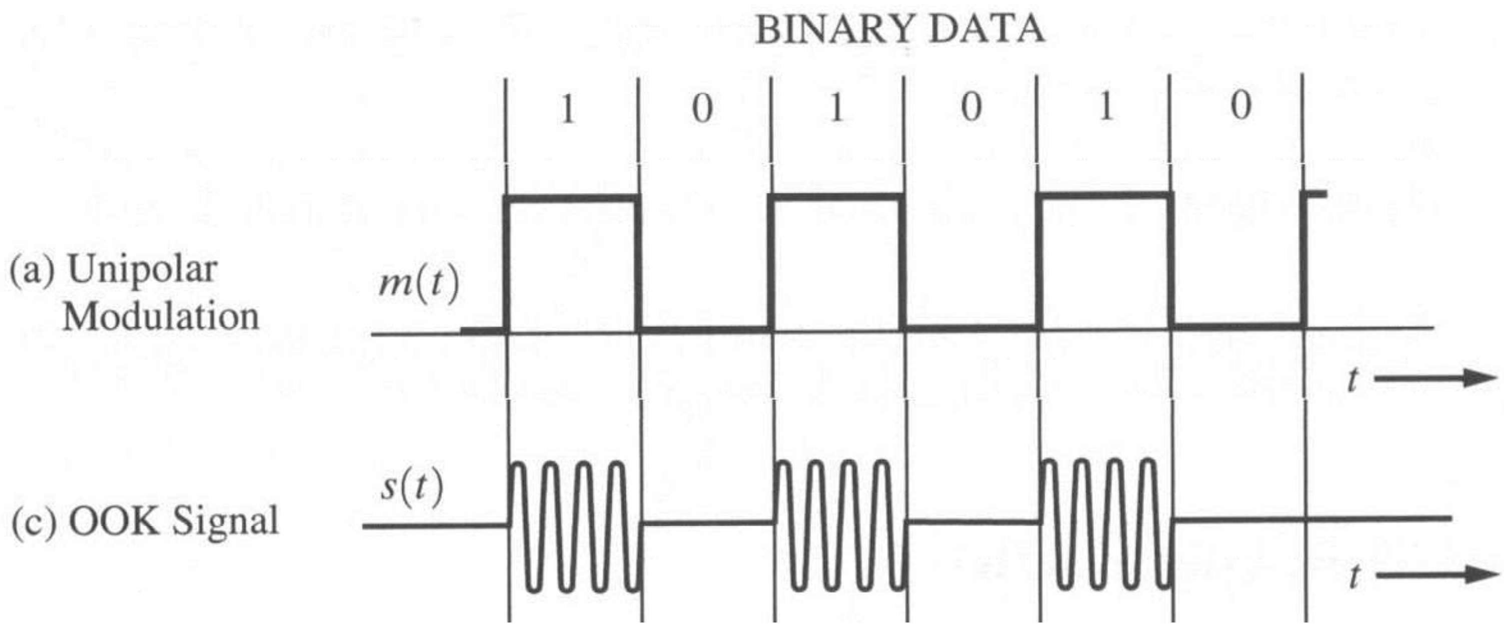
Why 2 channels?

L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Lecture 9

Amplitude Shift Keying (ASK)

- Switch on-off the carrier:



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

- Signal representation:
- Is it similar to something?
- Signal spectrum?

$$\boxed{x(t) = A_c m(t)\cos(2\pi f_c t)}$$

binary ASK: $m(t) = 1 \text{ or } 0$

general case: a fixed number of levels

Lecture 9

Amplitude Shift Keying (ASK)

- Signal spectrum (FT):

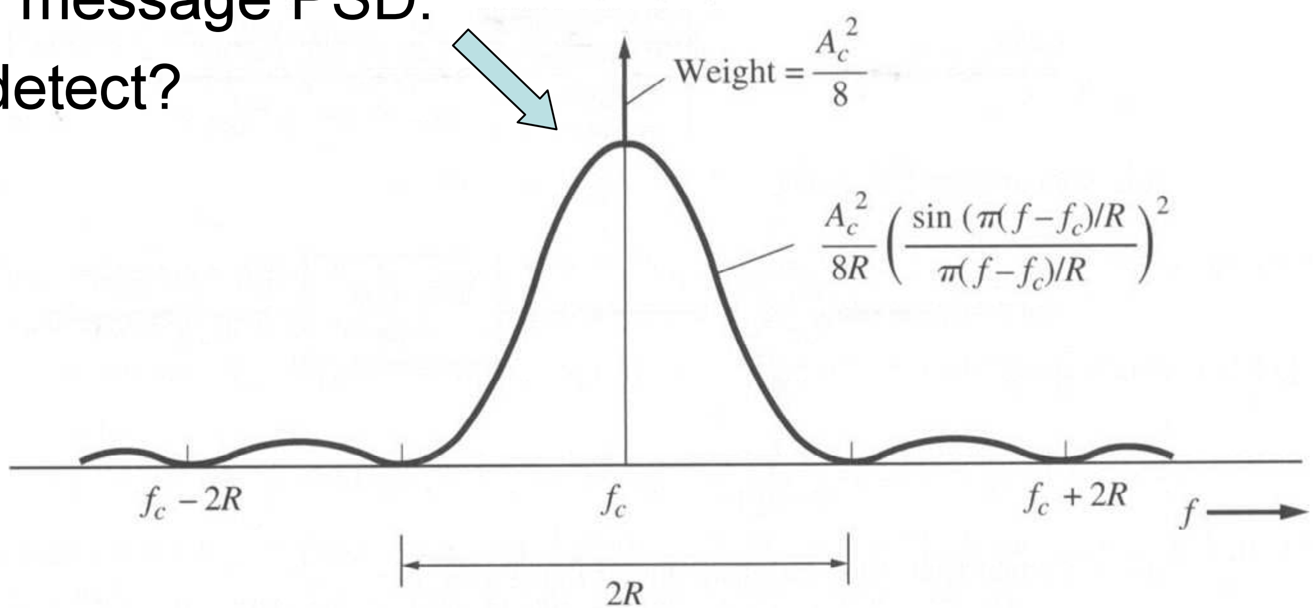
$$\boxed{S_x(f)=\frac{A_c}{2}\left[S_m(f-f_c)+S_m(f+f_c)\right]}$$

- Square-wave message:

$$S_m(f)=\sum_{n=-\infty}^{+\infty} c_n \delta(f-nf_0),$$

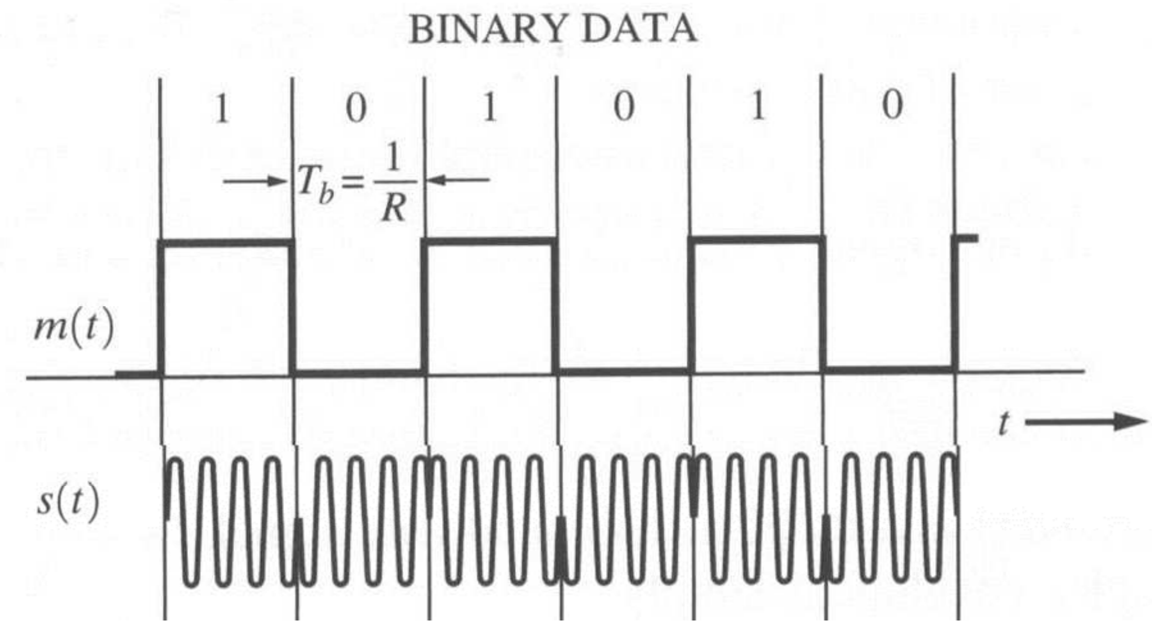
$$c_n=\frac{1}{2}\operatorname{sinc}\left(\frac{n}{2}\right)e^{j\frac{\pi}{2}n},\ f_0=\frac{1}{2T_b}=\frac{R}{2}$$

- Random message PSD:
- How to detect?



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Binary Phase Shift Keying



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

- BPSK signal representation: $\boxed{x(t) = A_c \cos\left(\omega_c t + \Delta\varphi \cdot m(t)\right)}$

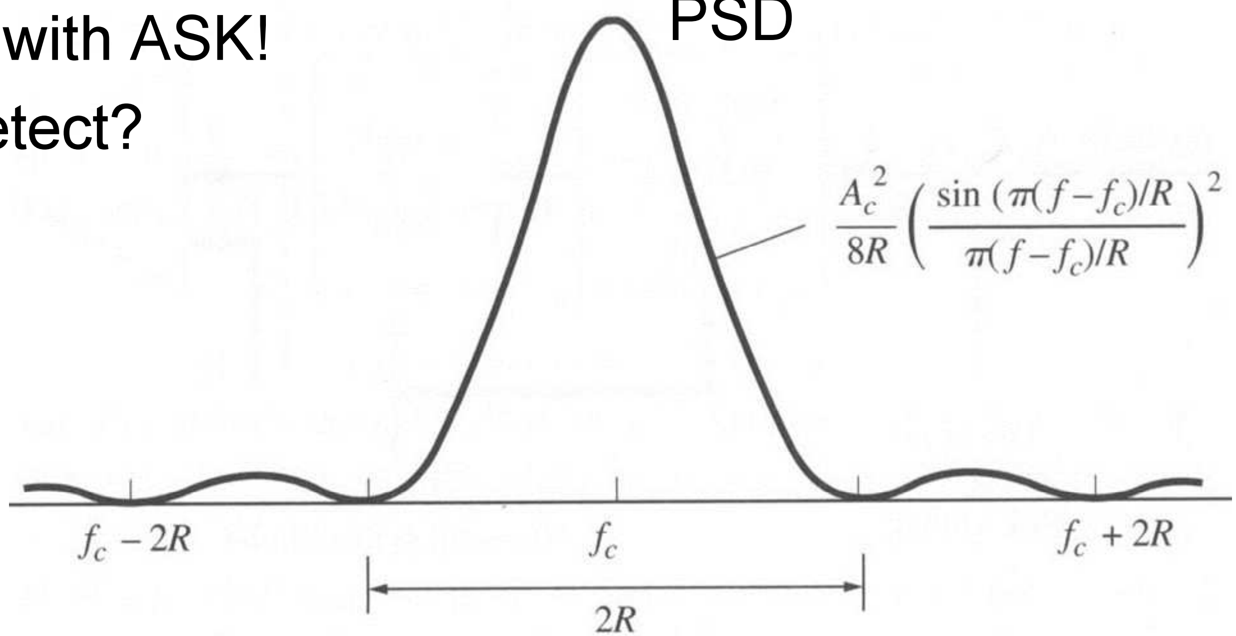
  where $m(t) = \pm 1$ is bipolar message.
- Another form of the BPSK signal -> $$\boxed{\begin{aligned} x(t) = {} & A_c \cos\Delta\varphi \cos\omega_c t - \\ & - A_c \sin\Delta\varphi \cdot m(t) \sin\omega_c t \end{aligned}}$$

Lecture 9

Binary Phase Shift Keying

- Digital modulation index: $\boxed{\beta_d = \frac{2\Delta\varphi}{\pi}}$
- Important special case $\beta_d = 1$ (random message)
- Compare with ASK!
- How to detect?

PSD

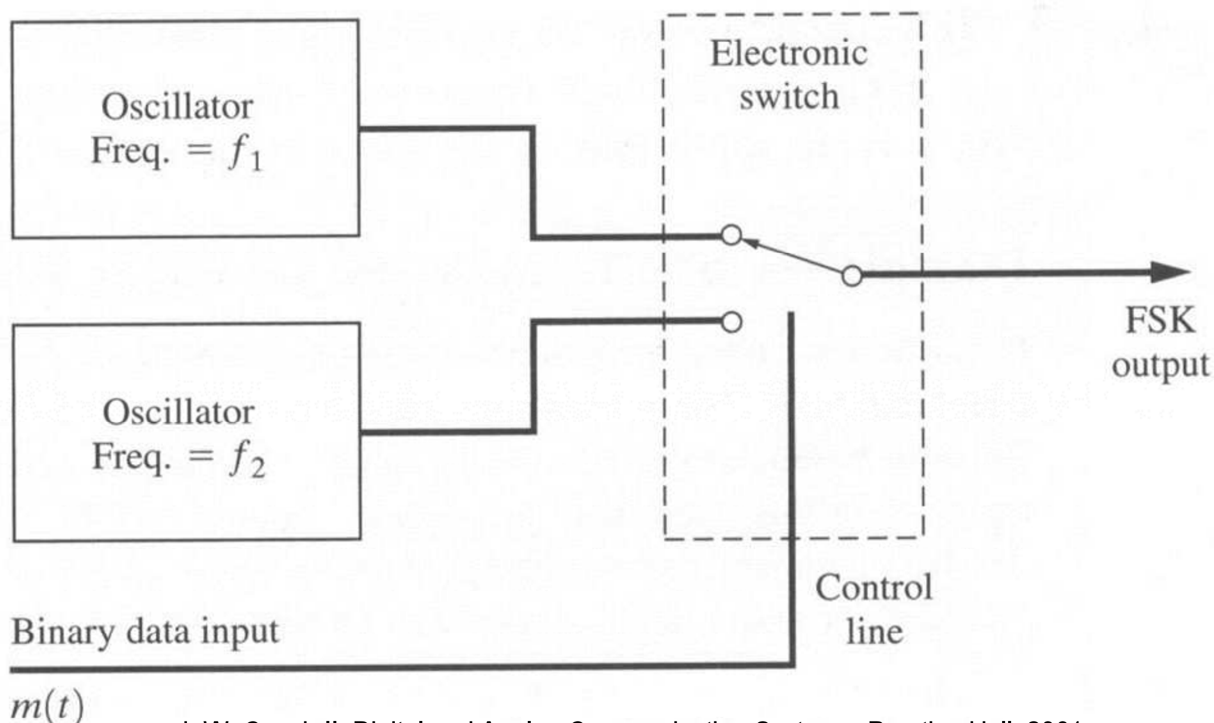


L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Lecture 9

Frequency Shift Keying

- Discontinuous FSK: $x(t) = \begin{cases} A_c \cos(\omega_1 t + \theta_1), \text{ mark (1)} \\ A_c \cos(\omega_2 t + \theta_2), \text{ space (0)} \end{cases}$



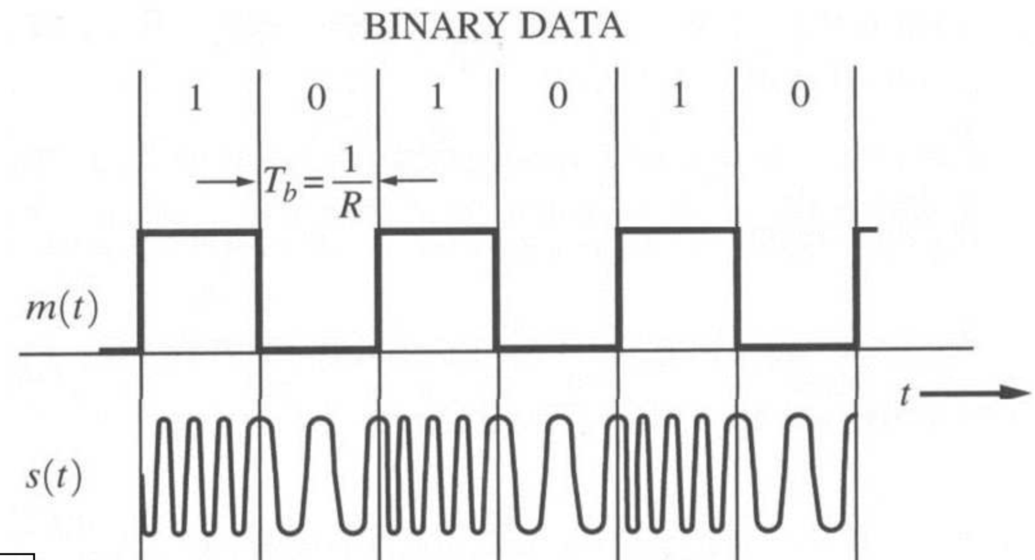
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Not popular (spectral noise + PLL problems)

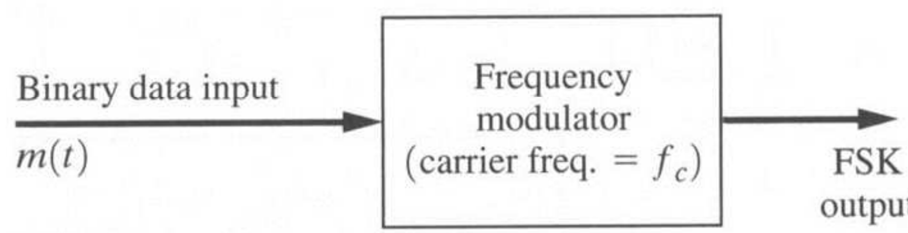
Lecture 9

Frequency Shift Keying

- Continuous FSK:



$$x(t) = A_c \cos\left(\omega_c t + \Delta\Omega \int_0^t m(\tau) d\tau\right)$$



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Appendix 2:

Autocorrelation and Power/Energy Spectral Density

This material was covered in ELG3175 and ELG3126. You are advised to consult your notes and the textbook of that course.

A good textbook to follow is this:

L.W. Couch II, Digital and Analog Communication Systems, 7th Edition, Prentice Hall, 2007. (other editions are OK as well)

Power and Energy

- Power P_x & energy E_x of signal x(t) are:

$$\boxed{P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt}$$

$$\boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

- Energy-type signals: $E_x < \infty$
- Power-type signals: $0 < P_x < \infty$
- Signal *cannot* be both energy & power type!
- Signal energy: if x(t) is voltage or current, E_x is the energy dissipated in 1 Ohm resistor
- Signal power: if x(t) is voltage or current, P_x is the power dissipated in 1 Ohm resistor.

Lecture 3

Energy-Type Signals (summary)

- Signal energy in time & frequency domains:

$$\boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |S_x(f)|^2 df = \frac{1}{2\pi}\int_{-\infty}^{\infty} |S_x(\omega)|^2 d\omega}$$

- Energy spectral density (energy per Hz of bandwidth):

$$\boxed{E_x(f) = |S_x(f)|^2}$$

- ESD is FT of autocorrelation function:

$$\boxed{R_x(\tau) = \int_{-\infty}^{+\infty} x(t)x^*(t-\tau)\,dt \leftrightarrow E_x(f)} \qquad \boxed{R_x(0) = E_x}$$

Power-Type Signals: PSD

- Definition of the power spectral density (PSD) (power per Hz of bandwidth):

$$\boxed{P_x(f) = \lim_{T\to\infty} \frac{|S_T(f)|^2}{T} \Rightarrow P_x = \int_{-\infty}^{\infty} P_x(f)\,df < \infty}$$

- where $x_T(t)$ is the truncated signal (to [-T/2,T/2]),

$$x_T(t) = x(t)\Pi\left(\frac{t}{T}\right) = \begin{cases} x(t), & -T/2 \le t \le T/2 \\ 0, & \text{otherwise} \end{cases}$$

- and $S_T(f)$ is its spectrum (FT),

$$S_T(f) = FT\{x_T(t)\}$$

Lecture 3

Power-Type Signals

- Time-average autocorrelation function:

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\, x^*(t-\tau)\, dt$$

- Power of the signal:

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt = R_x(0)$$

- Wiener-Khintchine theorem :

$$P_x = \int_{-\infty}^{\infty} P_x(f)\, df \Rightarrow P_x(f) = FT\{R_x(\tau)\}$$

Periodic Signals

- Power of a periodic signal:

$$\boxed{P_x = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2 = R_x(0)} \leftarrow x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$$

- Autocorrelation function:

$$\boxed{R_x(\tau) = \frac{1}{T}\int_T x(t)\, x^*(t-\tau)\, dt = \sum_{n=-\infty}^{\infty} |c_n|^2 e^{jn\omega_0\tau}}$$

- Power spectral density (PSD):

$$\boxed{P_x(f) = FT\{R_x(\tau)\} = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta\left(f - \frac{n}{T}\right)}$$

Prove these properties!