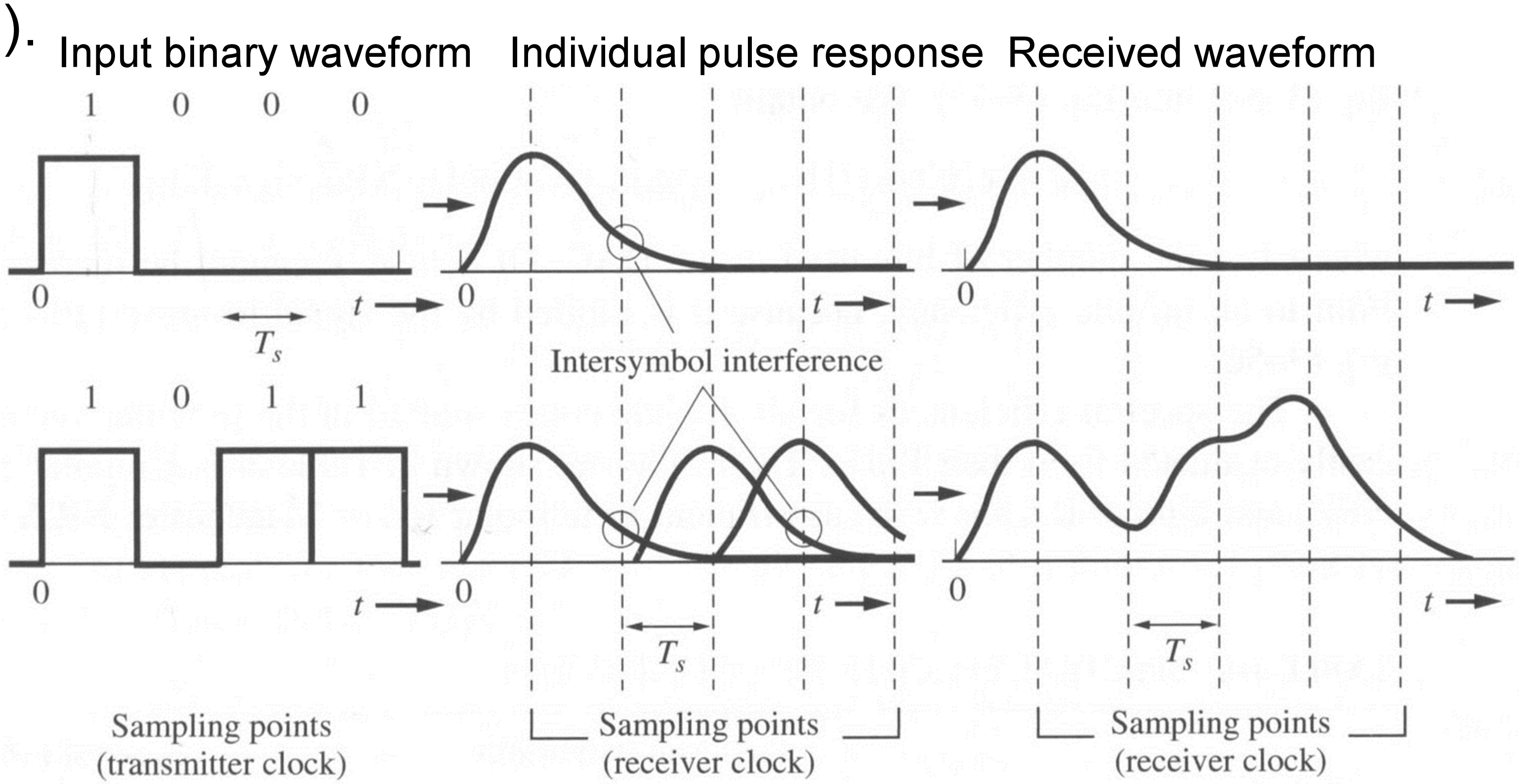
Band-limited Channels and Intersymbol Interference

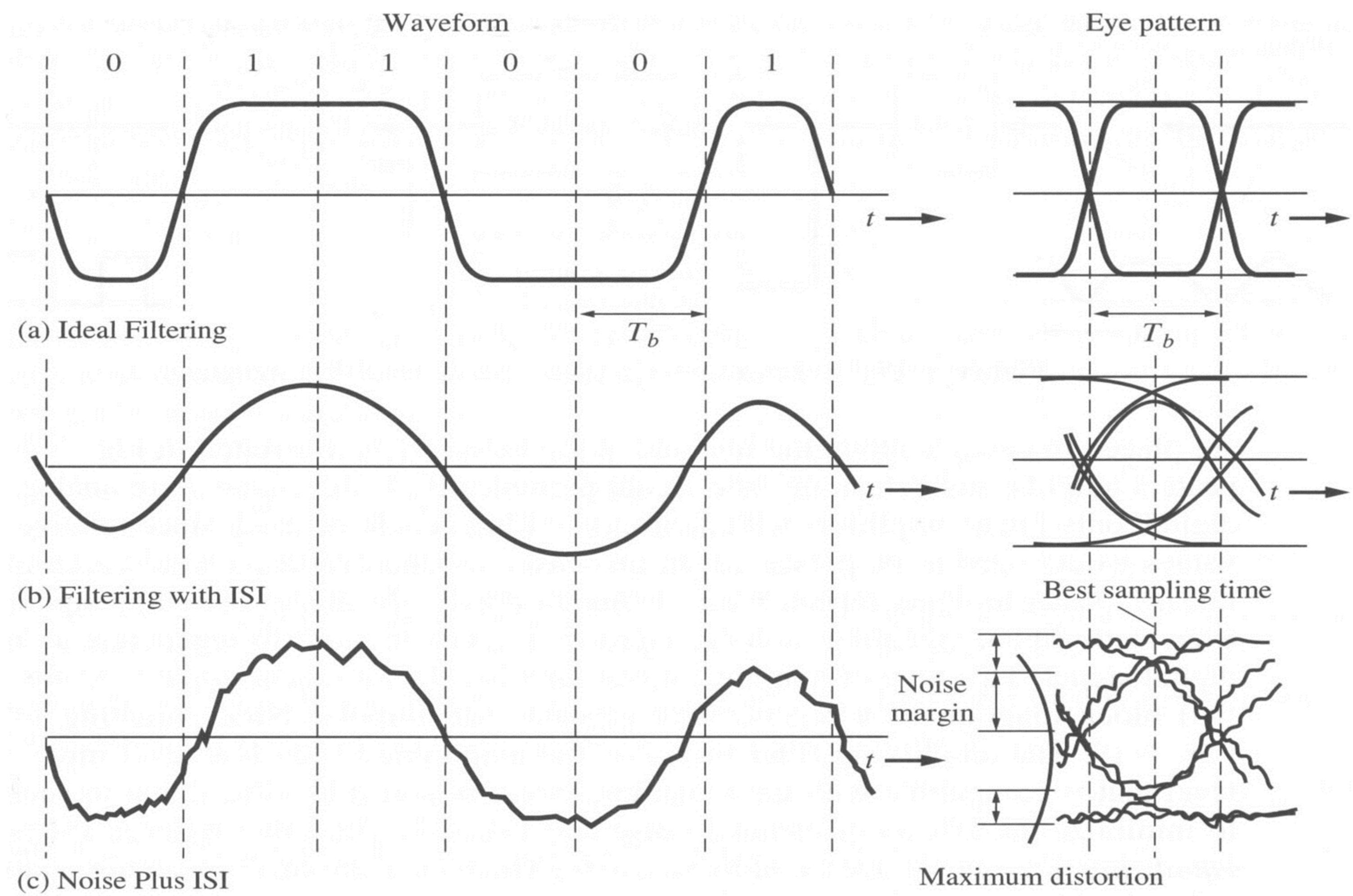
- Rectangular pulses are suitable for infinite-bandwidth channels (practically – wideband).
- Practical channels are band-limited -> pulses spread in time and are smeared into adjacent slots. This is intersymbol interference (ISI).



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Eye Diagram

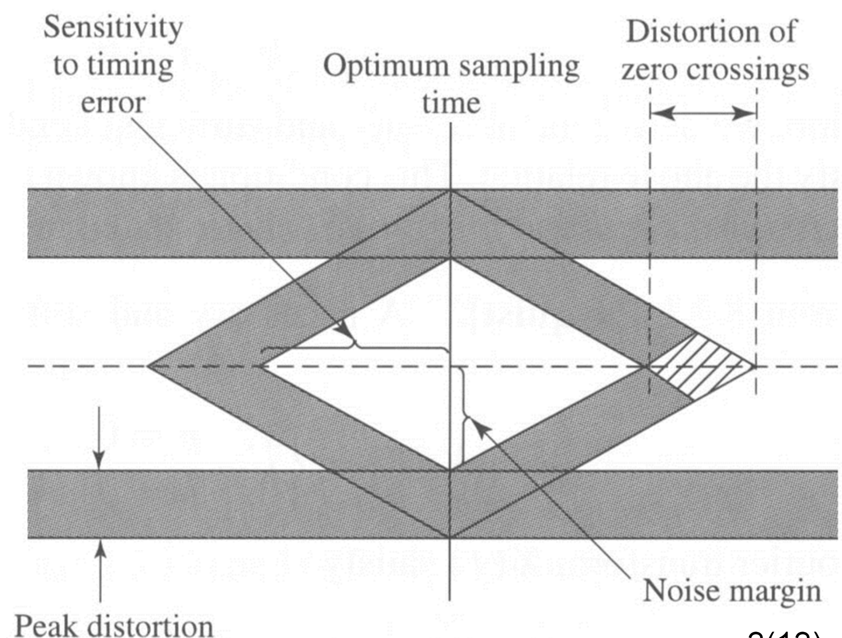
- Convenient way to observe the effect of ISI and channel noise on an oscilloscope.



Eye Diagram

- Oscilloscope presentations of a signal with multiple sweeps (triggered by a clock signal!), each is slightly larger than symbol interval.
- Quality of a received signal may be estimated.
- Normal operating conditions (no ISI, no noise) -> eye is open.
- Large ISI or noise -> eye is closed.

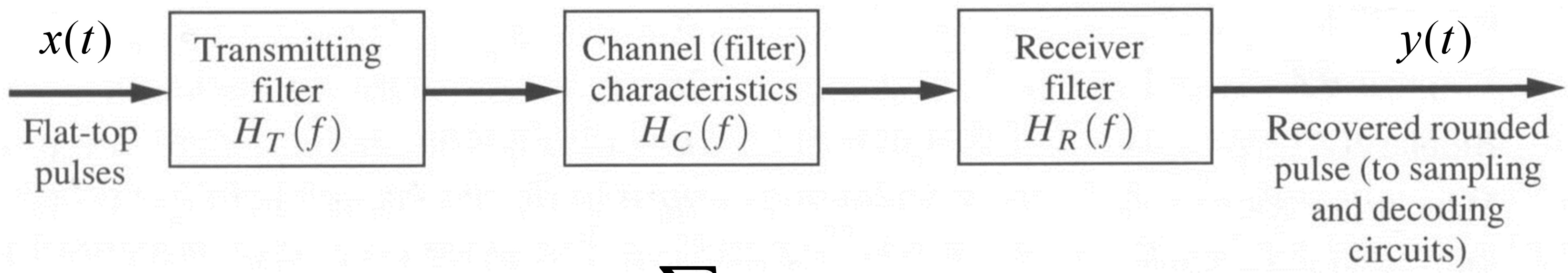
- Timing error allowed – width of the eye, called eye opening (preferred sampling time – at the largest vertical eye opening).
- Sensitivity to timing error -> slope of the open eye evaluated at the zero crossing point.
- Noise margin -> the height of the eye opening.



J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Transmission over a Band-Limited Channel

baseband transmission system



- Input PAM signal: $x(t) = \sum_n a_n s_T(t - nT)$

T – symbol interval

f_0=1/T – symbol rate

- Output signal: $y(t) = \sum_n a_n s(t - nT),$

$$s(t) = s_T(t) * h_T(t) * h_C(t) * h_R(t) = s_T(t) * h(t)$$

- Sampled (at *t=mT*) output:

transmitted symbol

$$y_m = \sum_n a_n s_{m-n} = a_m s_0 + \sum_{n \neq m} a_n s_{m-n},$$

$$y_m = y(mT),\ s_{m-n} = s(mT - nT)$$

sampled output

ISI

Pulse Shaping to Eliminate ISI

- Nyquist (1928) discovered 3 methods to eliminate ISI
  - zero ISI pulse shaping
  - controlled ISI (eliminated later on by, say, equalizer)
  - zero average ISI (negative and positive areas under the pulse in an adjacent interval are equal)
- Zero ISI pulse shaping:

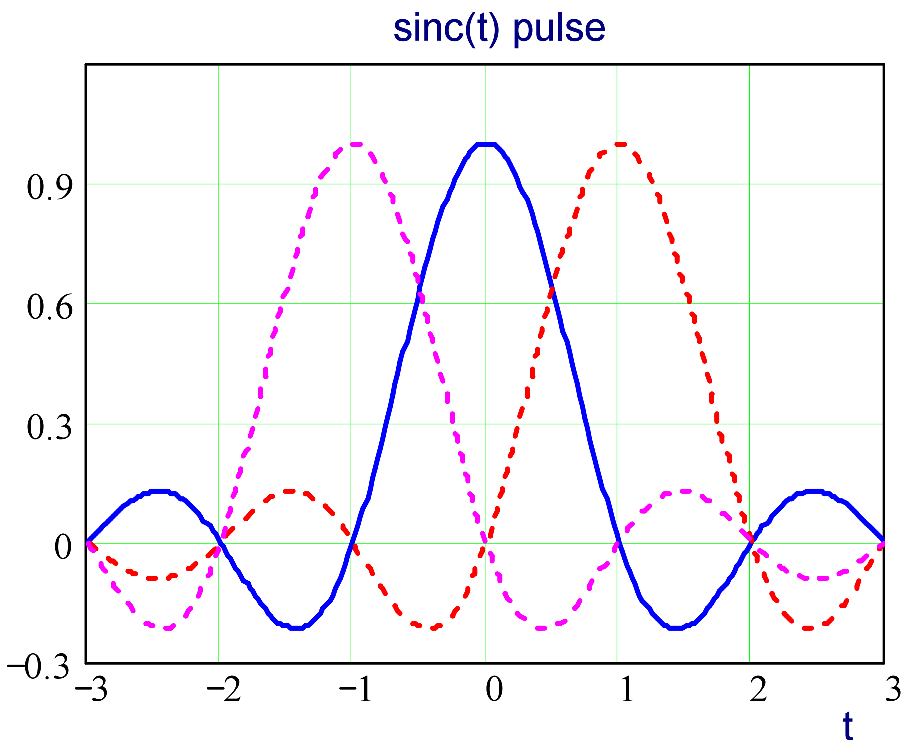
$$s(nT) = \begin{cases} s_0, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$y_m = a_m s_0 + \sum_{n \neq m} a_n s_{m-n} \Rightarrow y_m = a_m s_0$$

- Example: $s(t) = \text{sinc}(f_0 t) \Rightarrow s(nT) = \text{sinc}(n) = 0,\ n \neq 0$

Note: f_0=1/T – transmission rate [symbols/s]

Zero ISI: sinc Pulse



$S_s(f) = ?$

- Example: $s(t) = \text{sinc}(f_0 t) \Rightarrow s(nT) = \text{sinc}(n) = 0, \; n \neq 0$
- Hence, *sinc* pulse allows to eliminate ISI at sampling instants. However, it has some (2) serious drawbacks.

Nyquist Criterion for Zero ISI

- Provides a generic solution to the zero ISI problem.
- A necessary and sufficient condition for s(t) to satisfy

$$s(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \quad \Leftrightarrow \quad \text{is} \quad \sum_{m=-\infty}^{\infty} S_s\left(f + \frac{m}{T}\right) = T$$

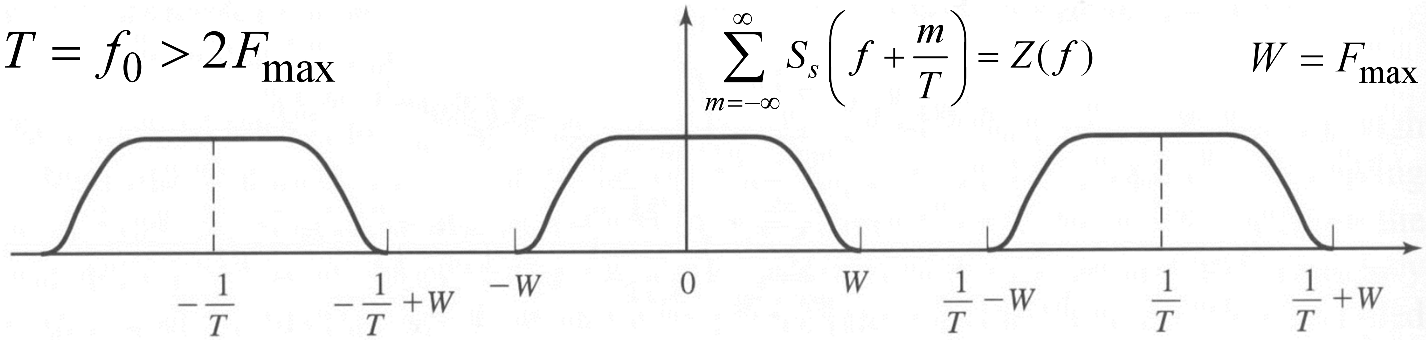
- Proof: homework (see the text by Proakis and Salehi, Sec.8.3.1).
  - Hint: consider ideal sampling of s(t) and find its FT
- Consider a channel badndlimited to $[0, F_{max}]$. Three cases:

  1) $1/T = f_0 > 2F_{\max}$ -> no way to eliminate ISI.
  2) $f_0 = 2F_{\max}$ -> only $\text{sinc}(f_0 t)$ eliminates ISI. The highest possible transmission rate f_0 for transmission with zero ISI is $2F_{max}$, not F_{max} (what a surprise!)
  3) $f_0 < 2F_{\max}$ -> many signals may eliminate ISI in this case.

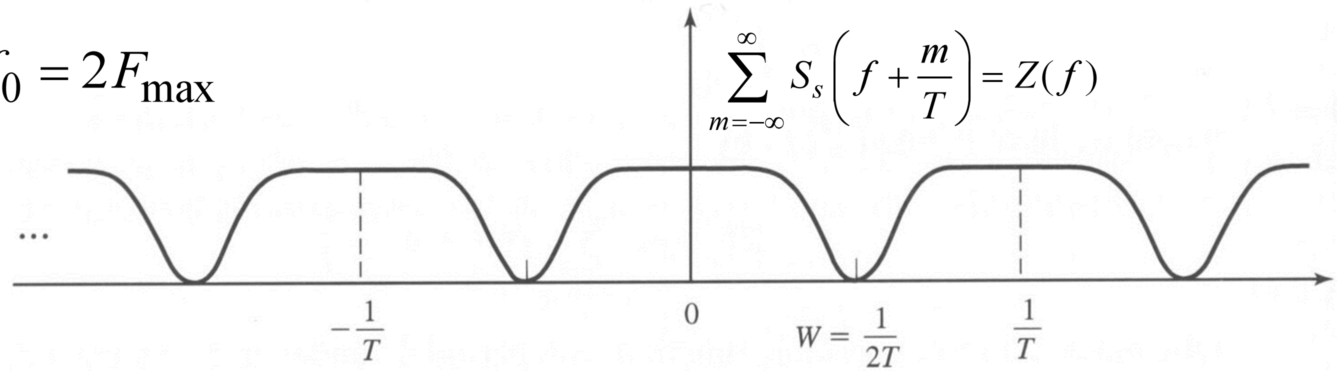
Note: $f_0 = 1/T$ – transmission rate [symbols/s]

Nyquist Criterion for Zero ISI

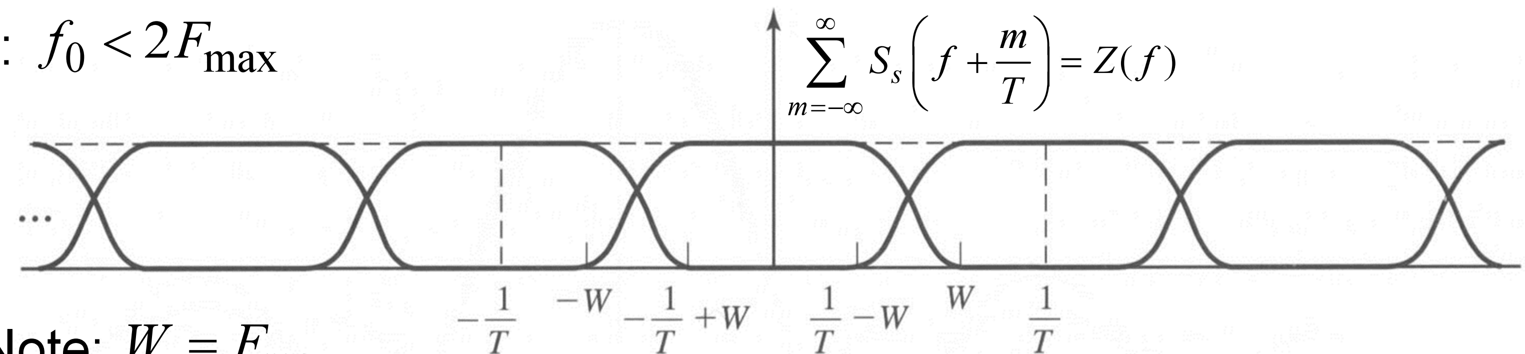
Case 1: $1/T = f_0 > 2F_{\max}$



Case 2: $f_0 = 2F_{\max}$



Case 3: $f_0 < 2F_{\max}$



Note: $W = F_{\max}$

Raised Cosine Pulse

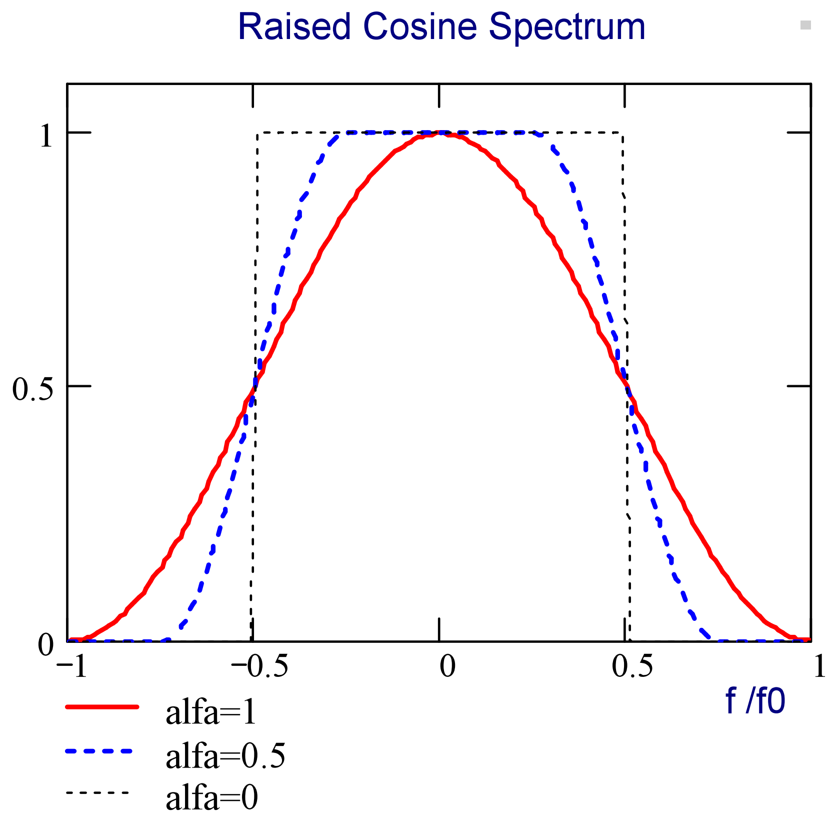
- When $f_0 < 2F_{\max}$, raised cosine pulse is widely used.
- Its spectrum is

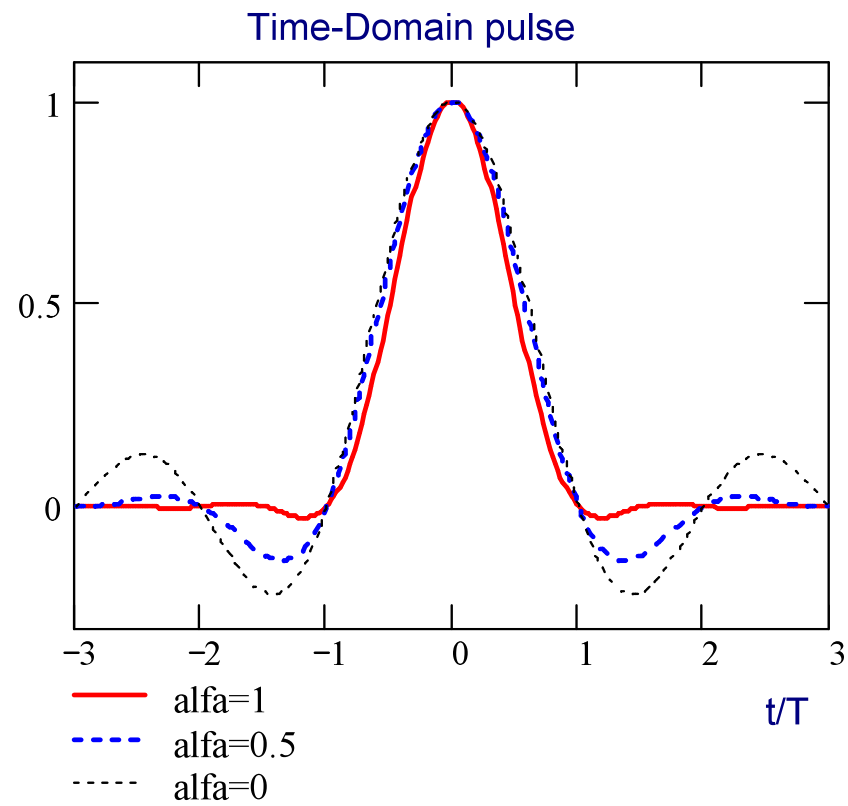
$$S_{rc}(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha) f_0 / 2 \\ \frac{T}{2}\left[1+\cos\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2} f_0\right)\right], & \frac{1-\alpha}{2} f_0 \le |f| \le \frac{1+\alpha}{2} f_0 \\ 0, & |f| > \frac{1+\alpha}{2} f_0 \end{cases}$$

- where α is roll-off factor, $0 \le \alpha \le 1$
- The bandwidth above the Nyquist frequency $f_0/2$ is called excess bandwidth.
- Time-domain waveform of the pulse:

$$s(t) = \text{sinc}(t/T) \frac{\cos(\pi \alpha t / T)}{1 - 4\alpha^2 t^2 / T^2}$$

Raised Cosine Pulse (RCP)





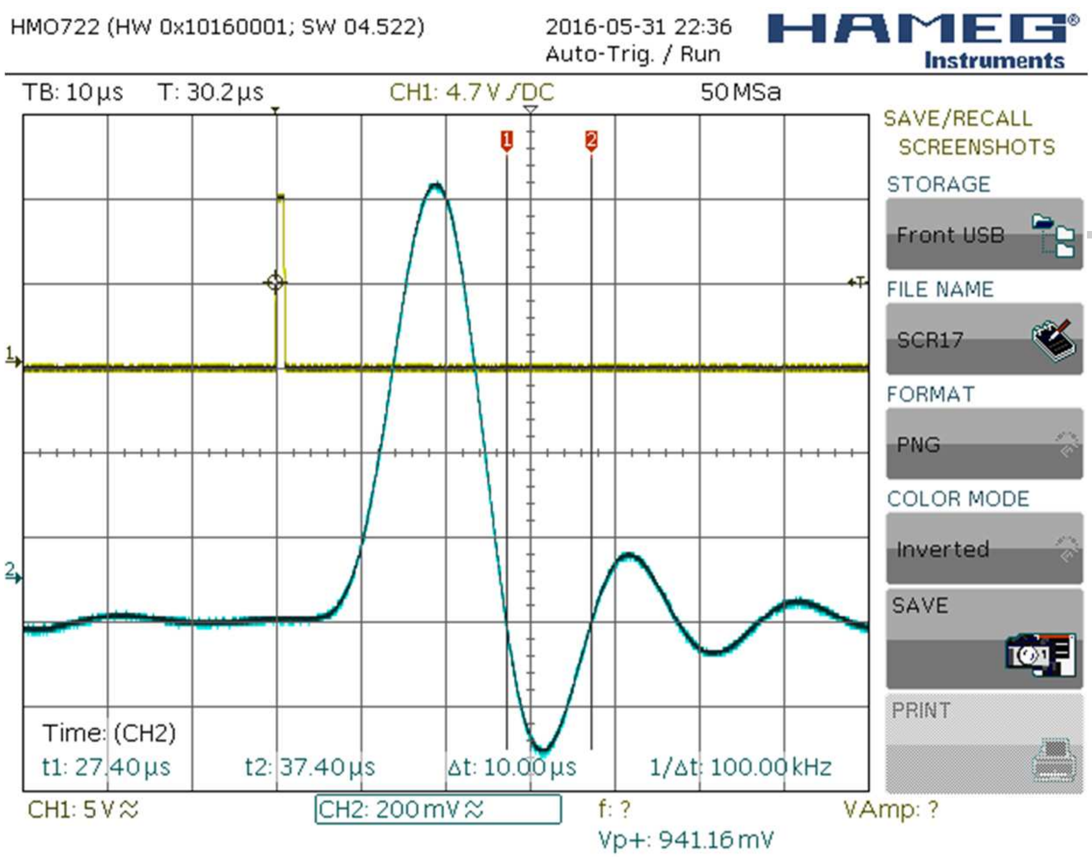
Note that: (1) $\alpha = 0$, pulse reduces to $\mathrm{sinc}(tf_0)$ and $f_0 = 2F_{\max}$

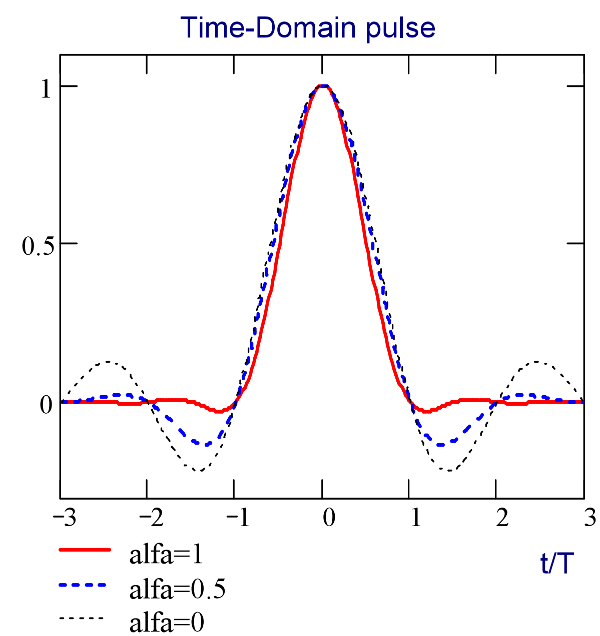
(2) $\alpha = 1$, symbol rate $f_0 = F_{\max}$

(3) tails decay as t^3 -> mistiming is not a big problem

(4) smooth shape of the spectrum -> easier to design filters

Practical sinc(t):
(as measured in Lab 1)





RCP: Example

- R_b=1Mb/s, binary PAM (BPAM) or BPSK, RCP with $\alpha = 1$
- Bandwidth = ?
- Solution: BPAM

$$R_b = R_s = f_0 = 1 \text{ Msymb./s}$$

$$\Delta f_{BPAM} = \frac{1+\alpha}{2} f_0 = 1 \text{ MHz}$$

- Solution: BPSK

$$\Delta f_{BPSK} = 2\Delta f_{BPAM} = 2 \text{ MHz}$$

- 4-PAM, 4-PSK (QPSK): bandwidth = ?

Summary

- Signal design for bandlimited channels.
- Eye diagram. Timing error, sensitivity to timing error, noise margin.
- Transmission over bandlimited channels and intersymbol interference.
- Nyquist criterion for zero ISI. Sinc pulse.
- Raised cosine pulse.
- **Homework**: Reading, Couch, 3.5, 3.6. Study carefully all the examples and make sure you understand and can solve them with the book closed. Do some end-of-chapter problems. Students' solution manual provides solutions for many of them.