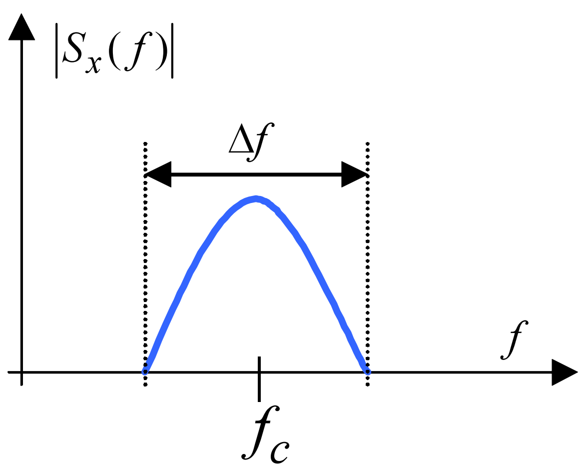
Baseband & Bandpass Signals

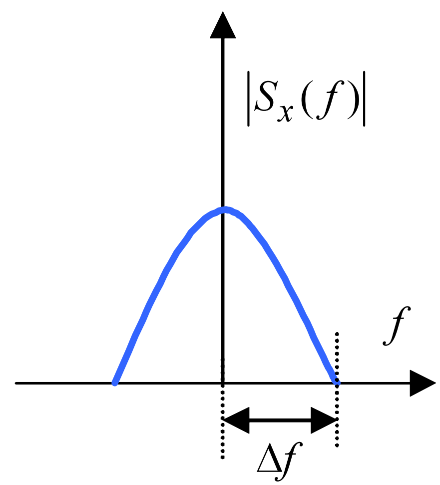
- *Baseband* (lowpass) signal: spectrum is (possibly) nonzero around the origin (f=0) and zero (negligible) elsewhere:

$$S_x(f) = 0, \quad |f| > f_{\max}$$

- *Bandpass* (narrowband) signal: spectrum is (possibly) nonzero around the carrier frequency f_c and zero (negligible) elsewhere:

$$S_x(f) = 0, \quad |f - f_c| > B$$





Complex Envelope Representation

- Any narrowband (bandpass) signal can presented as

$$x(t) = \mathrm{Re}\left\{C(t)e^{j\omega_c t}\right\} = A(t)\cos\left(\omega_c t + \varphi(t)\right)$$

where $C(t) = A(t)e^{j\varphi(t)}$ is complex envelope (phasor), $A(t) = \left|C(t)\right|$ is amplitude, and $\varphi(t) = \angle C(t)$ is phase.

- Amplitude and phase vary in time, but much slower than the carrier. This is a generalization of the harmonic signal.
- Equivalent form (in-phase (I) & quadrature (Q)):

$$x(t) = a_I(t)\cos\left(\omega_c t\right) - a_Q(t)\sin\left(\omega_c t\right)$$

where
$$a_I(t) = \mathrm{Re}\left\{C(t)\right\} = A(t)\cos\left(\varphi(t)\right),$$
$$a_Q(t) = \mathrm{Im}\left\{C(t)\right\} = A(t)\sin\left(\varphi(t)\right)$$

Complex Envelope Representation

- $C(t), A(t), \varphi(t), a_I(t), a_Q(t)$ - are baseband signals.
- Proof of the complex envelope representation – multiplication property of FT.
- Some additional relations:

$$C(t) = a_I(t) + j a_Q(t)$$

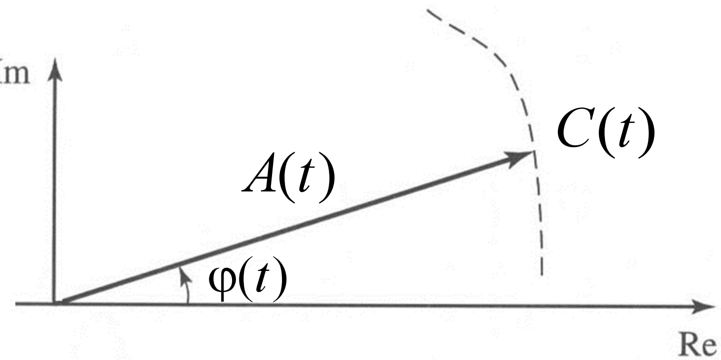
$$A(t) = \sqrt{a_I^2(t) + a_Q^2(t)}$$

$$\varphi(t) = \tan^{-1}\left(\frac{a_Q(t)}{a_I(t)}\right)$$

- Very useful for analysis & simulation of modulated signals

Geometric Viewpoint of Narrowband Signals

Complex envelope



- $A(t)$ is rotating at $d\varphi(t)/dt$ (rad/s)
- $z(t)$ is rotating at $2\pi f_c$ (rad/s) w.r.t. *A(t)*

Distortionless Transmission

- When a signal is not distorted by a filter?
- Output is a shifted and scaled copy of the input:

$$y(t) = \mathbf{L}\left[x(t)\right] = a \cdot x(t - t_0)$$

- In the frequency domain:

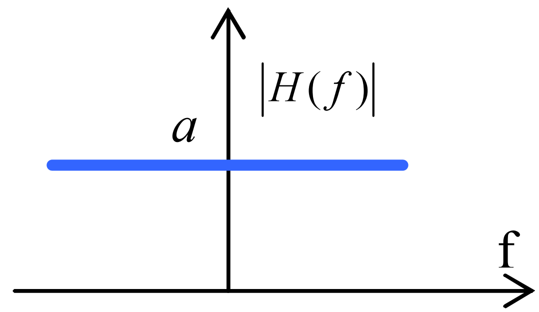
$$S_y(f) = a \cdot e^{-j2\pi f t_0} S_x(f)$$

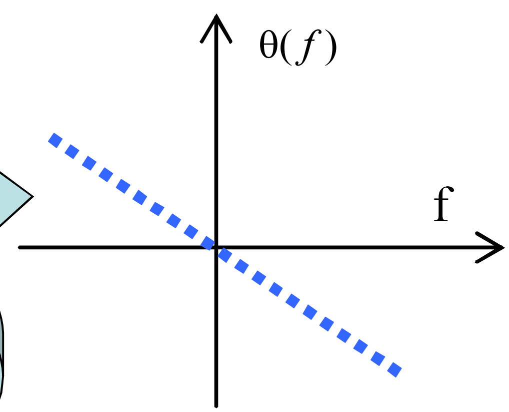
- Filter frequency response:

$$H(f) = a \cdot e^{-j2\pi f t_0}$$

$$\begin{cases} |H(f)| = const \\ \theta(f) = -2\pi f t_0 \end{cases}$$

$$t_0 = -\theta(f)/(2\pi f)$$

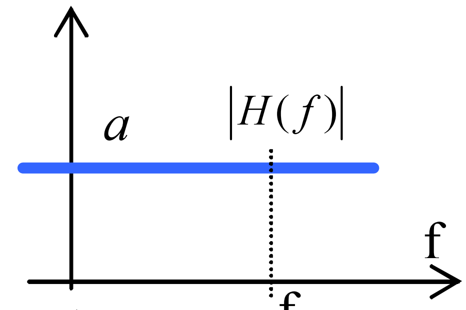




Distortionless Transmission: Narrowband Signals

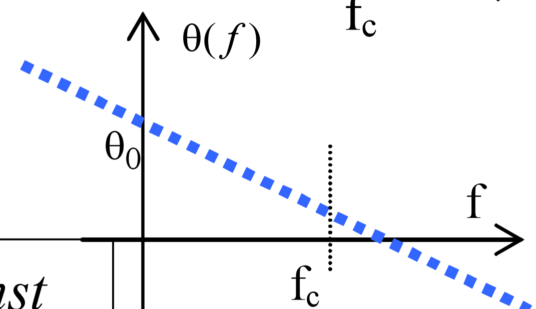
- Output is a shifted and scaled copy of the input + *constant phase shift* of the carrier is permitted:

$$x(t) = A(t)\cos\left(\omega_c t + \varphi(t)\right) \Rightarrow$$
$$y(t) = a \cdot A(t-t_0)\cos\left(\omega_c (t-t_0) + \varphi(t-t_0) + \theta_0\right)$$



- In the frequency domain:

$$S_y^+(f) = a \cdot e^{j(-2\pi f t_0 + \theta_0)} S_x^+(f)$$



- Filter frequency response (over the signal bandwidth):

$$\begin{cases} |H(f)| = const \\ \theta(f) = -2\pi f t_0 + \theta_0 \end{cases}$$

group time delay:
carrier time delay:

$$\begin{cases} t_g = -\dfrac{1}{2\pi}\dfrac{d\theta(f)}{df} \\ t_c = t_0 - \theta_0/(2\pi f_c) \end{cases}$$

Summary

- Baseband (lowpass) & narrowband (bandpass) signals & systems
- Complex envelope representation. Time-varying amplitude & phase.
- Geometric representation of narrowband signals.
- Distortionless transmission.

- **Homework**: Couch, 4.1-4.6. Study carefully all the examples and make sure you understand them. Attempt some end-of-chapter problems (there are answers to selected problems at the end!).