Modulation Process

- Modulation: transforming an information-carrying signal m(t) (lowpass) into a narrowband signal x(t). m(t) is also called the *modulating signal* or *message*.
- Start with a sinusoidal signal (carrier) $x(t) = A\cos\left(2\pi f t + \varphi_0\right)$
- Varying $A = A(t)$ according to m(t) – amplitude modulation (AM)
- Varying $\varphi = \varphi(t)$ according to m(t) – phase modulation (PM)
- Varying $f = f(t)$ according to m(t) – frequency modulation (FM)
- FM and PM can be viewed as angle modulation.
- General form of a modulated signal:

$$\boxed{x(t) = A(t)\cos\left(\omega_c t + \int_0^t \Delta\omega(\tau)d\tau + \varphi(t)\right)}$$

Amplitude Modulation (AM)

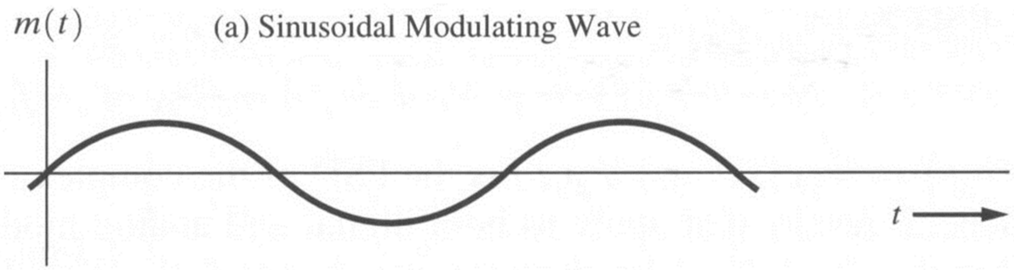
- Information-bearing signal m(t) is impressed onto the carrier amplitude.
- Four types of AM:
  - conventional,
  - double sideband suppressed carrier (DSB-SC)
  - single sideband (SSB); can be lower or upper (LSB/USB)
  - vestigial sideband (VSB)
- Spectral characteristics & bandwidth
- Modulation index
- Power efficiency

Conventional AM

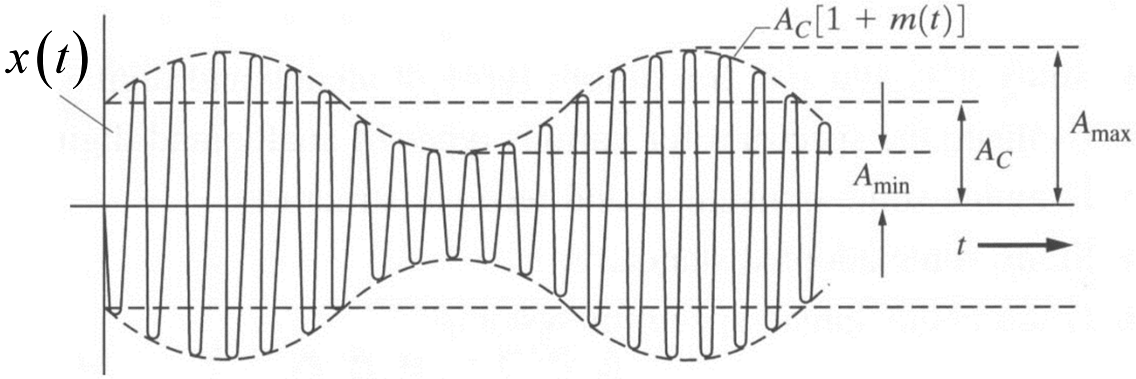
- General form: $$x(t) = \underbrace{A_c\left[1+m(t)\right]}_{A(t)}\cos\left(2\pi f_c t + \phi_c\right)$$
- m(t) must be constrained: $-1 \le m(t)$
- A_c – carrier amplitude without modulation;
- $A_c\left[1+m(t)\right]$ - time-varying (modulated) carrier amplitude.
- Example: sinusoidal modulating signal:

$$m(t) = a\cos(\Omega t)$$

(a) Sinusoidal Modulating Wave



(b) Resulting AM Signal



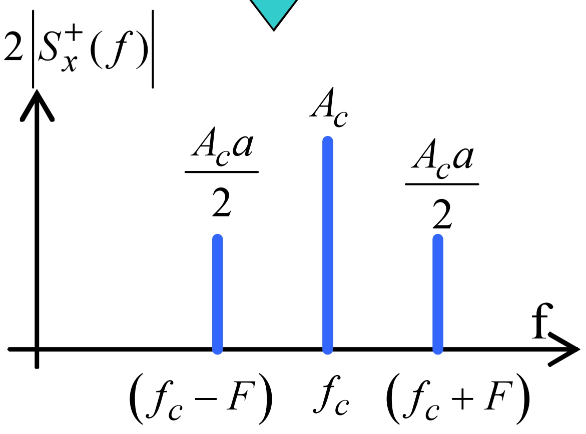
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

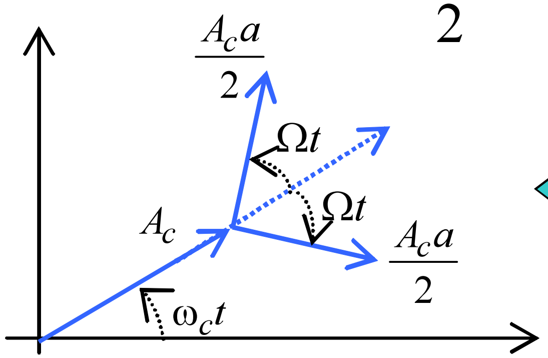
Conventional AM: Sinusoidal Modulation

- Modulated signal: $\boxed{x(t) = A_c\left[1 + a\cos(2\pi Ft)\right]\cos\left(2\pi f_c t + \phi_c\right)}$
- Minimum & maximum carrier amplitudes: $\boxed{\begin{aligned} A_{\min} &= A_c\left[1 - a\right] \\ A_{\max} &= A_c\left[1 + a\right] \end{aligned}}$
- Modulation index: $\boxed{M = \frac{A_{\max} - A_{\min}}{2A_c} \leq 1}$
- % mod.=M*100%
- x(t) spectrum: $x(t) = A_c\cos\left[2\pi f_c t + \phi_c\right] + \frac{A_c a}{2}\cos\left[2\pi\left(f_c - F\right)t + \phi_c\right]$

$$+\frac{A_c a}{2}\cos\left[2\pi\left(f_c + F\right)t + \phi_c\right]$$

Bandwidth?





Geometrical representation

Conventional AM: Sinusoidal Modulation

- Signal power (average):

$$\boxed{P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt = \overline{x^2(t)} = \frac{A_c^2}{2} + \frac{a^2 A_c^2}{4}}$$

carrier sidebands

- Power efficiency:

$$\boxed{\eta = \frac{P_{SB}}{P_{tot}} = \frac{a^2}{2+a^2}}$$

- Bandwidth:

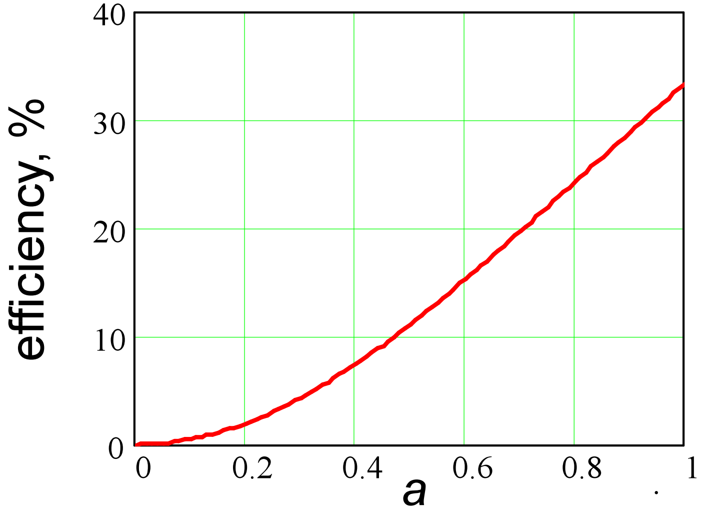
$$\boxed{\Delta f = 2F}$$

- Peak power:

$$\boxed{P_{peak} = \frac{[A_c(1+a)]^2}{2}}$$

- In general: (no DC in m(t))

$$\boxed{\eta = \frac{P_m}{1+P_m}}$$


Power efficiency of AM

- What is the best power efficiency?

Conventional AM: General Case

- General form: $\boxed{x(t) = A_c\left[1+m(t)\right]\cos\left(2\pi f_c t + \phi_c\right)}$
- Modulated signal spectrum:

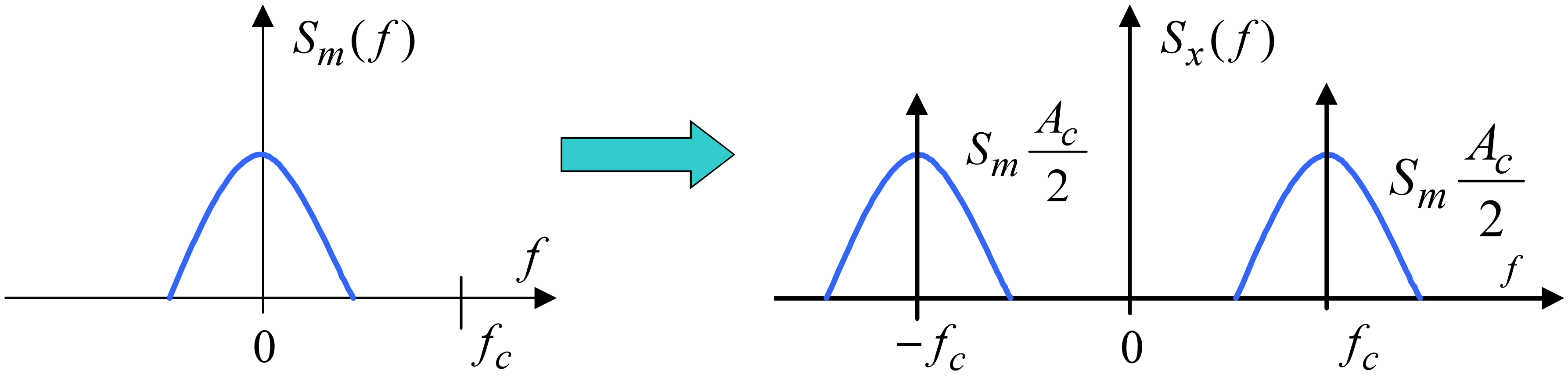
$$\boxed{S_x(f) = \frac{A_c}{2}\left[\delta(f-f_c)e^{j\phi_c} + \delta(f+f_c)e^{-j\phi_c} + S_m(f-f_c)e^{j\phi_c} + S_m(f+f_c)e^{-j\phi_c}\right]}$$

$$\boxed{2\left|S_x^+(f)\right| = A_c\left[\delta(f-f_c) + \left|S_m(f-f_c)\right|\right]}$$

- Power ?
- Power efficiency?

- Measured by spectrum analyzer: no inf. height for delta function in practice, $\delta(f-f_c) \leftrightarrow \Delta(f-f_c)$

Bandwidth???



Example

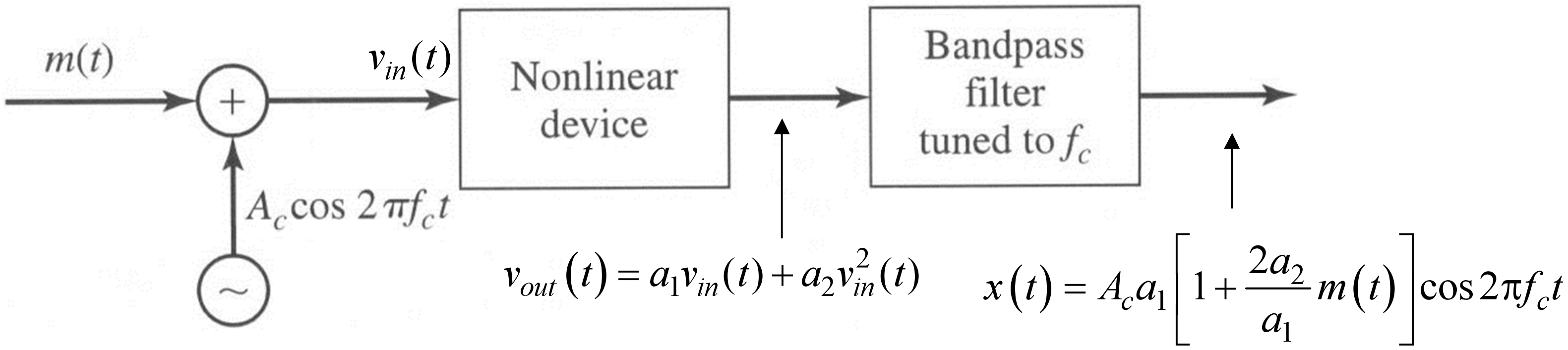
- Conventional AM signal with a sinusoidal message has the following parameters:

$$A_c = 10,\ M = 0.5,\ f_c = 1MHz,\ F = 1kHz$$

1. Find time-domain expression x(t) of the signal
2. Find its Fourier transform
3. Sketch its spectrum as it appears on the spectrum analyzer
4. Find the signal power, peak power and the power efficiency
5. Find the signal bandwidth

Generation of Conventional AM

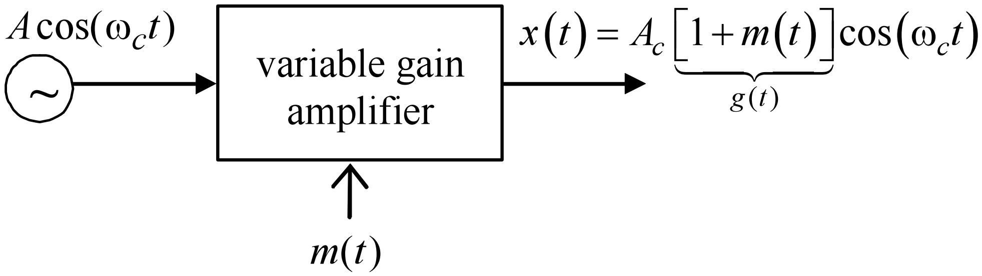
- Power-law modulator:



$$v_{out}(t) = a_1 v_{in}(t) + a_2 v_{in}^2(t)$$

$$x(t) = A_c a_1 \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t$$

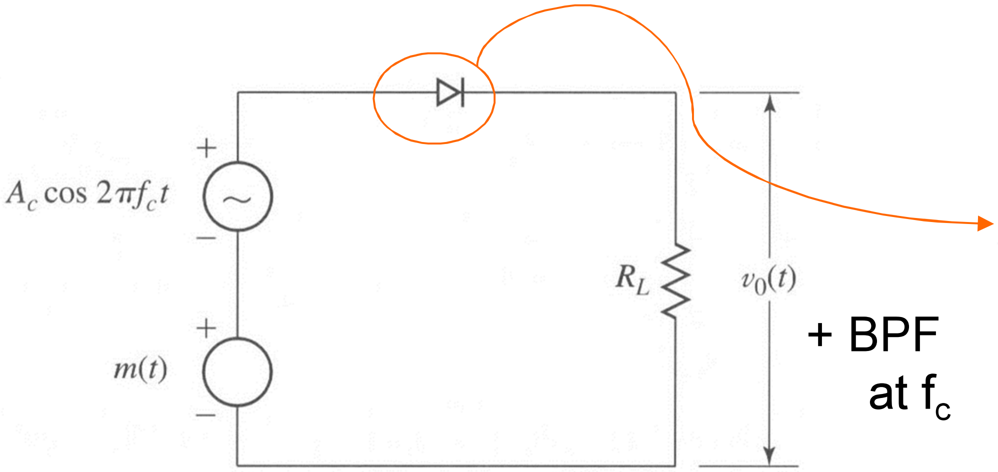
- Using variable-gain amplifier (modulator):



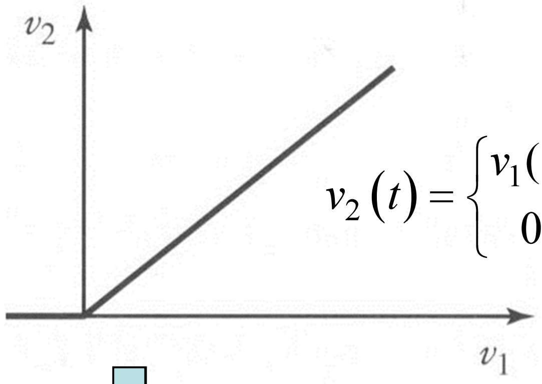
$$x(t) = A_c \underbrace{\left[1 + m(t)\right]}_{g(t)} \cos(\omega_c t)$$

Generation of Conventional AM

- Switching modulator:



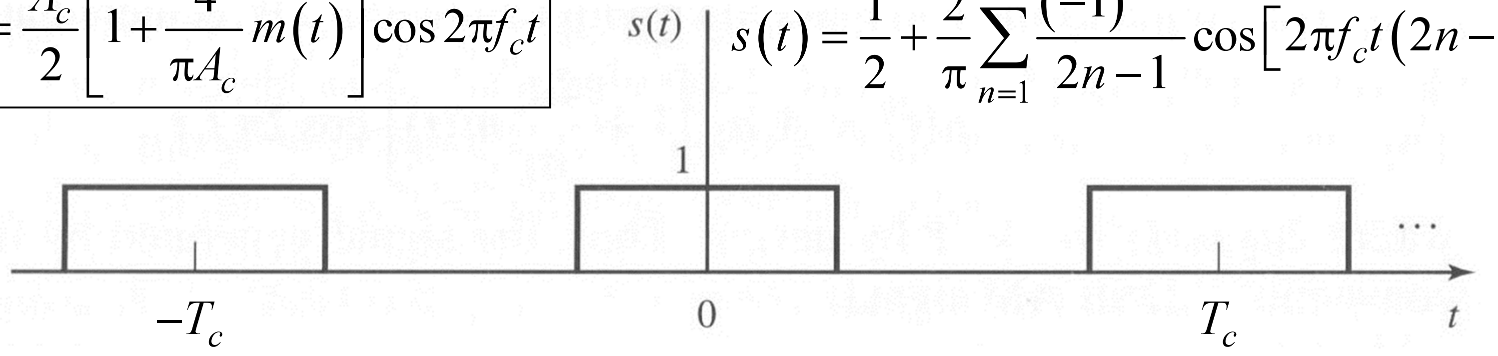
+ BPF at f_c



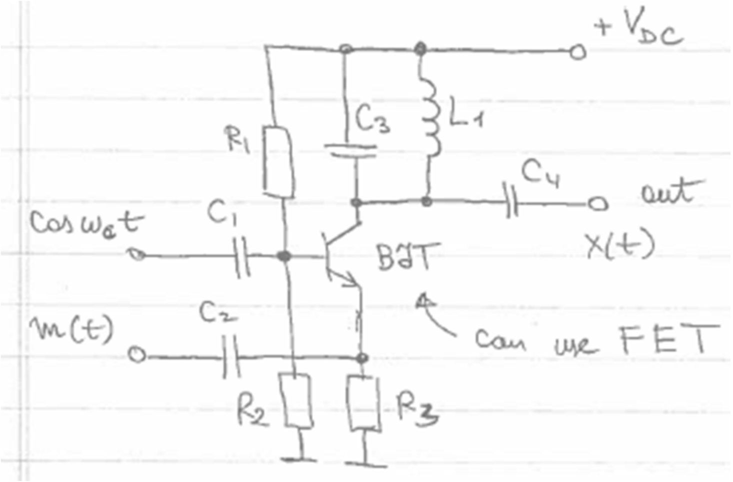
$$v_2(t) = \begin{cases} v_1(t), v_1(t) \geq 0 \\ 0, v_1(t) < 0 \end{cases}$$

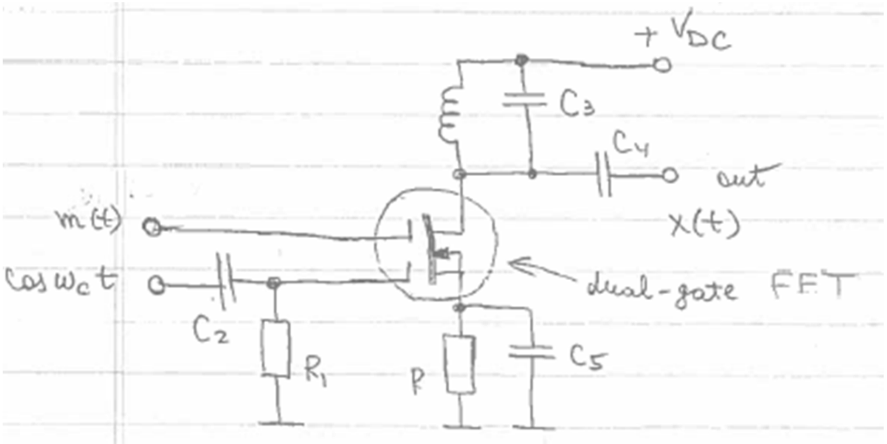
$$\boxed{\begin{aligned} x(t) &= \mathrm{BPF}\left\{\left(m(t) + A_c \cos 2\pi f_c t\right) s(t)\right\} \\ &= \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos 2\pi f_c t \end{aligned}}$$

$$s(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t (2n-1)\right]$$



Examples of Modulators for Conventional AM

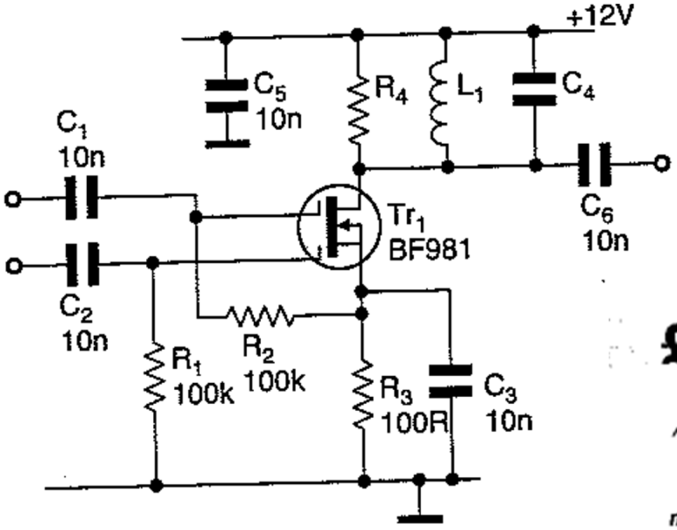




For more info, see e.g.

- B. Razavi, RF Microelectronics, Prentice Hall, 2012.
- U.L. Rohde, D.P. Newkirk, RF/Microwave Circuit Design for Wireless Applications, Wiley, 2000.
- B. Leung, VLSI for Wireless Communications, Prentice Hall, 2002.

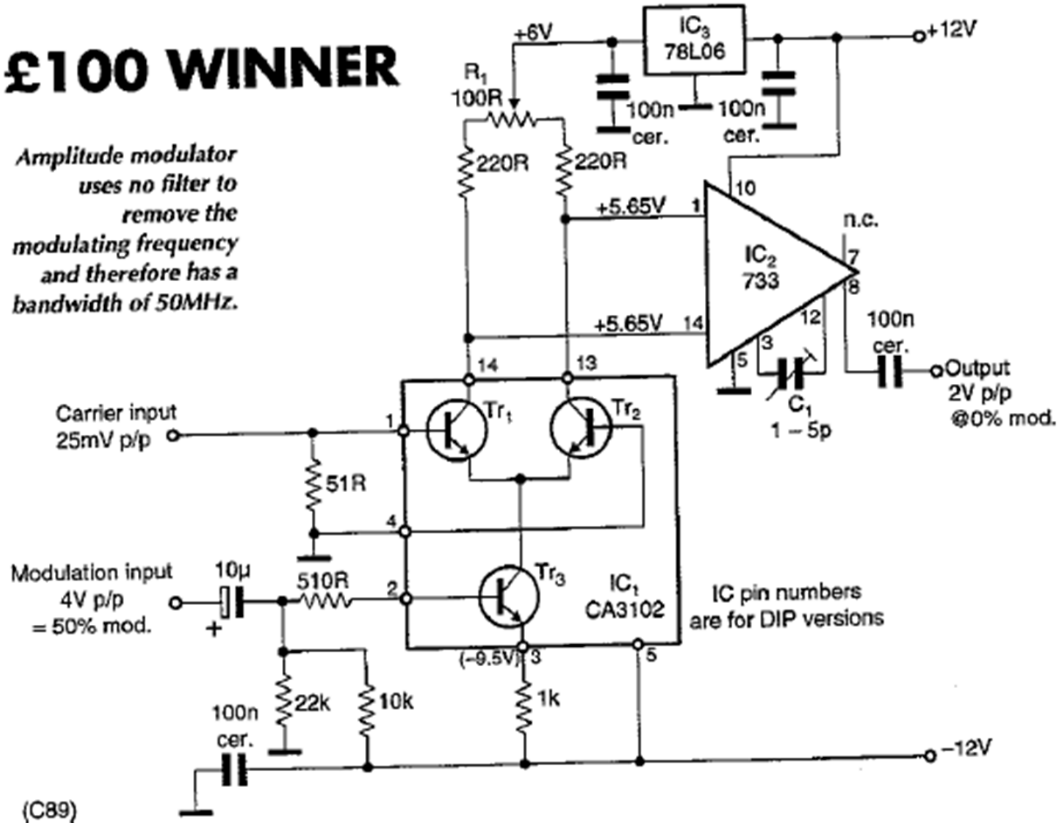
Examples of Modulators for Conventional AM



adopted from Electronics World,
July 1998.

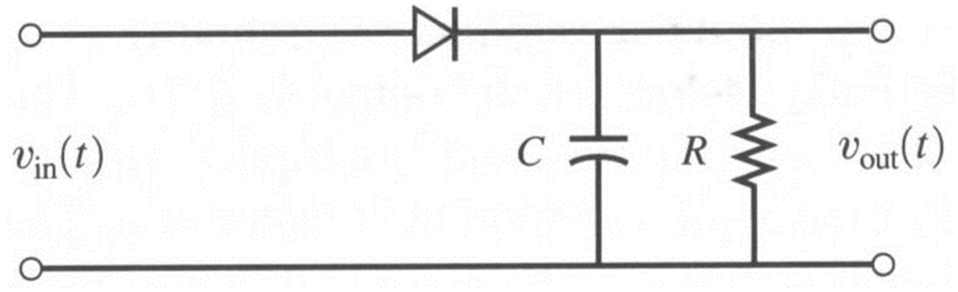
£100 WINNER

Amplitude modulator uses no filter to remove the modulating frequency and therefore has a bandwidth of 50MHz.



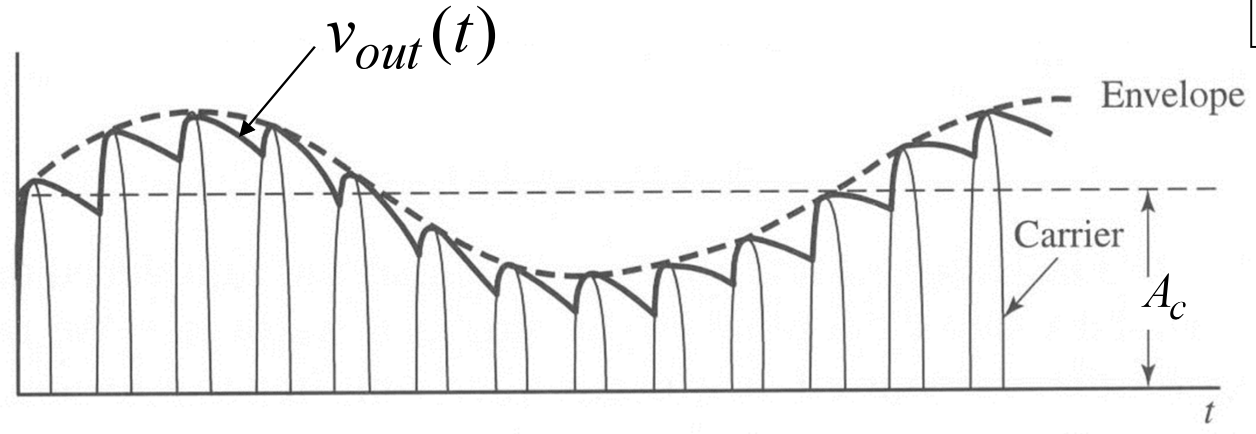
Demodulation of Conventional AM

- Envelope detector
- RC – lowpass filter
- Attenuates carrier, passes modulating signal



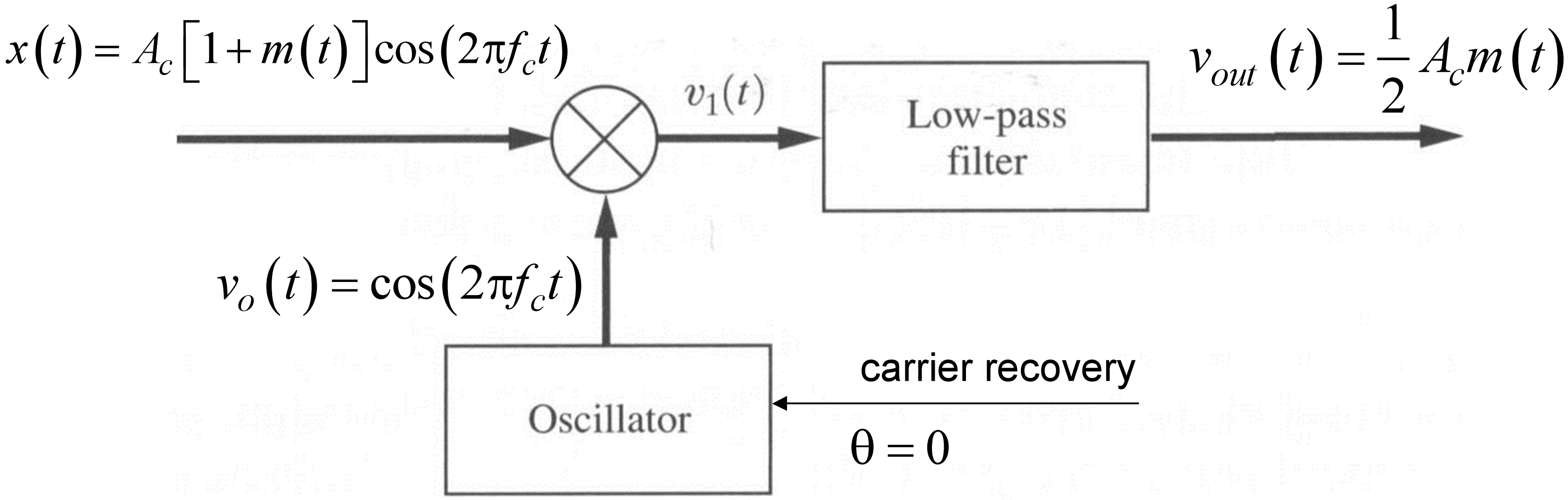
$$\boxed{\Delta f << \frac{1}{2\pi RC} << f_c}$$

$$\boxed{T_c << \tau = RC << T_m}$$



Demodulation of Conventional AM

- Product detector:



$x(t)=A_c\left[1+m(t)\right]\cos(2\pi f_c t)$

$v_{out}(t)=\frac{1}{2}A_c m(t)$

$v_o(t)=\cos(2\pi f_c t)$

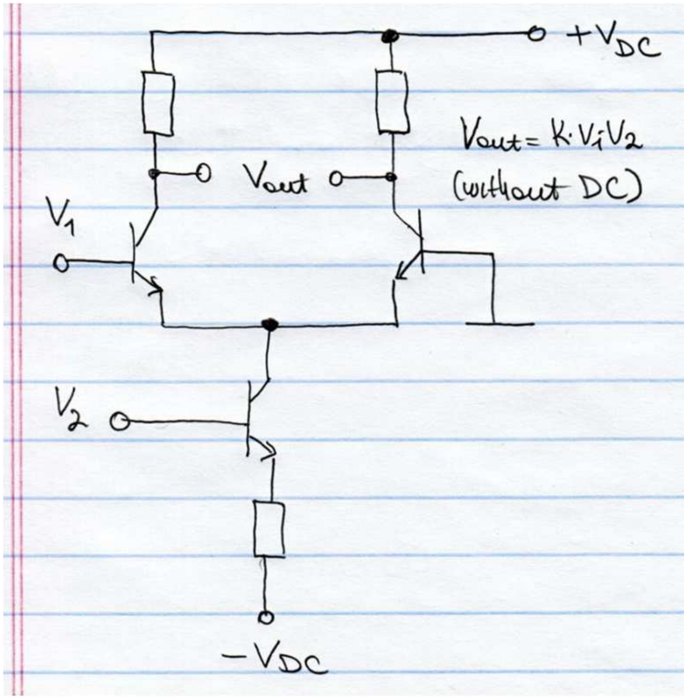
carrier recovery

$\theta=0$

- What happens if $\theta\neq0$?

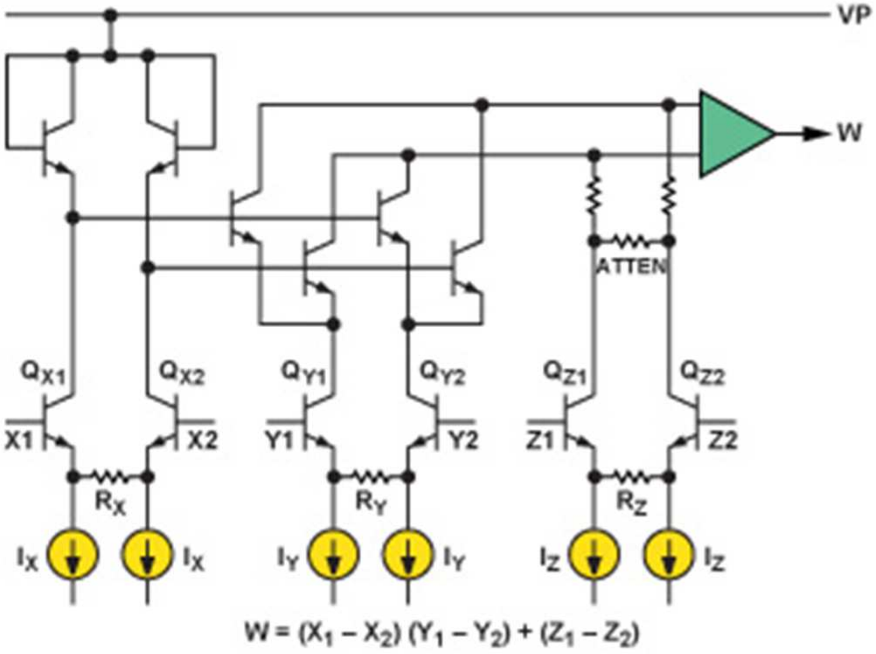
Multiplier Implementation

simplified



more realistic

AD534: A Four-Quadrant Translinear Multiplier



Adopted from “Considering Multipliers ” by B. Gilbert

Advantages/Disadvantages of Conventional AM

- Advantages
  - – Very simple demodulation (envelope detector)
  - – “Linear” modulation*
- Disadvantages
  - – Low power efficiency
  - – Doubles the baseband bandwidth

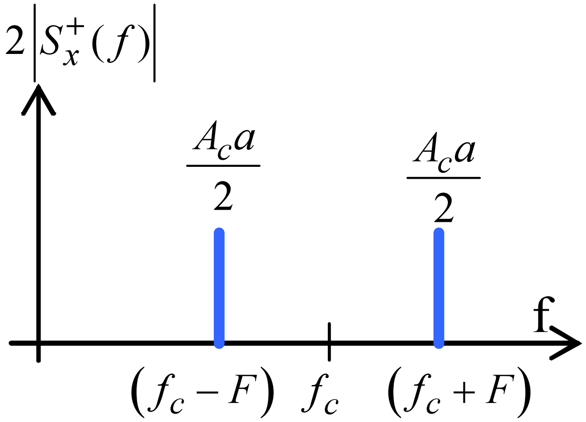
*Q.: is the conventional AM modulator an LTI system?

Double-Sided AM: Suppressed Carrier (DSB-SC)

- How to increase power efficiency?
- DSB-SC signal: $\boxed{x(t) = A_c m(t)\cos(2\pi f_c t)}$
- Example: sinusoidal modulation,

$$\boxed{x(t) = A_c a\cos(2\pi F t)\cos(2\pi f_c t)}$$

- Spectrum: $\boxed{x(t) = \frac{aA_c}{2}\left[\cos(2\pi(f_c - F)t) + \cos(2\pi(f_c + F)t)\right]}$



Bandwidth???

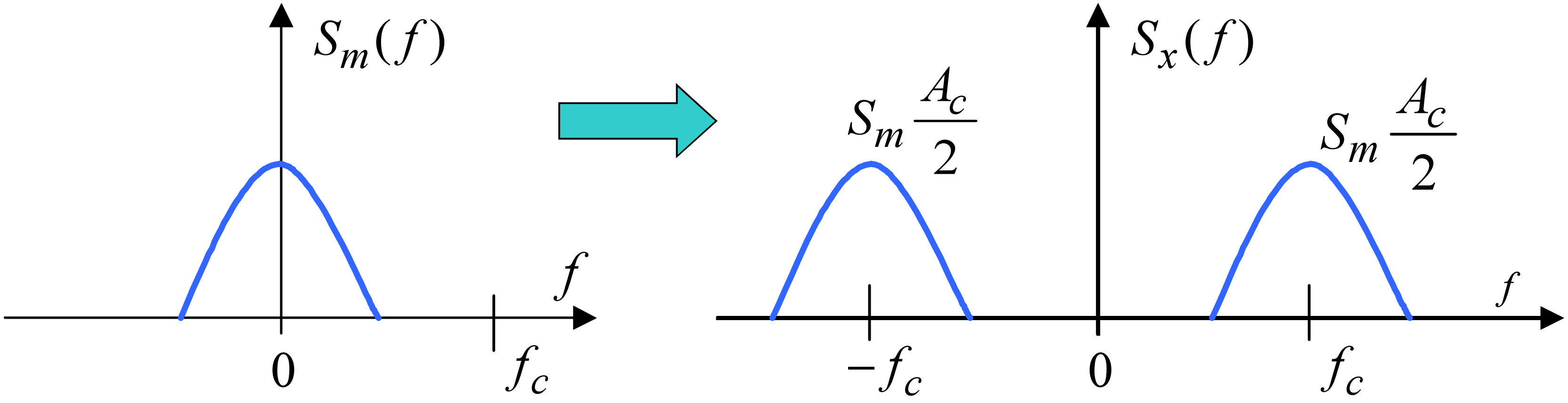
Geometrical representation???

Power efficiency???

Modulation index???

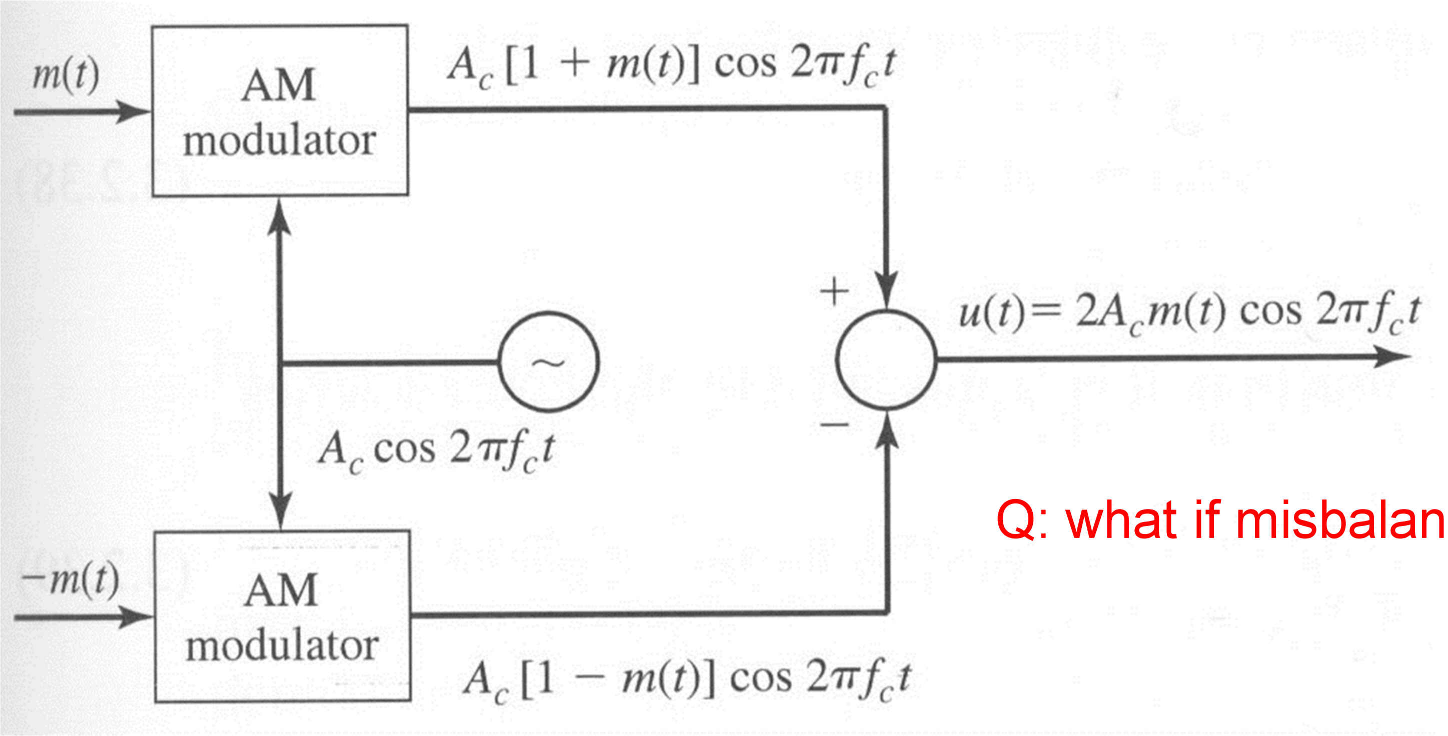
DSB-SC: General Case

- DSB-SC signal: $\boxed{x(t) = A_c m(t)\cos(2\pi f_c t)}$
- Spectrum: $\boxed{S_x(f) = \frac{A_c}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right]}$
- What do you see on a spectrum analyzer?
- Bandwidth ? Power efficiency? PSD?



Generation of DSB-SC

- Generation:
  - Mixer. Not practical in many cases.
  - Filtered conventional AM. Not practical.
- Balanced modulator:

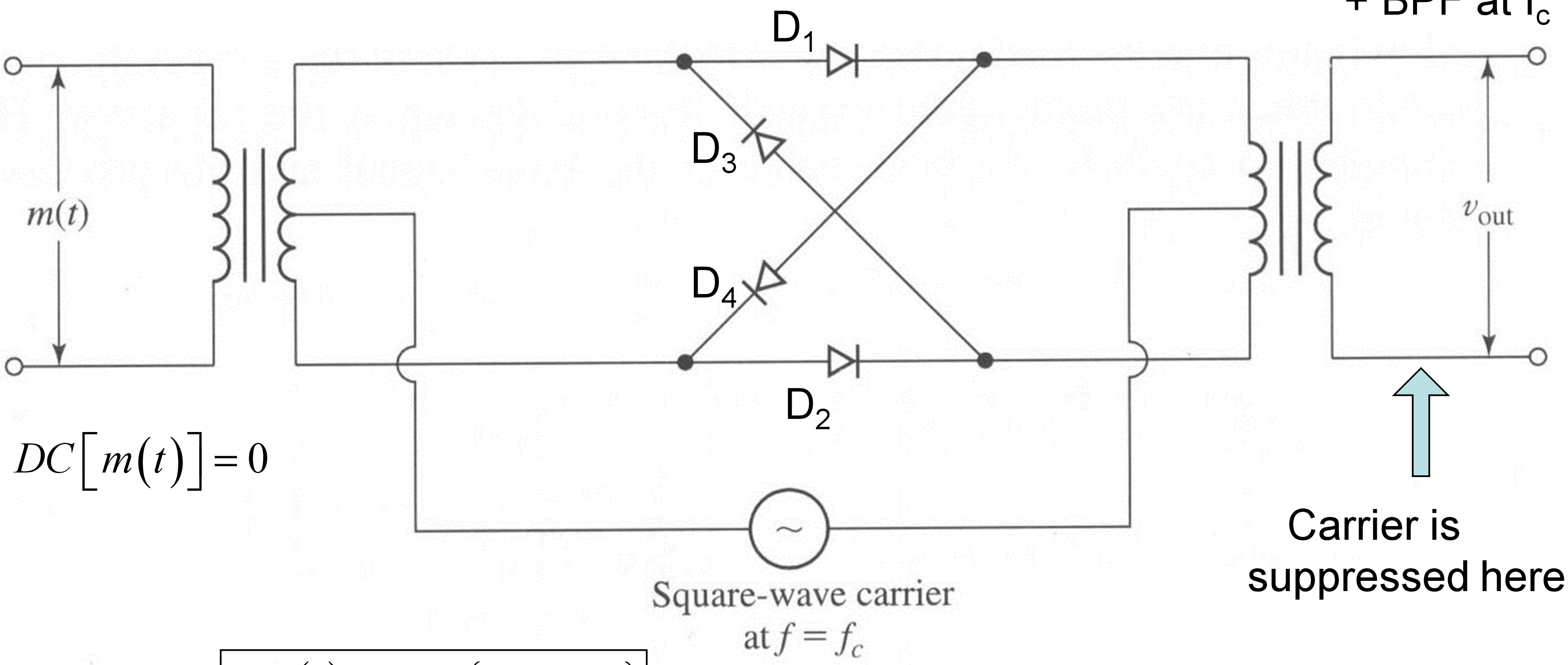


Q: what if misbalanced?

J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Generation of DSB-SC

Ring modulator



+ BPF at f_c

$$DC\left[m(t)\right] = 0$$

Carrier is suppressed here

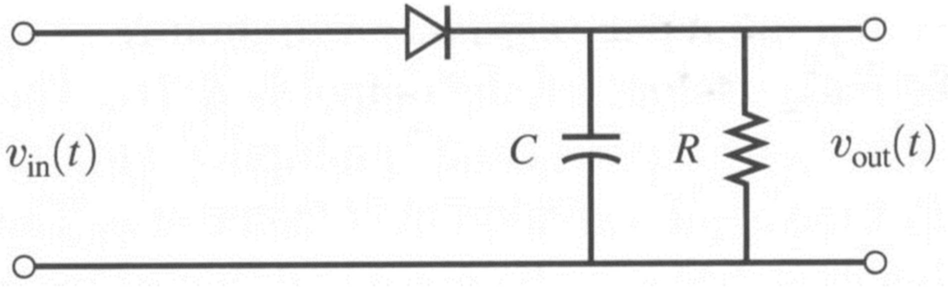
$$\boxed{\begin{aligned} v_{out}(t) &= \text{BPF}\{m(t)s(t)\} \\ &= \frac{2A_c}{\pi} m(t)\cos 2\pi f_c t \end{aligned}}$$

$$s(t) = A_c \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right]$$

- Large-amplitude sinusoidal signal may be used instead of the square wave

Demodulation of DSB-SC

- Why will the envelope detector not work?



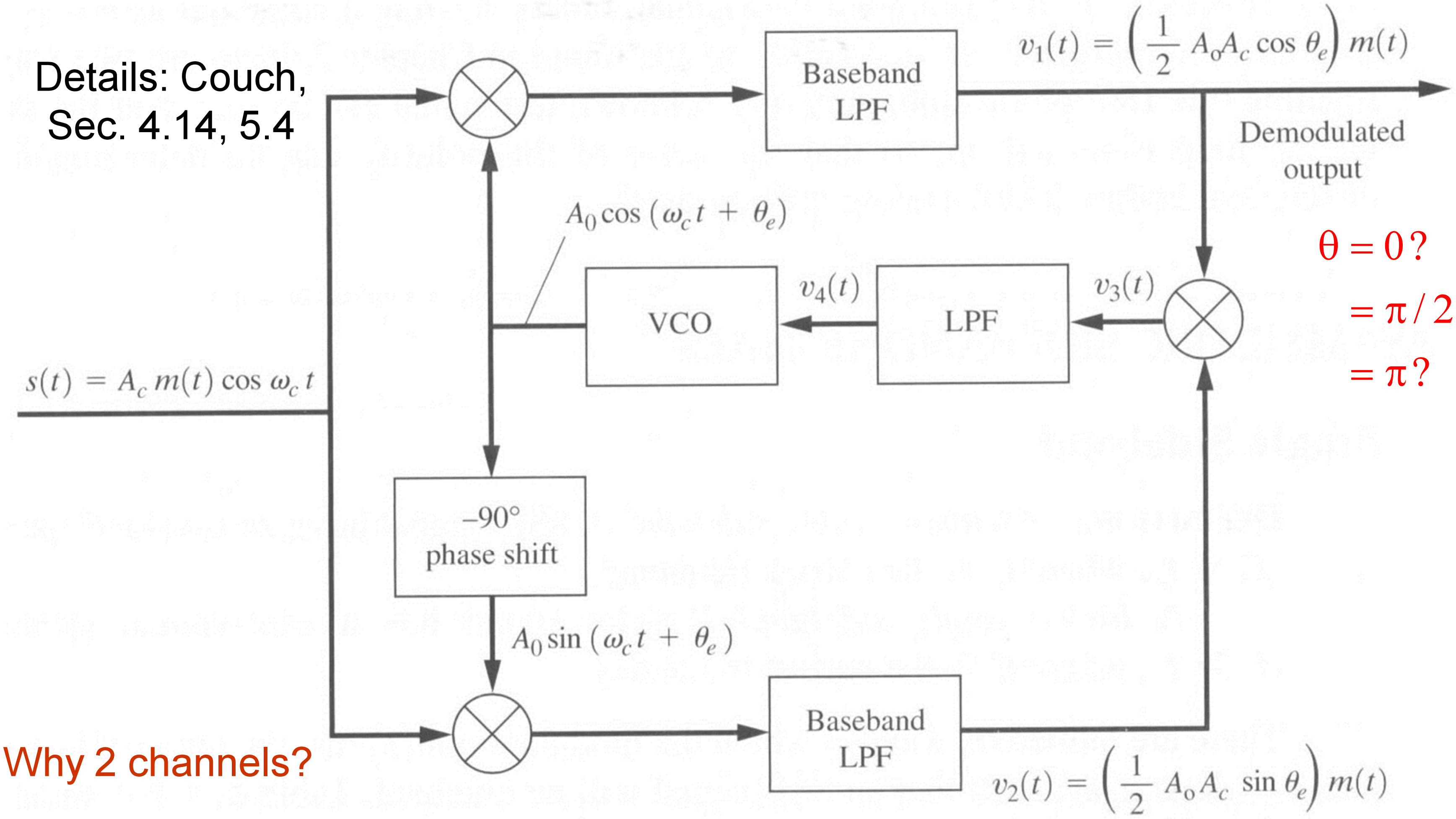
$$x_{in}(t) = m(t)\cos(2\pi f_c t)$$

$$x_{out}(t) = ?$$

Demodulation of DSB-SC

- Demodulation - Costas loop:

Details: Couch, Sec. 4.14, 5.4



$\theta = 0?$

$= \pi/2?$

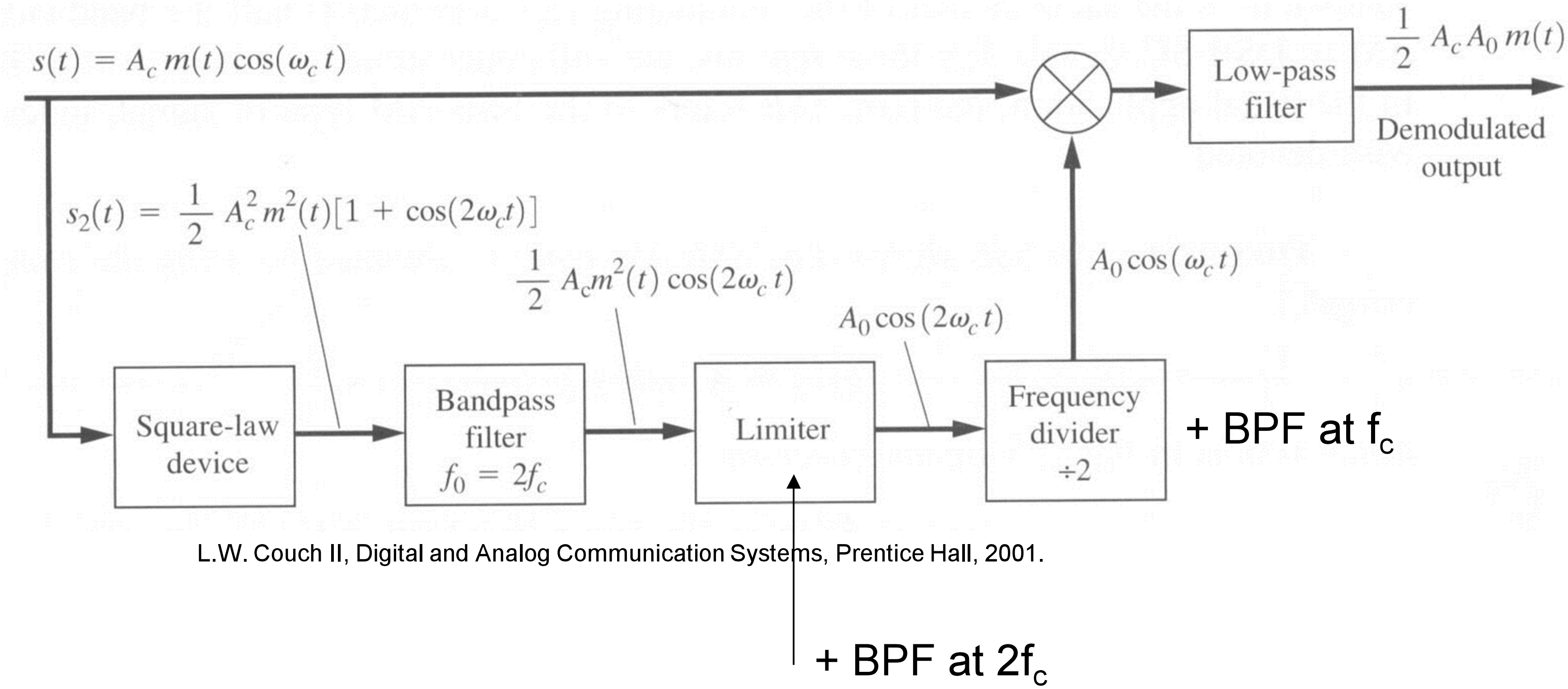
$= \pi?$

Why 2 channels?

L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Demodulation of DSB-SC

- Product detector + squaring carrier recovery loop:



+ BPF at f_c

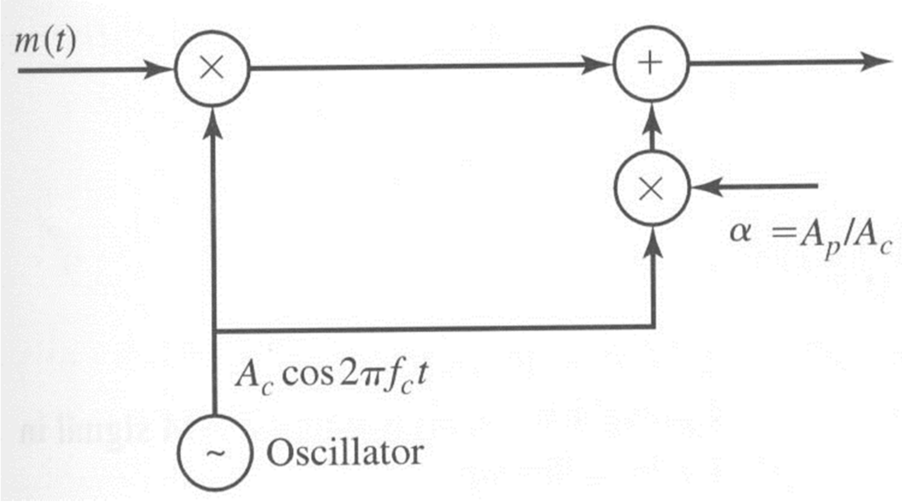
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

+ BPF at $2f_c$

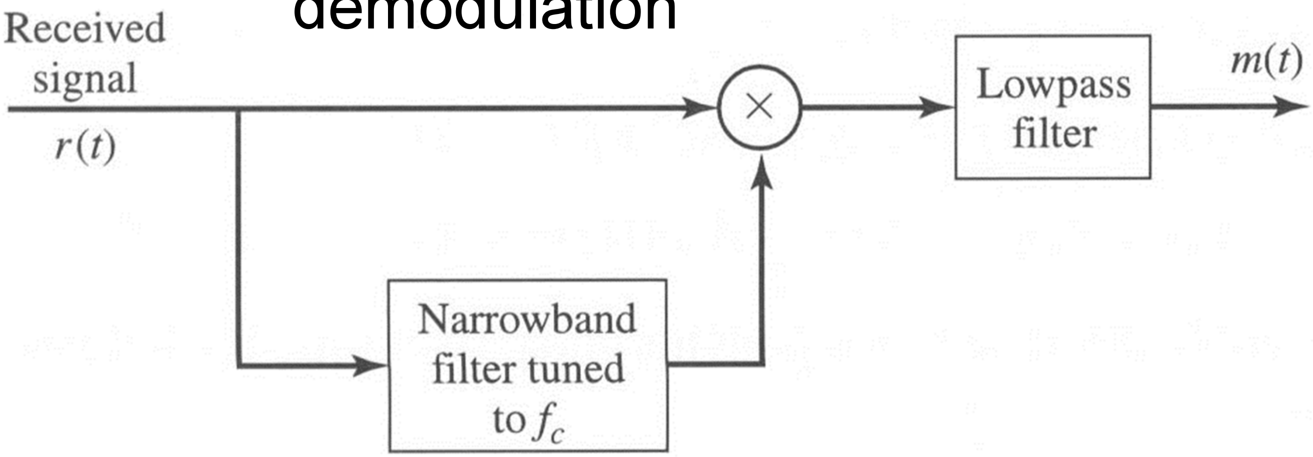
Demodulation of DSB-SC

- Using a pilot tone:

modulation



demodulation



J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Advantages/Disadvantages of DSB-SC

- Advantages
  - High power efficiency
  - If message m(t)>0, envelope detection is possible
- Disadvantages
  - Doubles the baseband bandwidth
  - Complex modulation/demodulation (some form of carrier recovery is required)
  - Pilot tone may be required to simplify demodulation

Summary

- Modulation process. Types of analog modulation.
- Conventional AM. Time-domain & frequency-domain representations. Power efficiency & bandwidth.
- Generation (modulation) & demodulation of conv. AM.
- Double sideband suppressed carrier (DSB-SC). Spectrum. Bandwidth. Generation & demodulation of DSB-SC.
- Advantages/disadvantages of conventional & DSB-SC AM.
- **Homework**: Reading: Couch, 5.1-5.4, 4.4., 4.11-4.13. Study carefully all the examples, make sure you understand and can solve them with the book closed.
- Do some end-of-chapter problems. Students' solution manual provides solutions for many of them.