SER/BER in a Fading Channel

Major points for a fading channel:

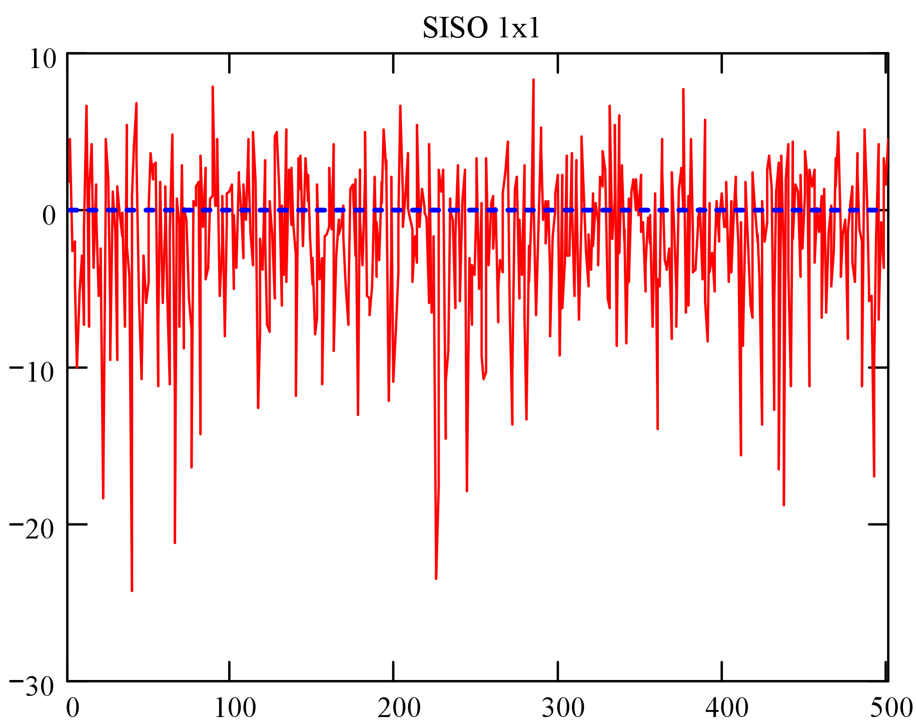
* SNR is a R.V. or R.P.

* SER(BER) depends on the SNR → conditional SER(BER).

* Two performance measures: outage probability and average SER(BER).

* Overall, 3 effects in a mobile channel: fading, delay spread and Doppler spread affect the BER/SER. Concentrate on 1st one.

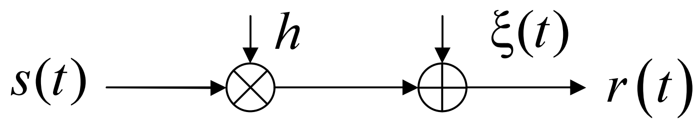
Example: Rayleigh Fading Channel



Received power (SNR) norm. to the average [dB] vs distance (location, time)

Channel model:

Consider first slow (quasi-static) flat-fading channel,



$$r(t) = \alpha e^{j\varphi} \cdot s(t) + \xi(t) \tag{10.1}$$

where $\xi(t)$ is AWGN, $h = \alpha e^{j\varphi}$ is the complex channel gain (an RV for a fading channel), α and φ are its magnitude and phase.

After the matched filter and sampler, the model is discrete-time,

$$r_i = h \cdot s_i + \xi_i \tag{10.2}$$

Slow block-fading: h stays the same for many symbol intervals and then changes to a new random value.

Introduce conditional SER/BER $P_e(\gamma)$ (i.e., for a fixed α), and $\gamma = \alpha^2 E_b / N_0$ is instantaneous SNR.

Introduce the average (over fading) SNR:

$$\gamma_0 = \bar{\gamma} = \overline{\alpha^2} E_b / N_0 \tag{10.3}$$

For Rayleigh fading, α is a Rayleigh RV, and α^2 or γ is χ_2^2 (or exponential), its pdf is

$$\rho(\gamma) = \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} \qquad (10.4)$$

When $\overline{\alpha^2} = 1 \Rightarrow \gamma_0 = E_b / N_0$ is the AWGN channel SNR (reference point).

The pdf of α^2 is $\rho_{\alpha^2}(x) = e^{-x}$.

<u>Average SER/BER</u> is

$$\overline{P_e} = \int_0^{\infty} P_e(\gamma)\rho(\gamma)d\gamma \qquad (10.5)$$

The expression is general, can be used for any flat-fading channel.

Using the SER/BER expressions of various modulation formats above, average SER/BER in Rayleigh channel can be found.

Note: average SER/BER is not the only performance measure! Outage probability is important as well.

"Slow"/"fast" fading: P_{out} vs. $\overline{P_e}$

<u>Outage probability</u>: a probability that the instantaneous SER exceeds a given threshold ε,

$$P_{out} = \Pr\left\{P_e > \varepsilon\right\} = \Pr\left\{\gamma < \gamma_{th}\right\} \qquad (10.6)$$

where γ_{th} = threshold SNR, such that $P_e\left(\gamma_{th}\right) = \varepsilon$.

P_{out} can be expressed via the CDF F_γ of instantaneous SNR γ,

$$P_{out} = \int_0^{\gamma_{th}} \rho\left(\gamma\right) d\gamma = F_\gamma(\gamma_{th}) \qquad (10.7)$$

Average BER Expressions in Rayleigh Fading:

Coherent BPSK:
$$\overline{P_e} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right) \approx \frac{1}{4\gamma_0} \qquad (10.8)$$

Coherent BFSK:
$$\overline{P_e} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{2+\gamma_0}}\right) \approx \frac{1}{2\gamma_0} \qquad (10.9)$$

DPSK:
$$\overline{P_e} = \frac{1}{2(1+\gamma_0)} \approx \frac{1}{2\gamma_0} \qquad (10.10)$$

Non-coherent orthogonal BFSK:
$$\overline{P_e} = \frac{1}{2+\gamma_0} \approx \frac{1}{\gamma_0} \qquad (10.11)$$

Approximate expressions hold for large SNR ($\gamma_0 \gg 1$).

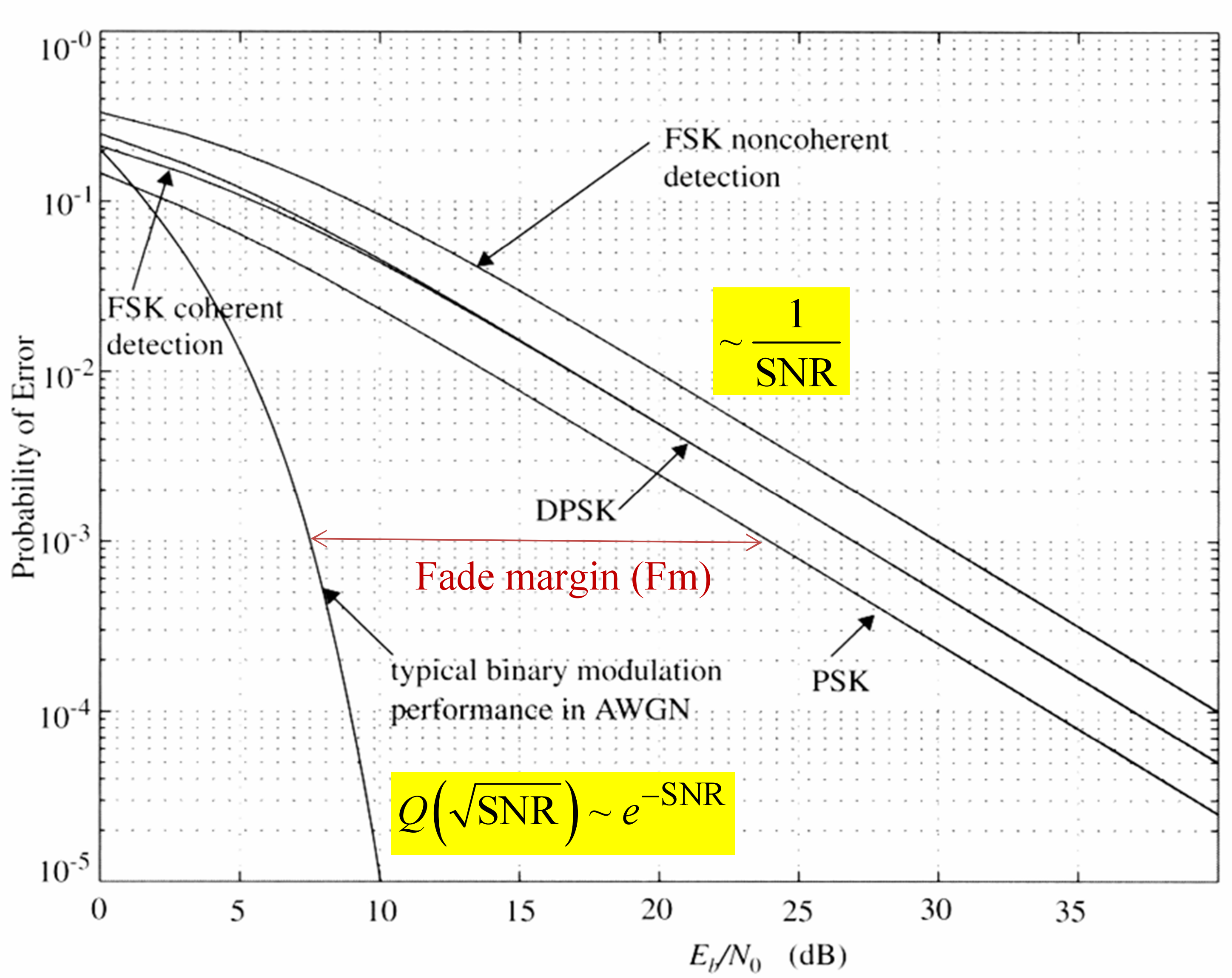
Q.: Compare with the AWGN channel BER!

Deep fade events dominate the average error rate at high SNR,

$$\overline{P_e} \sim P_{out} \sim \frac{1}{\gamma_0} \qquad (10.12)$$

See [Tse, Ch. 3] for a detailed discussion.

BER in a Fading Channel

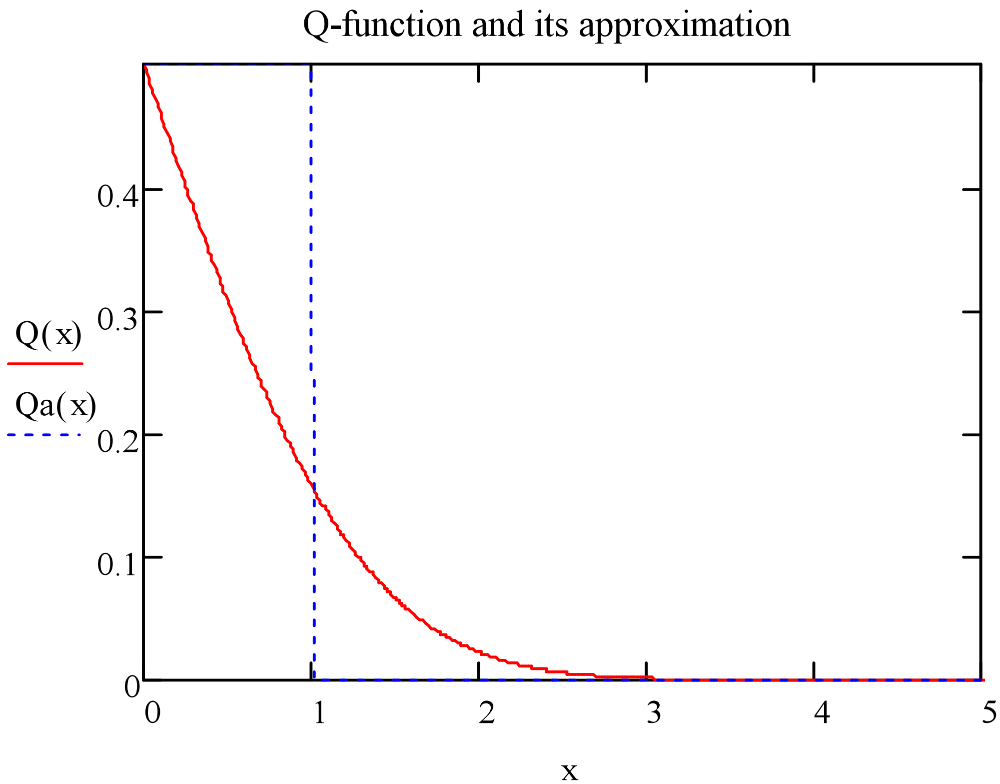


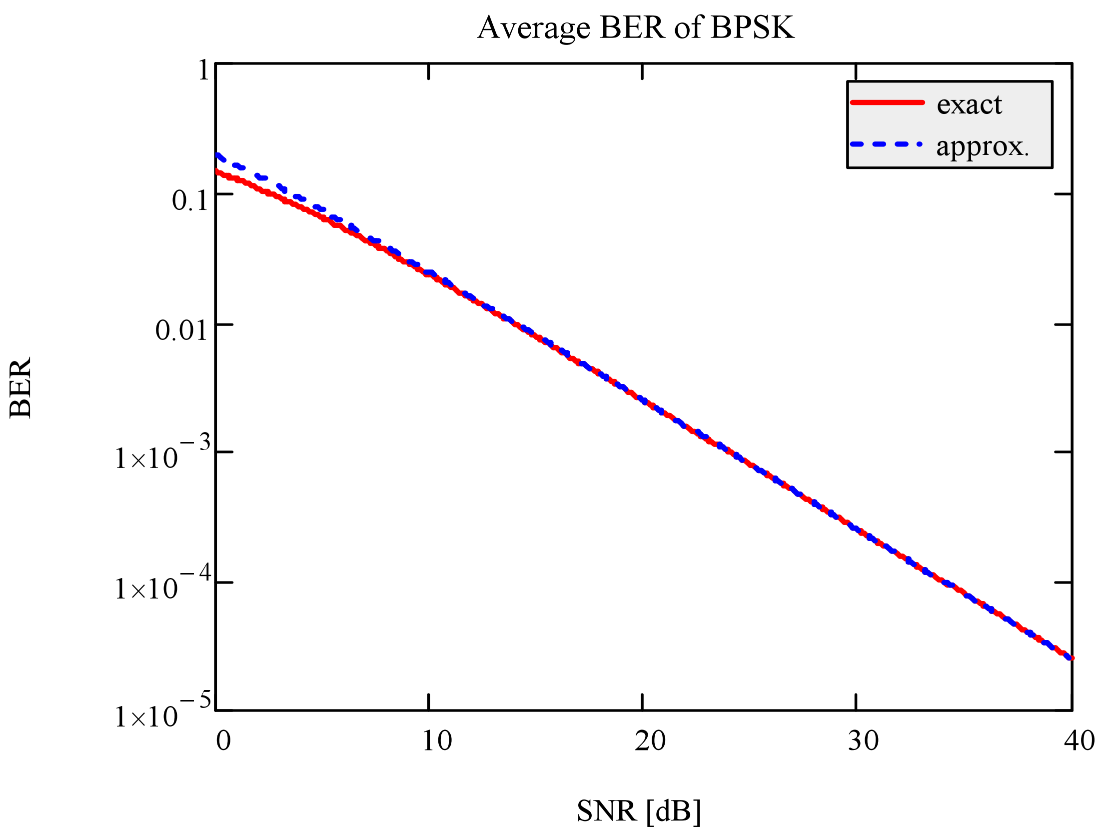
T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

The difference is dramatic: the average BER is inverse in γ_0 rather than exponential, as for AWGN channel.

<u>Fade margin</u>: additional increase in SNR to keep the same average BER as in a fixed AWGR channel.

Large increase in BER is due to deep fades. Some techniques are required to mitigate the effect of fading.


Q-function and its approximation
Q(x)
Qa(x)
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
x


Average BER of BPSK
exact
approx.
BER
1
0.1
0.01
1×10−3
1×10−4
1×10−5
0
10
20
30
40
SNR [dB]

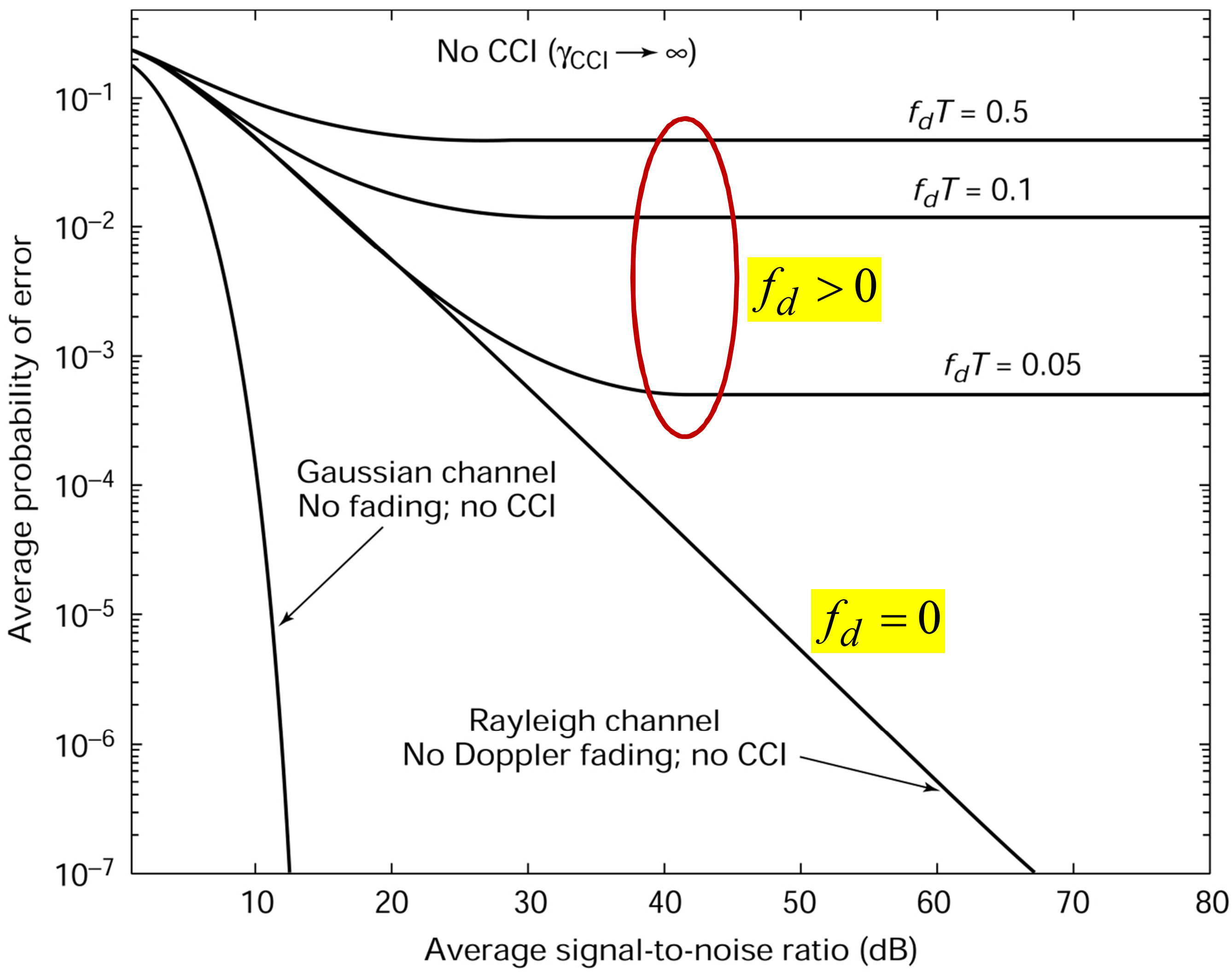
BER in a Fast Fading Channel

Fast Fading: $f_d T_s > 1$ or $T_s > T_c$

Error floor: BER cannot be decreased below a certain level.

Average BER of differential BPSK* is

$$\overline{P_e} = \frac{1+\gamma_0\left[1-J_0\left(2\pi f_d T_s\right)\right]}{2\left(1+\gamma_0\right)} \qquad (10.13)$$



P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002.

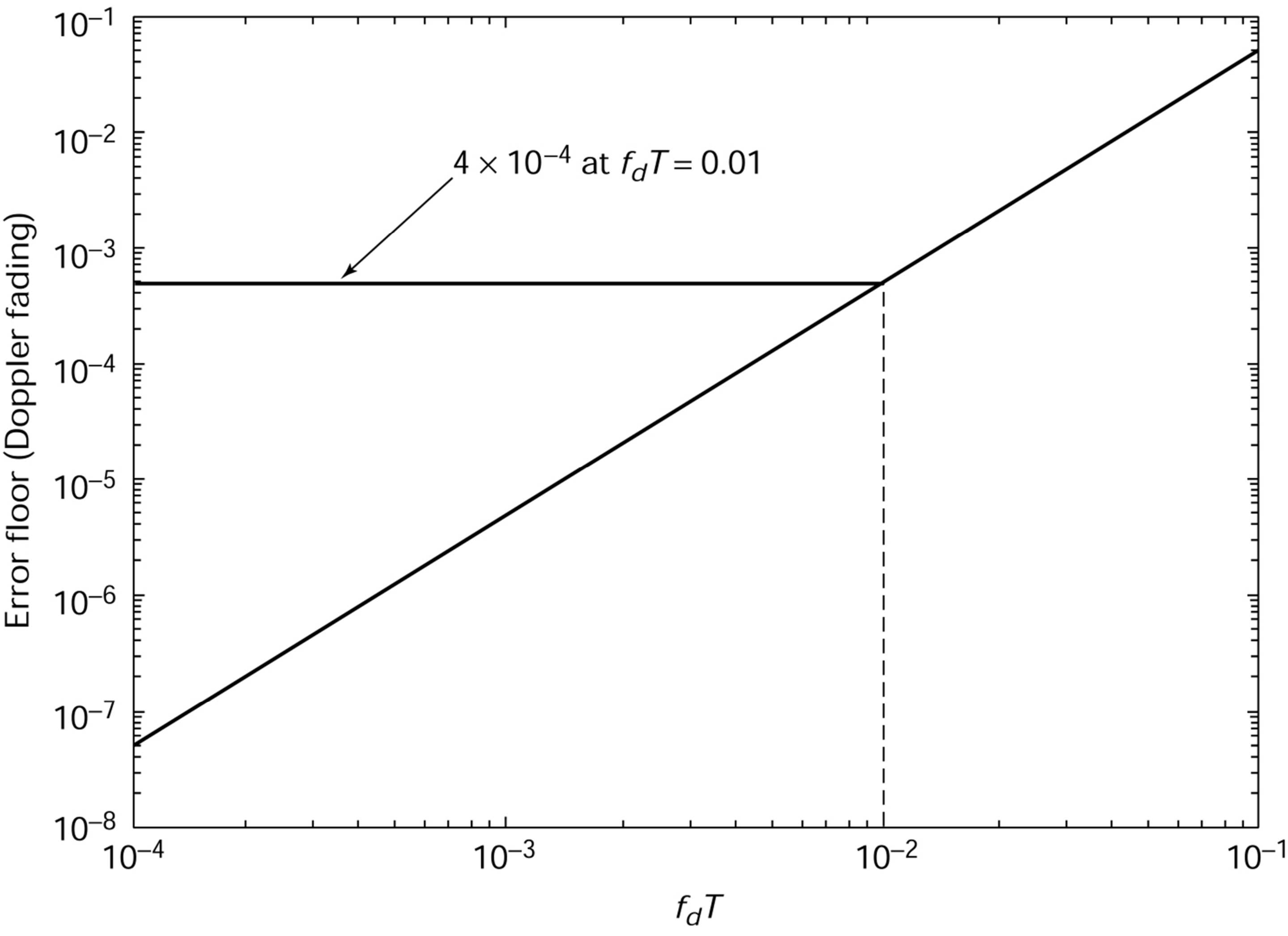
*Q: explain why coherent BPSK cannot be used.

The error floor is obtained by $\gamma_0 \to \infty$

$$\overline{P_{ef}} = \left[1 - J_0\left(2\pi f_d T_s\right)\right]/2 \qquad (10.14)$$

For small $f_d T_s << 1$,

$$\overline{P_{ef}} \approx 5\left(f_d T_s\right)^2 \text{ or } f_d T_s \le \sqrt{\overline{P_{ef}}/5} \qquad (10.15)$$



P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002.

For co-channel interference (I) and slow fading ($f_d T_s \approx 0$),

$$\overline{P_e} = \frac{1}{2}\left[1 - \frac{\gamma_0 \gamma_I}{\sqrt{(\gamma_0\gamma_I + \gamma_0 + \gamma_I)^2 - (\gamma_0/\pi)^2}}\right] \quad (10.16)$$

$$\approx \frac{1}{2}\left(\frac{1}{\gamma_0} + \frac{1}{\gamma_I}\right)$$

where γ_I=SIR (signal-to-interference ratio).

Interpretation: total noise under Gaussian assumption,

$$P_{tN} = P_N + P_I \sim \frac{1}{\gamma_0} + \frac{1}{\gamma_I} \quad (10.17)$$

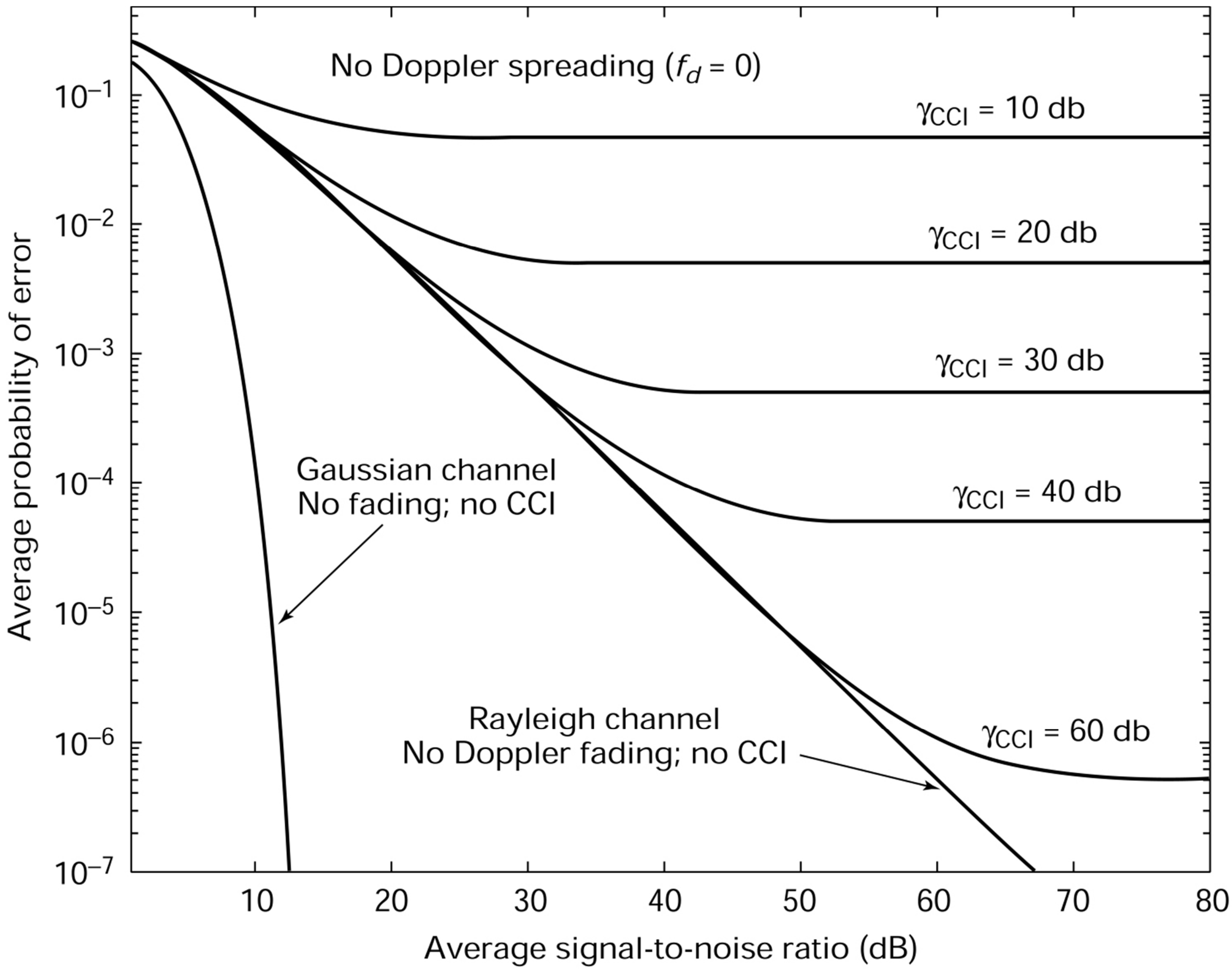
The error floor:

$$\overline{P_{ef}} \approx \frac{1}{2\gamma_I} \text{ as } \gamma_0 \to \infty \text{, or } \gamma_I \geq \frac{1}{2\overline{P_{ef}}} \quad (10.18)$$

Co-channel interference has a profound impact on the average BER.

These conclusions are true for other modulations as well, i.e. error floor due to fast fading and I, frequency –selective fading.

The difference is in when the floor is achieved. For details, see W.C. Jakes, Jr.: ‘Microwave Mobile Communications’, John Wiley and Sons, New York, 1974.



P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002.

BER in a Frequency–Selective Fading Channel

Recall: the channel is frequency selective when $\Delta f_c < \Delta f_s$ or $\Delta\tau > T_s$.

There is an <u>error floor</u> even if $\Delta\tau < T_s$!

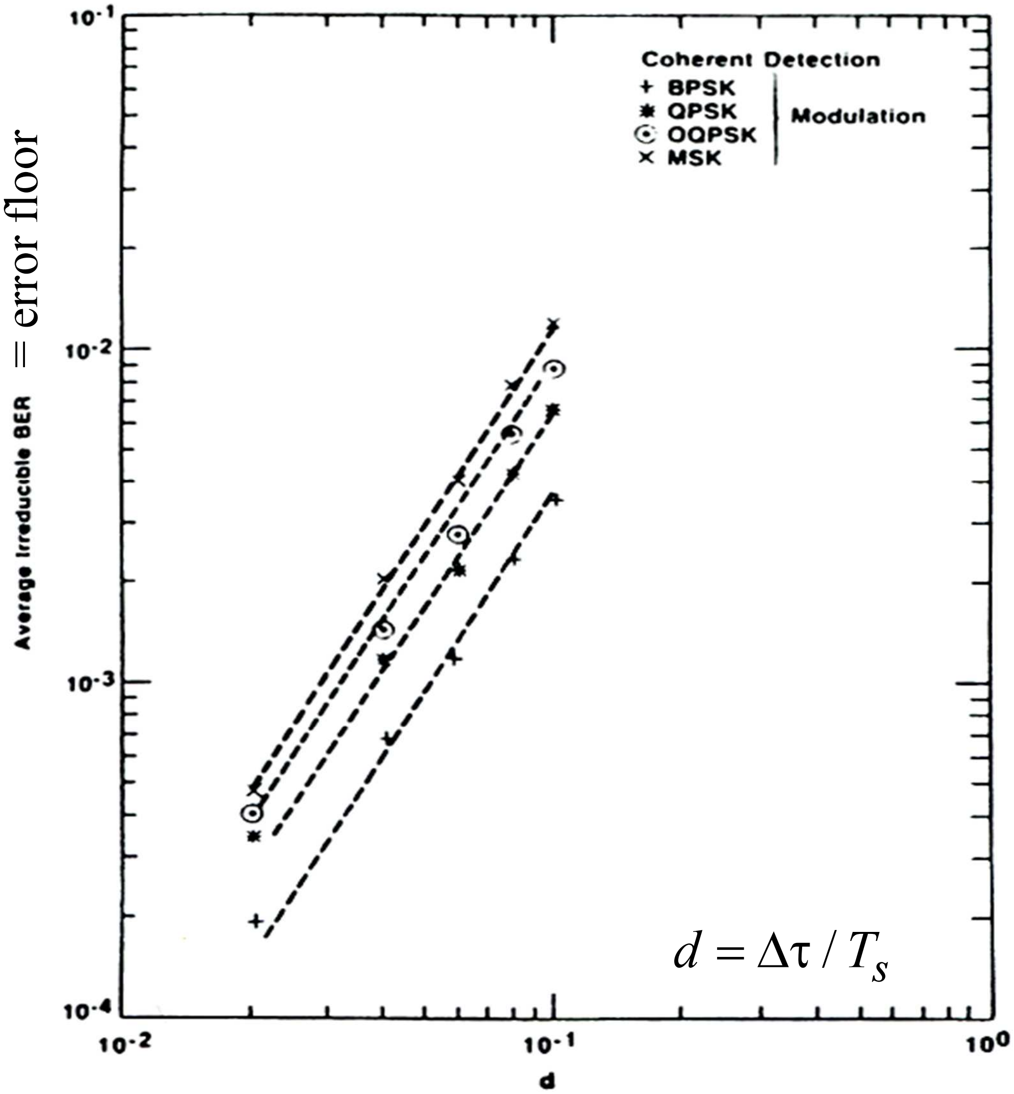


Figure 6.54 The irreducible BER performance for different modulations with coherent detection for a channel with a Gaussian shaped power delay profile. The parameter *d* is the rms delay spread normalized by the symbol period [from [Chu87] © IEEE].

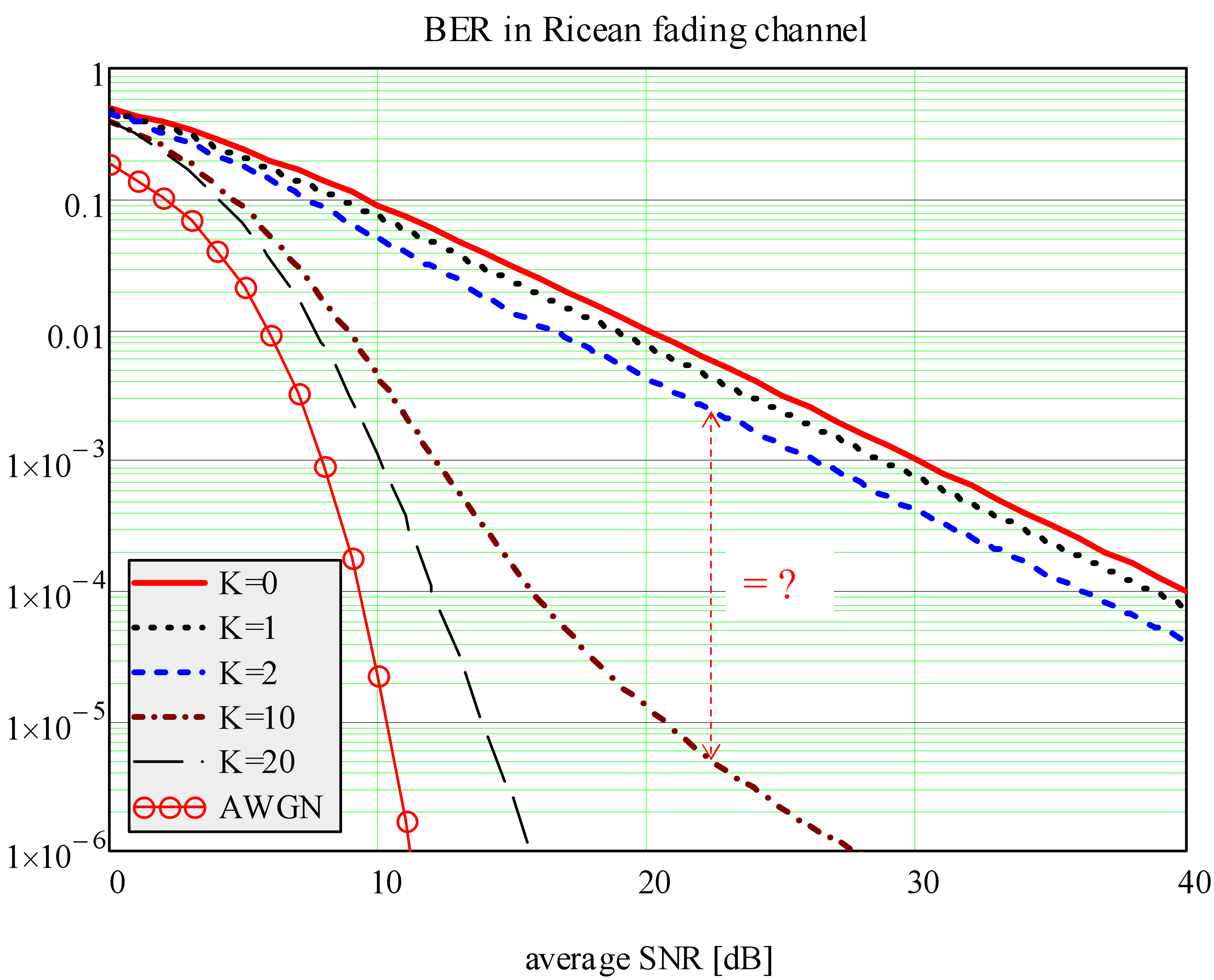
T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

BER in a Ricean Fading (LOS)

For the DPSK,

$$\overline{P}_e = \frac{1+K}{2(1+\gamma_0+K)} \exp\left(-\frac{K\gamma_0}{1+\gamma_0+K}\right) \qquad (10.19)$$

where $K = A^2/2\sigma^2$ is the key-factor of Ricean distribution, i.e. specular-to-scattered power ration.



Summary

- BER in a slow flat fading channel. Dramatic consequence of the fading.
- BER in fast-fading and frequency-selective channel.
- Impact of interference.
- Error floors.
- Impact of LOS (Ricean fading).

Reading:

- Rappaport, Ch. 6.
- Any other text that covers the topics above.

Note: Do not forget to do end-of-chapter problems. Remember the learning efficiency pyramid!

Fundamentals of digital communications and information theory: strongly recommended references

1. J.M. Wozencraft, I.M. Jacobs, Principles of communication engineering, Wiley: New York, 1965.
2. D. Tse, P. Viswanath, Fundamentals of Wireless Communications, Cambridge, 2005. – Chapters 3, 5, Appendices A and B.

- Other books (see the reference list).

Note: Do not forget to do end-of-chapter problems. Remember the learning efficiency pyramid!