Frequency-Selective and Time-Varying Channels

Amplitude fluctuations are not the only effect.

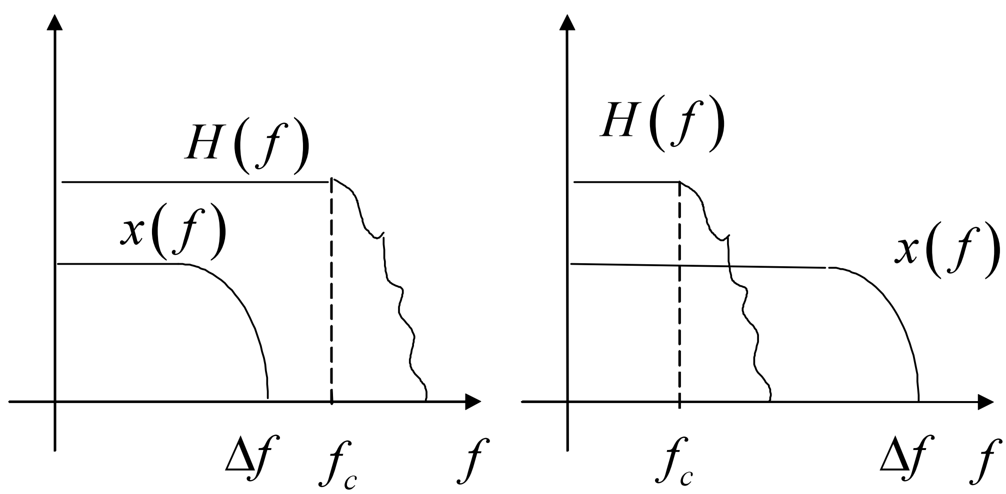
Wireless channel can be frequency selective (i.e. not flat) and time-varying.

Frequency –flat/frequency-selective channels

- Frequency response of the channel:

$$h(\tau) \xleftrightarrow{FT} H(f) \tag{5.1}$$

Channel as a linear filter



$H(f)$ - channel frequency response

$X(f)$ - signal's spectrum

f_c - channel coherence bandwidth

Δf - signal bandwidth

a) $\Delta f < f_c$ - frequency flat; b) $\Delta f > f_c$--frequency selective

Distortionless transmission:

$$H(f) = a \cdot e^{-j2\pi f \tau} \tag{5.2}$$

Impulse Response of a Wireless Channel

The cause of frequency selective channel: delay spread. Consider impulse response of the channel.

Given the input signal $s(t)$, the signal $x(t)$ at the channel output is

$$x(t) = \sum_{i=1}^{N} A_i s(t - \tau_i) \qquad (5.3)$$

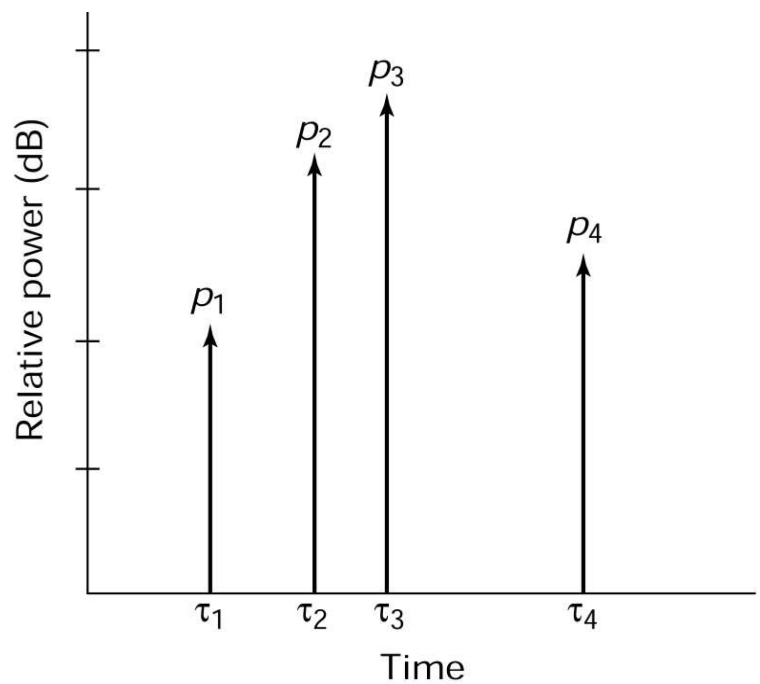
A_i - complex amplitude, $A_i = a_i e^{j\varphi_i}$

τ_i - delay of i-th multipath, there are N delayed components, LOS always arrives first.

The impulse response is

$$h(\tau) = \sum_{i=1}^{N} A_i \delta(\tau - \tau_i) \qquad (5.4)$$

One impulse at Tx -> many impulses at Rx (why?)



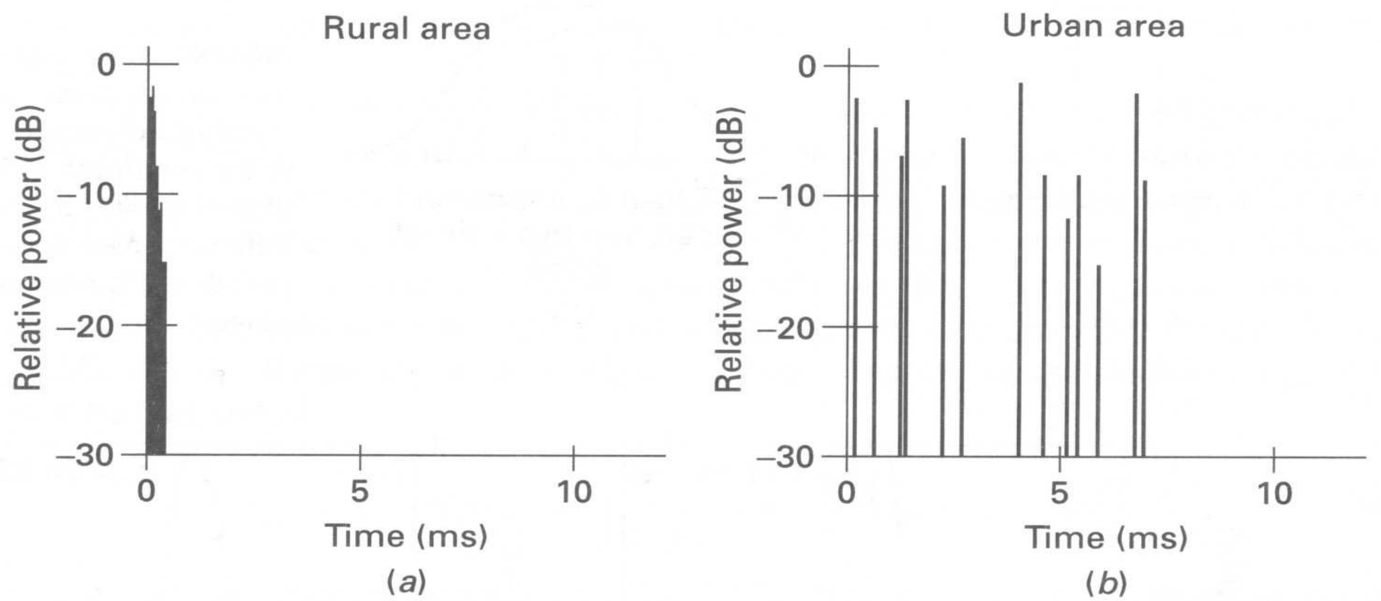


FIGURE 2.22 Impulse responses of two channels. (*a*) A typical rural area. (*b*) An urban area.

P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002.

Input-output relationship

$$x(t) = \int_0^{\infty} h(\tau) s(t-\tau)\, d\tau = \int_{-\infty}^{t} s(\tau) h(t-\tau)\, d\tau \quad (5.5)$$

Wireless channel can be modeled as a linear system (may be time-varying).

<u>Delay spread</u> is a key to FS channels. Average delay and mean-square delay are (weighted mean and mean square):

$$\bar{\tau} = \frac{\sum_i P_i \tau_i}{\sum_i P_i} \qquad \overline{\tau^2} = \frac{\sum_i P_i \tau_i^2}{\sum_i P_i} \quad (5.6)$$

where P_i - power of the i-th component.

Delay spread (RMS) is

$$\Delta\tau = \sqrt{\overline{\tau^2} - \left(\overline{\tau}\right)^2} = \sqrt{\overline{\left(\tau - \overline{\tau}\right)^2}} \qquad (5.7)$$

i.e. the standard deviation of the delay. $\Delta\tau$ characterizes time-spreading of the pulse in the channel.

Realistic example:

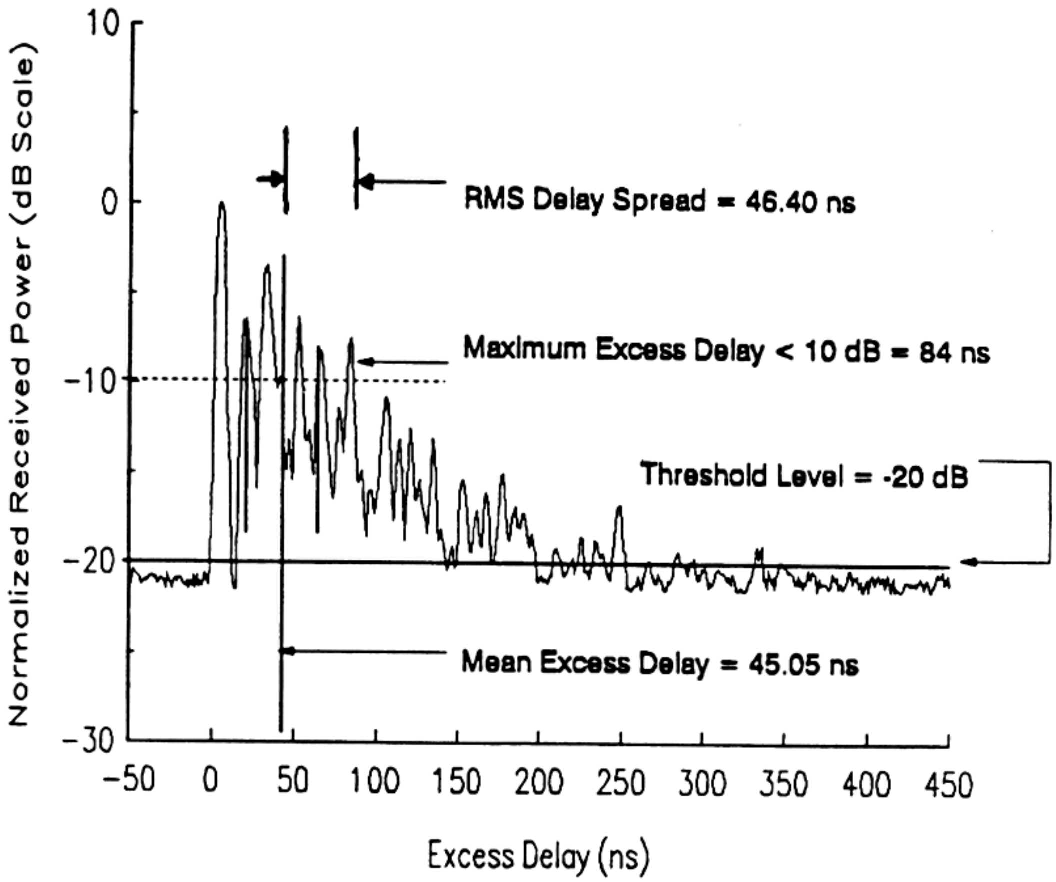


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

Table 5.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10–25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200–310 ns	Averaged typical case	[Cox72]
Suburban	910	1960–2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10–50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70–94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

Frequency-Selective Properties

Compare delay spread $\Delta\tau$ and symbol duration T_s:

Frequency-flat: $T_s >> \Delta\tau$ (5.8)

Frequency-selective : otherwise

not very precise ☹

Coherence bandwidth Δf_c of the channel: a frequency interval over which the frequency responds is almost constant,

$$\Delta f_c = \frac{c}{\Delta\tau} \approx \frac{1}{5\Delta\tau} \tag{5.9}$$

where $c < 1$; e.g. $c = 0.2$ for 0.5 correlation in a random channel.

Via differential phase shift, see the 2-ray example below:

$$\theta = 2\pi\Delta\tau\Delta f_c = 1 \leftrightarrow \Delta f_c = \frac{1}{2\pi\Delta\tau} \approx \frac{1}{6\Delta\tau} \tag{5.9a}$$

The same can be expressed using signal (RF) bandwidth $\Delta f_s \approx 2 / T_s$:

frequency-selective: $\Delta f_s > \Delta f_c$

frequency-flat: $\Delta f_s \le \Delta f_c \to T_s \ge 10\Delta\tau$ (5.10)

more precise now ☺

Error floor effect: to be discussed later.

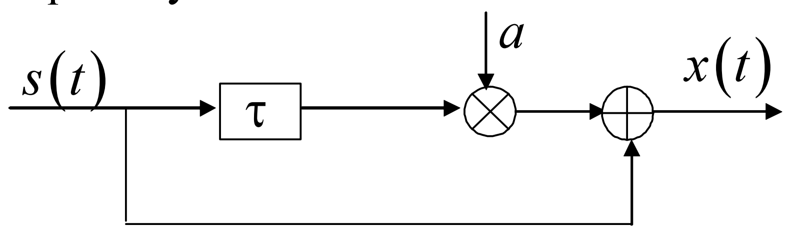
Example: two-ray model (baseband equivalent), deterministic

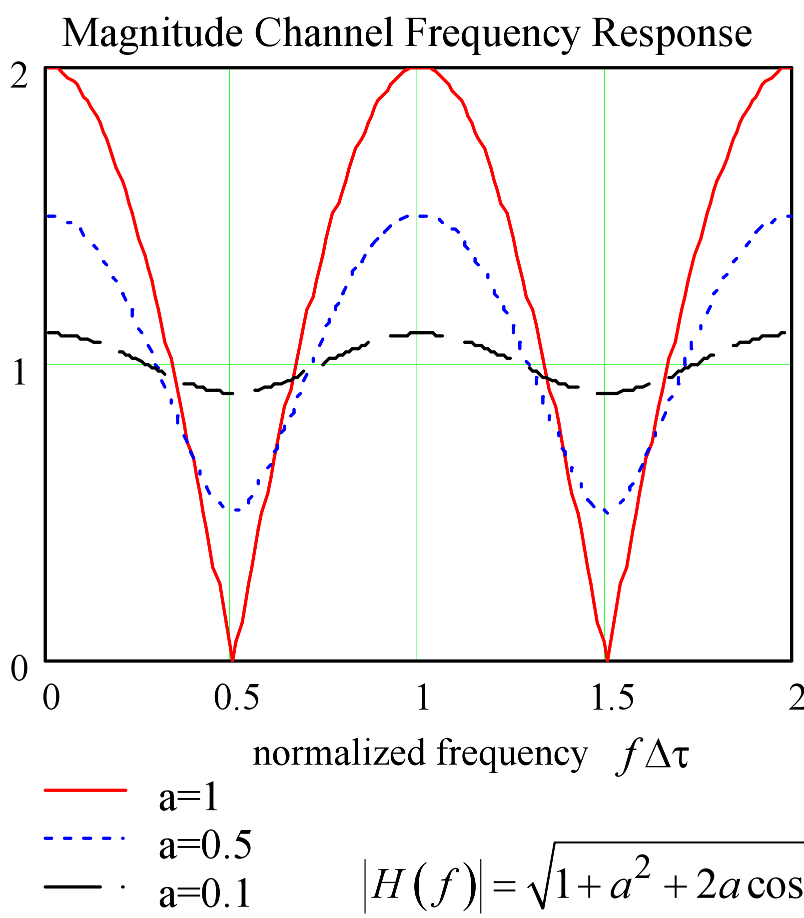
$$h(\tau)=\delta(\tau)+a\delta(\tau-\Delta\tau)\rightarrow H(f)=1+ae^{-j\omega\Delta\tau}$$

$$\begin{aligned}|H(f)|&=\sqrt{(1+a\cos\theta)^2+(a\sin\theta)^2}\\&=\sqrt{1+a^2+2a\cos\theta}\end{aligned}\qquad(5.11)$$

$$where\ \theta=\omega\Delta\tau=2\pi f\Delta\tau$$

Tap-delay model:





$$|H(f)|=\sqrt{1+a^2+2a\cos 2\pi f\Delta\tau}$$

Consider specific cases:

(1) **Frequency-flat** channel (strict):

$$2\pi f \Delta\tau << 1 \rightarrow \theta \ll 1 \rightarrow f << \frac{1}{2\pi\Delta\tau}, \text{ so that}$$

$\left|H\left(f\right)\right| \approx 1+a$ -> frequency-independent (flat).

In practice: less strict,

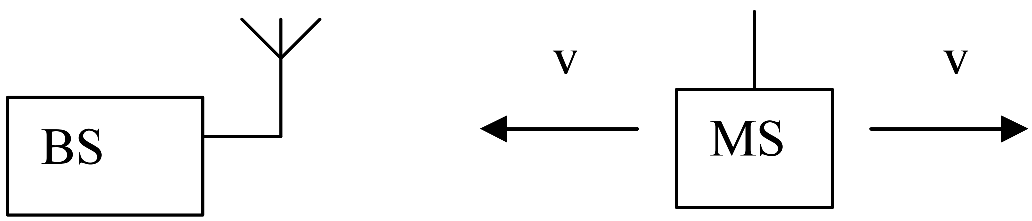
$$\theta = 2\pi f \Delta\tau \leq 1 \approx 60^0 \rightarrow f \leq \frac{1}{2\pi\Delta\tau} \rightarrow \Delta f_c = \frac{1}{2\pi\Delta\tau} \approx \frac{1}{6\Delta\tau}$$

(2) **Frequency-selective**: $f > \dfrac{1}{2\pi\Delta\tau}$

Q.: Using (5.7), find the delay spread (RMS) for the two-ray model.

Doppler Spread and Time-Varying Channels

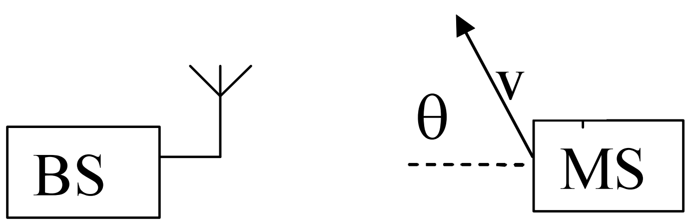
Consider moving MS:



Doppler effect: frequency shift by

$$f_d = f_0 \frac{v}{c} \rightarrow f_{MS} = f_0 \pm f_d \qquad (5.12)$$

Consider moving MS at angle:



$$f_d = f_0 \frac{v}{c} \cos\theta, \quad f_{MS} = f_0 \left(1 + \frac{v}{c} \cos\theta \right) \qquad (5.13)$$

Multipath channel:

i-th path: $$f_{di} = f_0 \frac{v}{c} \cos\theta_i \qquad (5.14)$$

Time-varying frequency response

If $e^{j\omega_0 t}$ is transmitted, the Rx signal (at MS) is

$$x(t) = e^{j\omega_0 t} \sum_{i=1}^{N} a_i e^{j(\varphi_i + 2\pi f_i t)} e^{-j\omega_0 \tau_i} \qquad (5.15)$$

where $f_i = f_0 \frac{v}{c} \cos\theta_i$. The **frequency response** is

$$H(f_0, t) = \sum_{i=1}^{N} a_i e^{j(\varphi_i - \omega_0 \tau_i)} e^{j2\pi f_i t} \leftrightarrow h(\tau, t) \qquad (5.16)$$

i.e., a function of time!

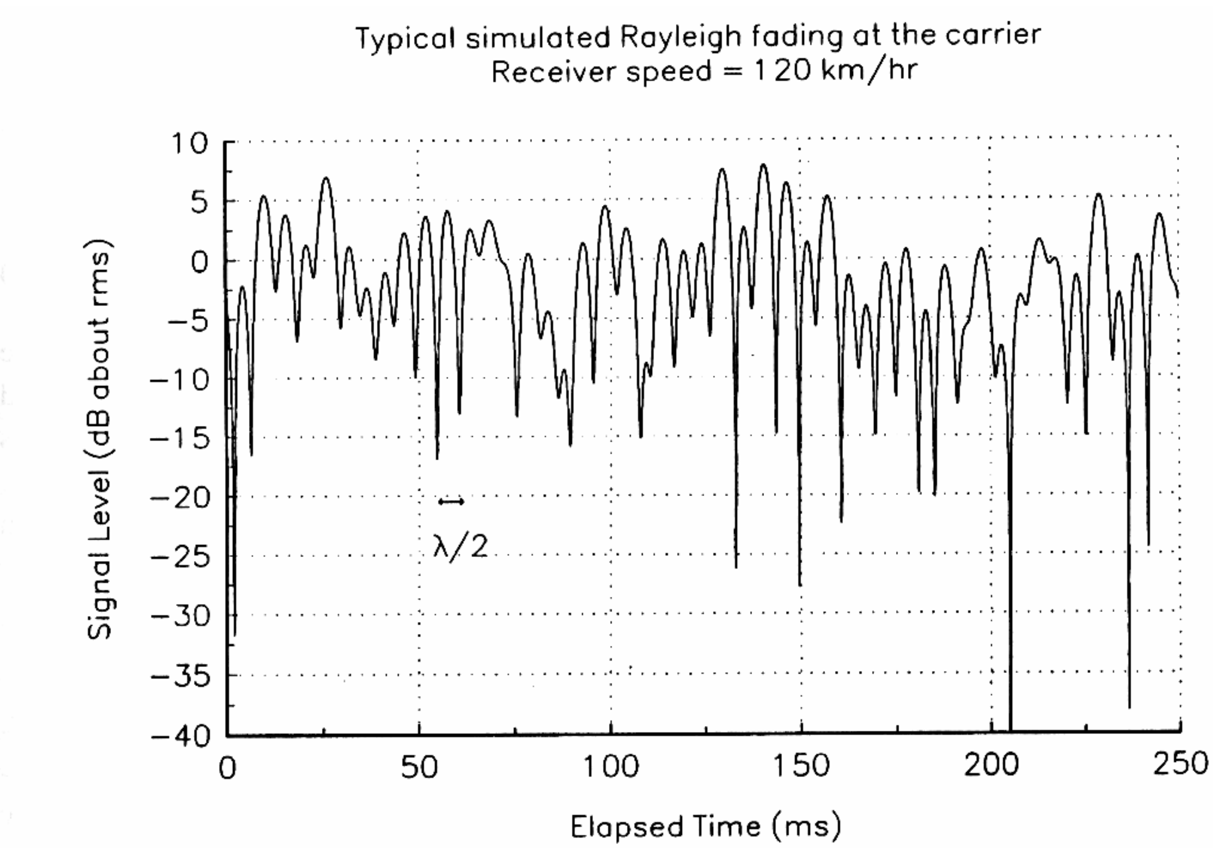


Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

Time-varying impulse response

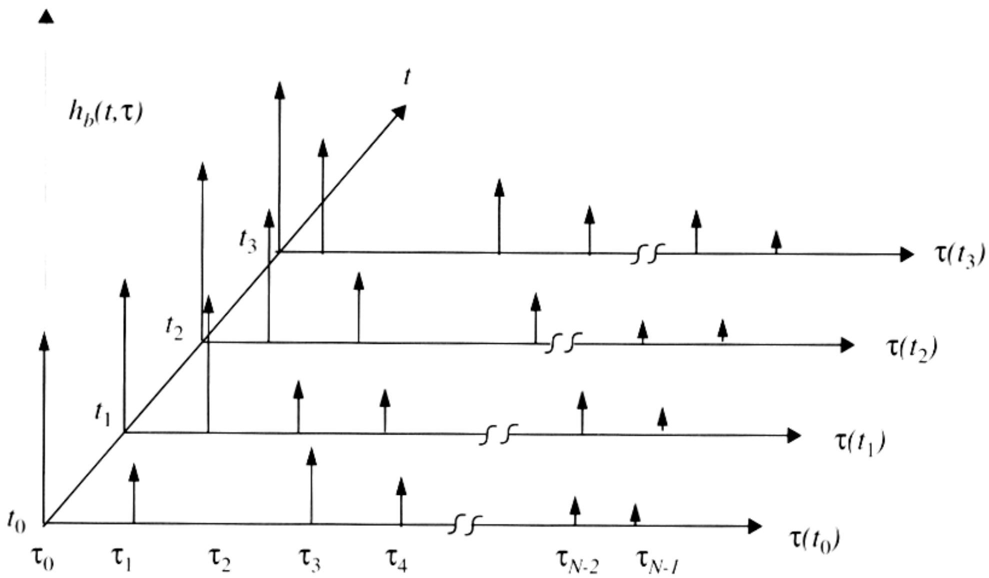


Figure 5.4 An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

If $f_i = 0$, the channel is fixed (not time-varying).

If f_i is large, the channel is fast-fading (varying).

If f_i is small, the channel is slow-fading.

How large (small) is large (small)?

Compare f_d with T_s :

$$\begin{cases} 2\pi f_d T_s > 1 \rightarrow \text{ fast fading} \\ 2\pi f_d T_s \le 1 \rightarrow \text{ slow fading} \end{cases} \quad (5.17)$$

where T_s is the symbol/block duration.

Q: how to decide fast/slow when all $\theta_i \in [\theta_0 + \Delta\theta]$?

Coherence time of the channel

Coherence time of the channel: the time interval over which the channel is approximately constant (static),

$$\theta = 2\pi f_d T_c = 1 \rightarrow T_c = \frac{1}{2\pi f_d} \approx \frac{1}{6 f_d} \qquad (5.18)$$

The channel is considered static if $\Delta t \leq T_c$ and time-varying for $\Delta t > T_c$.

Fast/slow fading can be expressed as

$$\begin{cases} T_s > T_c \rightarrow \text{ fast fading} \\ T_s \leq T_c \rightarrow \text{ slow fading} \end{cases} \qquad (5.19)$$

Note: the error floor effect exists for both cases.

*) another definition (less strict, may be unacceptable in some cases):

$$\theta = 2\pi f_d T_c = \frac{\pi}{2} \rightarrow T_c = \frac{1}{4 f_d}$$

*) yet another definition, not good at all (explain why)

$$T_c \approx \frac{1}{f_d}$$

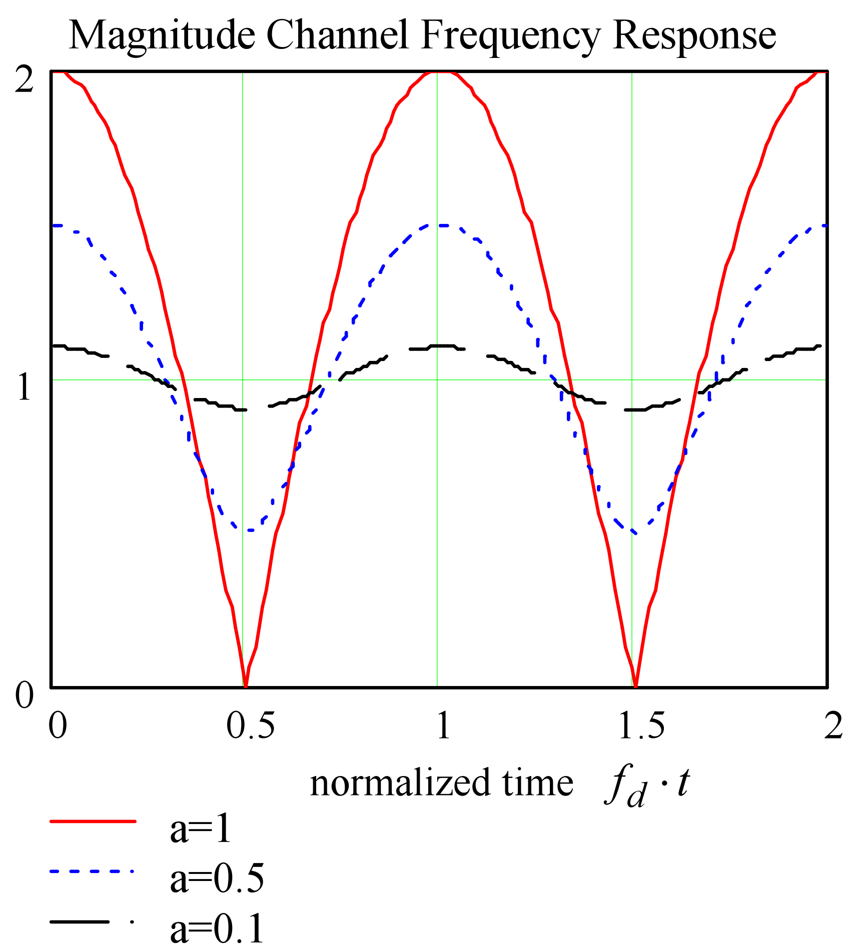
Example: two-ray model

$$H(f) = 1 + ae^{-j\omega\Delta\tau} e^{j\omega_d t} \qquad (5.20)$$

Consider $\Delta\tau = 0$:

$$H(f) = 1 + e^{j\omega_d t}$$
$$|H(f)| = \sqrt{1 + a^2 + 2a\cos\omega_d t} \qquad (5.21)$$

i.e. $|H(f,t)|$. It is the same as before if $\theta = \omega_d t = 2\pi f_d t$.
Recall that $\omega_d = \omega_0 v / c$



Doppler Spectrum

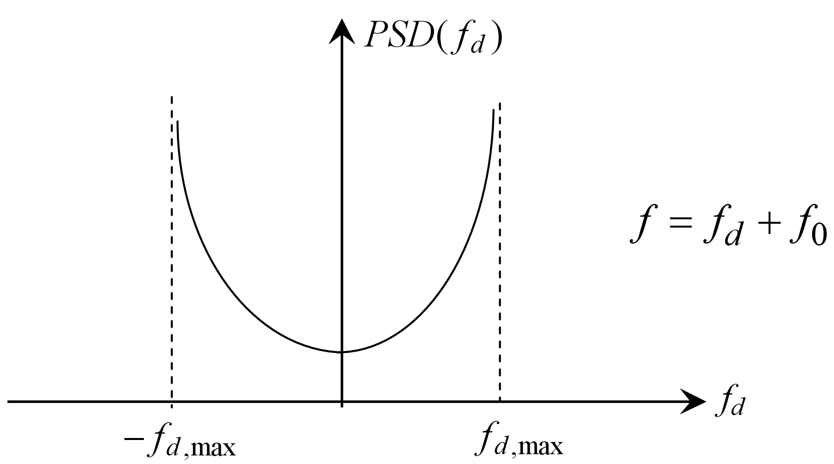
Consider many multipath components with uniform θ

$$\rho_\theta(\theta) = 1/2\pi,\ \theta \in [0, 2\pi] \qquad (5.22)$$

What is the pdf of $f_d = f_0 \frac{v}{c} \cos\theta$?

$$\rho_\theta(\theta) d\theta = \rho_f(f_d) df_d \rightarrow \rho_f(f_d) = \frac{a}{\sqrt{1 - \left(\frac{f_d}{f_{d,\max}}\right)^2}} \qquad (5.23)$$

where a is a normalization constant, and $f_{d,\max} = f_0 v / c$ is the maximum Doppler frequency. It can be shown that Doppler power spectrum $PSD(f_d)$ is the same as $\rho_f(f_d)$ (provided that uniform angular distribution holds).



Note that a single-tone Tx signal results in spread-out spectrum at Rx! ($\Delta f_d = 2 f_{d,\max}$)

Mobile wireless channel is a function of space and time!

Random channel: coherence time is defined as a time interval for which envelope correlation ≥ 0.5.

Coherence bandwidth: frequency interval for which envelope correlation ≥ 0.5.

Note: other correlation level (e.g. 0.9) can be used as well for some applications.

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading

1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading

1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

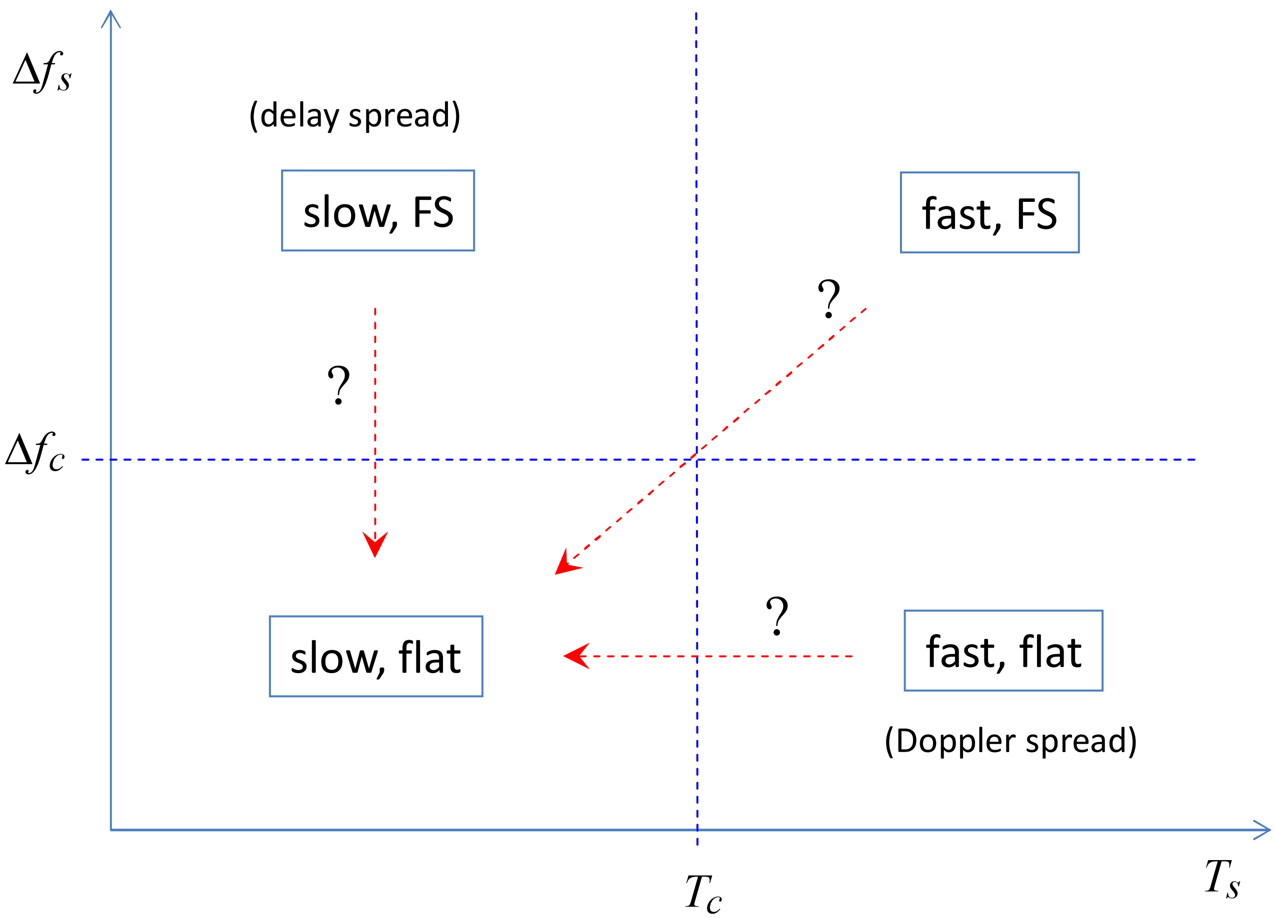
Slow Fading

1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

Different Forms of Multipath Fading



Overview of System-Level Propagation Effects

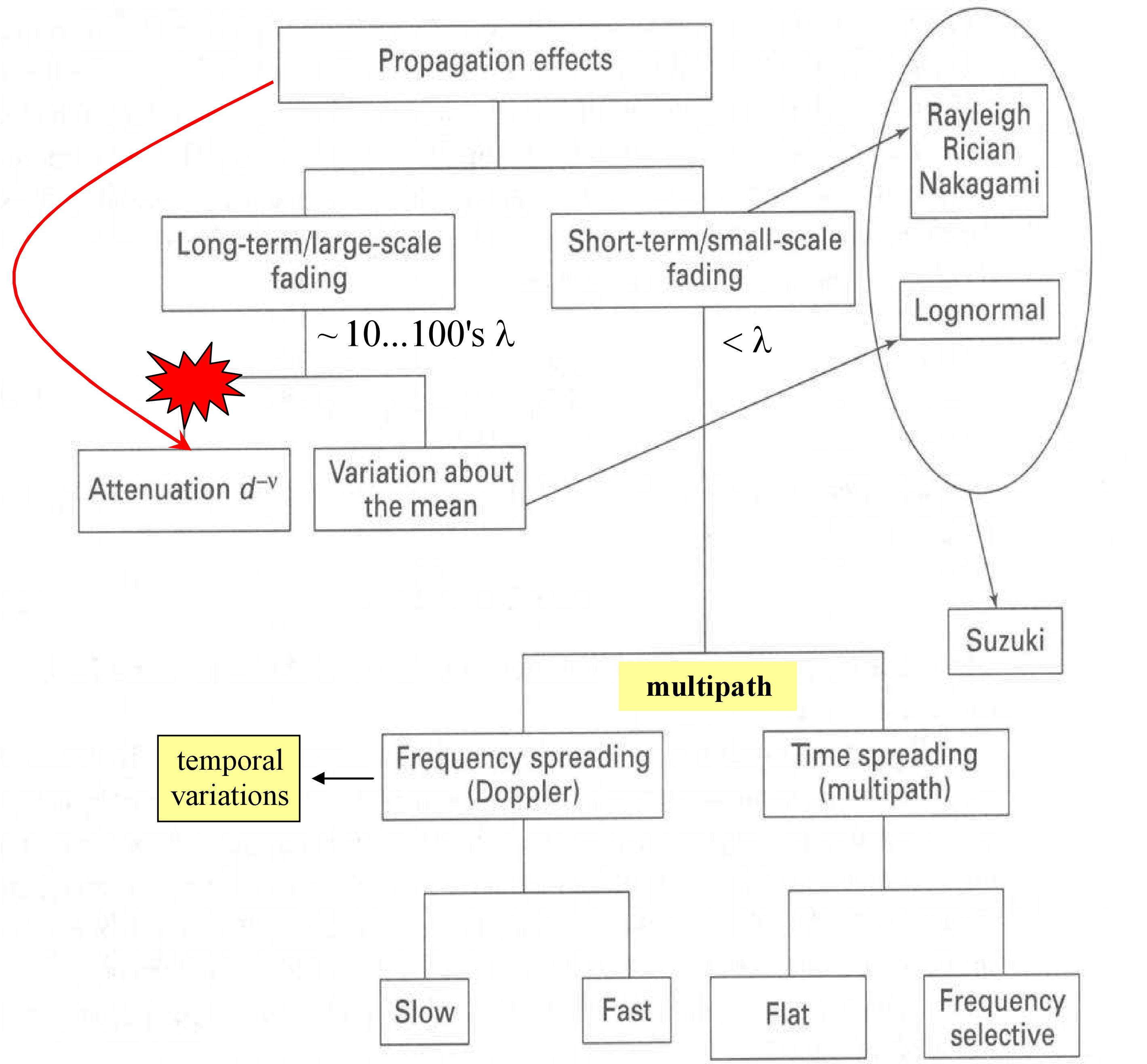


FIGURE 2.38 Overview of attenuation and fading. All forms of fading are shown along with their origins and relationships.

P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002. (modified)

Summary

- Impulse and frequency responses of a wireless channel.
- Delay spread and frequency selective channels
- Tap-delay model. Power delay profile.
- Doppler spread and time-varying channels.
- Envelope correlation. Coherence bandwidth and coherence time of the channel.
- Classification of fading and propagation effects

Reading:

- Rappaport, Ch. 5 (except 5.8).
- Other books (see the reference list).

Note: Do not forget to do end-of-chapter problems. Remember the learning efficiency pyramid!