Digital Modulation

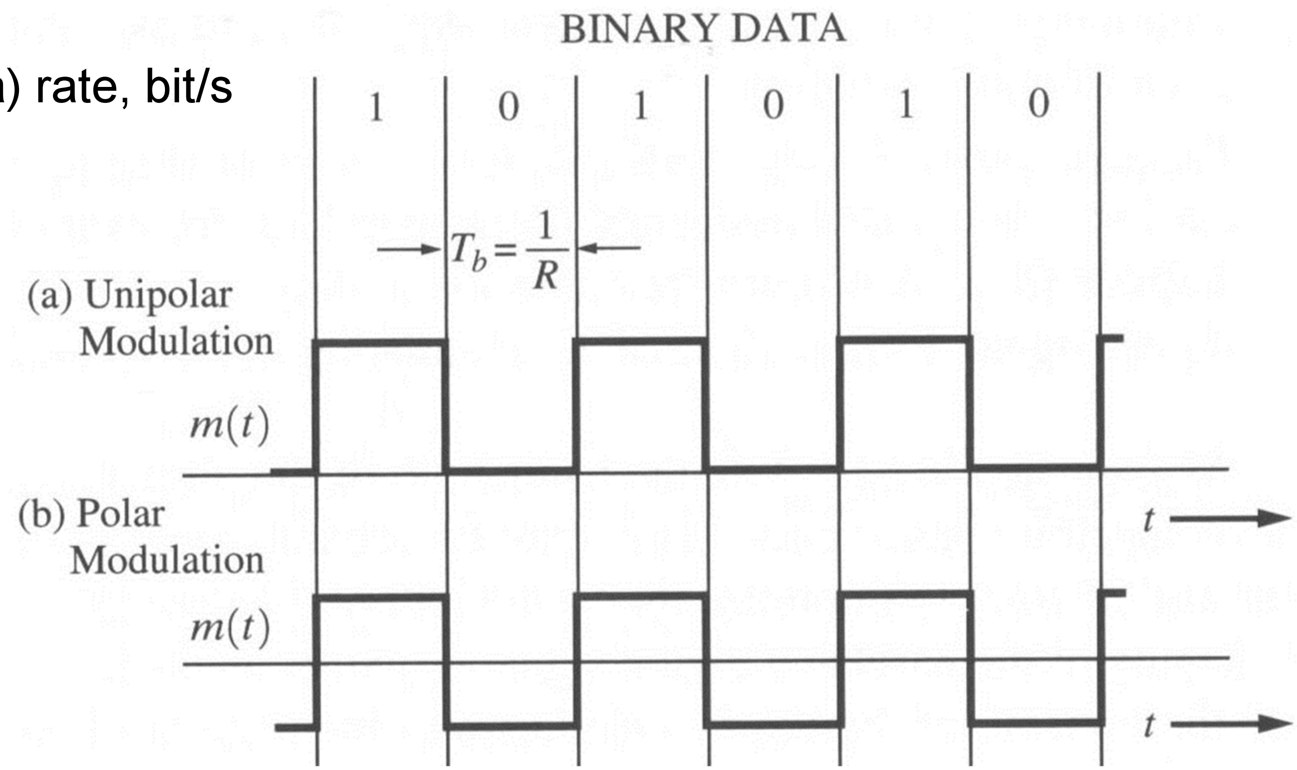
- On-Off keying (OOK), or amplitude shift keying (ASK)
- Phase shift keying (PSK), particularly binary PSK (BPSK)
- Frequency shift keying
- Typical spectra
- Modulation/demodulation principles
- Main difference between digital and analog systems: goal of transmission.
- Advantages of digital modulation:
  - ❑ More flexibility through DSP (processing, services, etc.)
  - ❑ Noise/interference immunity; security
  - ❑ Fits to computer/data communications

Baseband Binary Modulation

- Binary data representation: 0 or 1.
- Unipolar modulation: high level (e.g., 5V) / zero, or 1/0
- Bipolar modulation: +high level / -high level, or +1/-1

$R = \frac{1}{T_b}$ bit (data) rate, bit/s

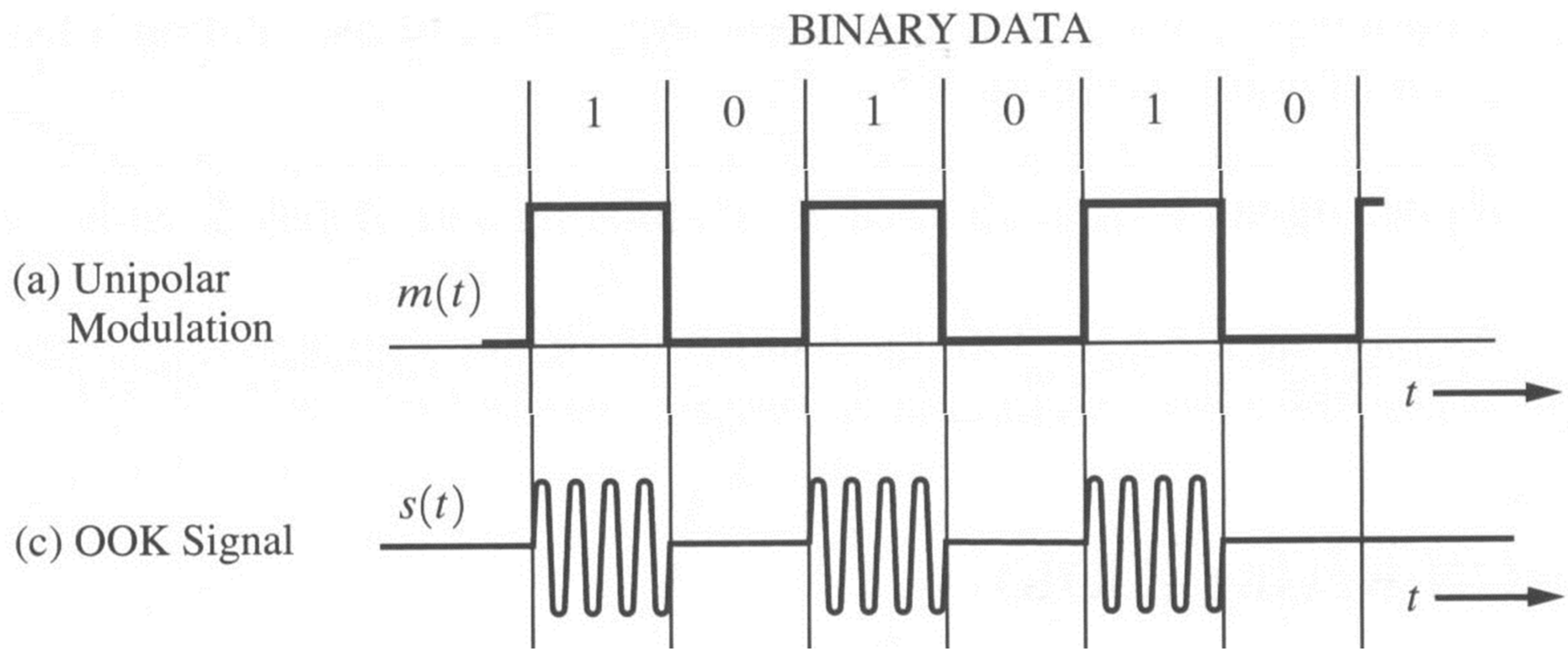
NRZ ⇨



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Amplitude Shift Keying (ASK)

- Switch on-off the carrier:



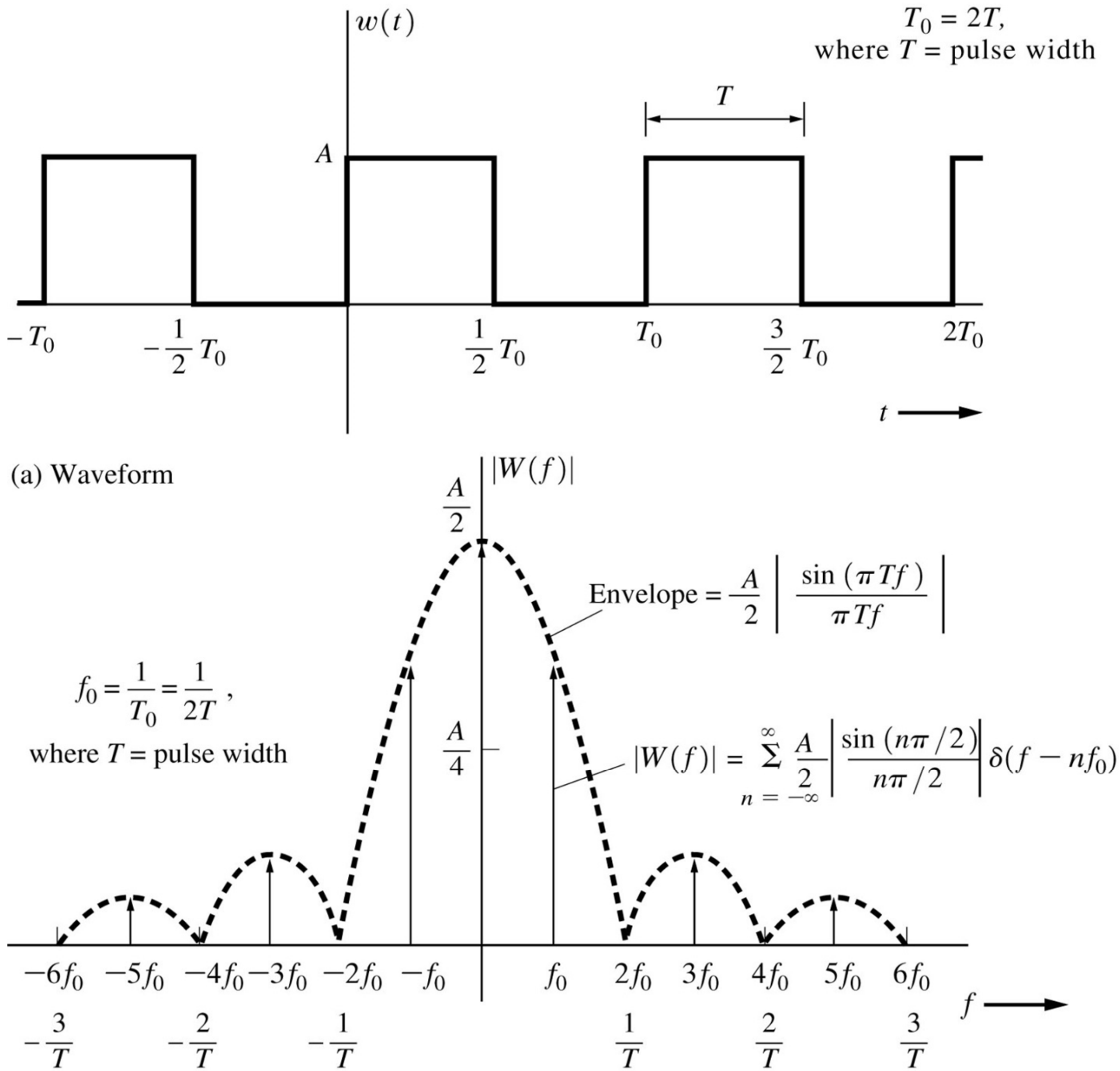
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

- Signal representation:
- Is it similar to something?
- Signal spectrum?

$$x(t) = A_c m(t)\cos(2\pi f_c t)$$

binary ASK: $m(t) = 1 \text{ or } 0$

general case: a fixed number of levels



(a) Waveform

(b) Magnitude Spectrum

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Amplitude Shift Keying (ASK)

- Signal spectrum (FT):

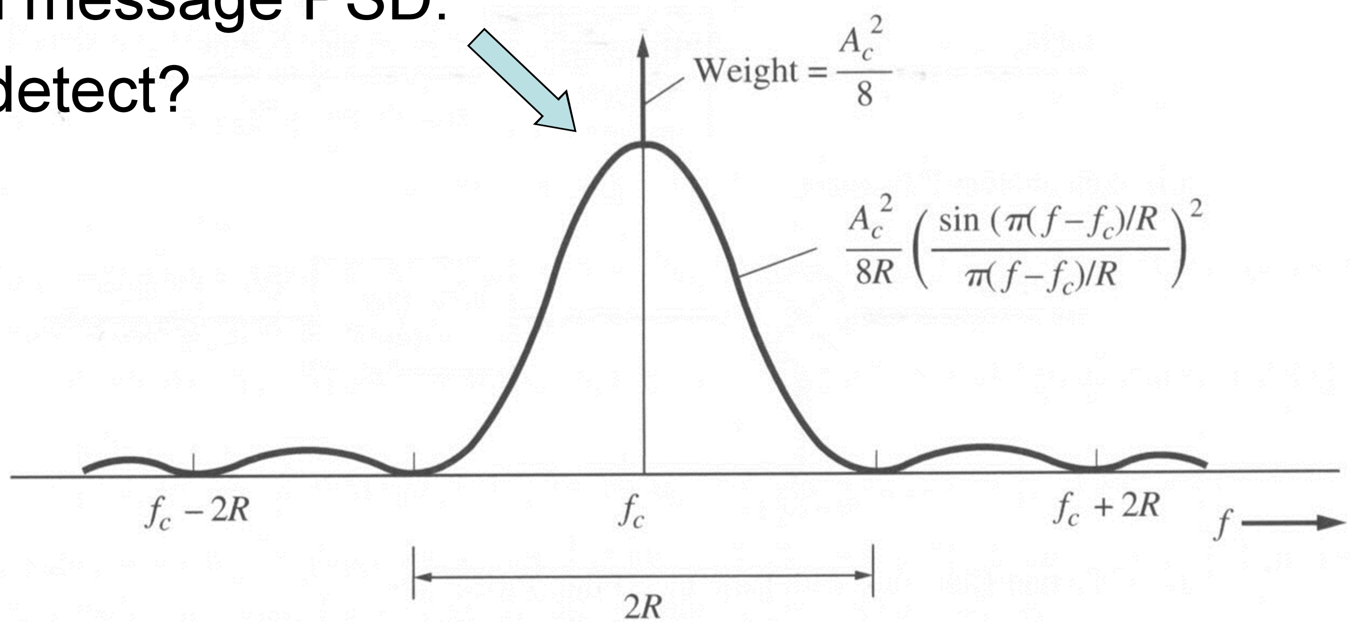
$$S_x(f) = \frac{A_c}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right]$$

- Square-wave message:

$$S_m(f) = \sum_{n=-\infty}^{+\infty} c_n \delta(f - nf_0),$$

$$c_n = \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right)e^{j\frac{\pi}{2}n},\ f_0 = \frac{1}{2T_b} = \frac{R}{2}$$

- Random message PSD:
- How to detect?



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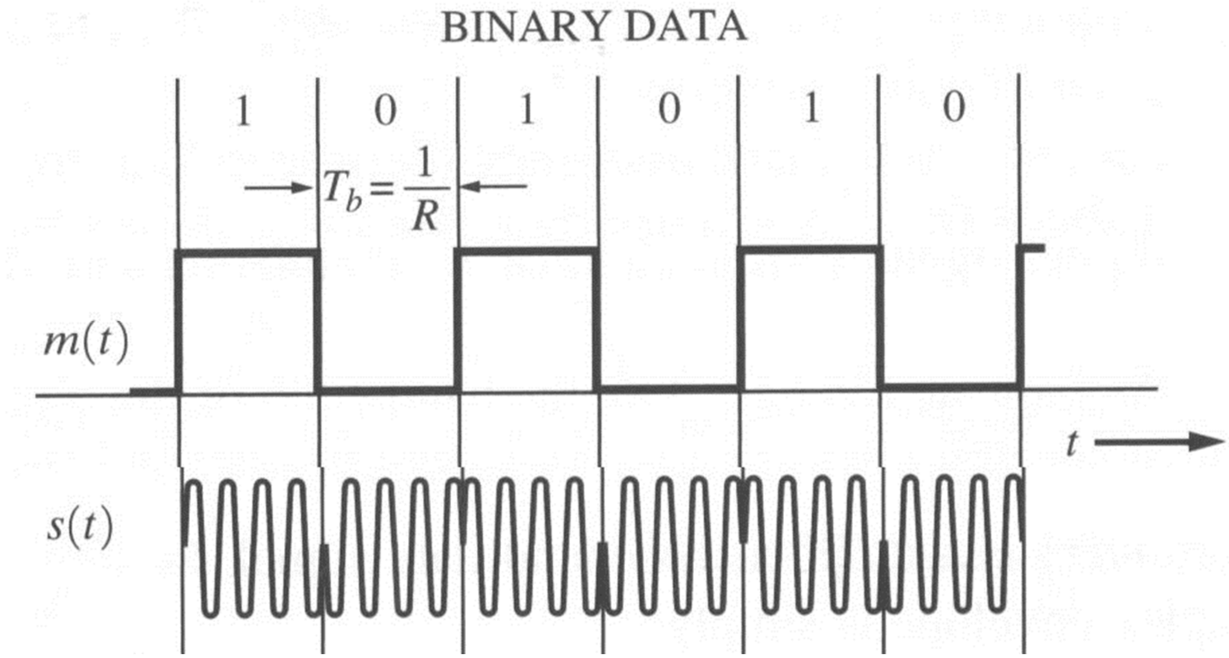
Binary Phase Shift Keying



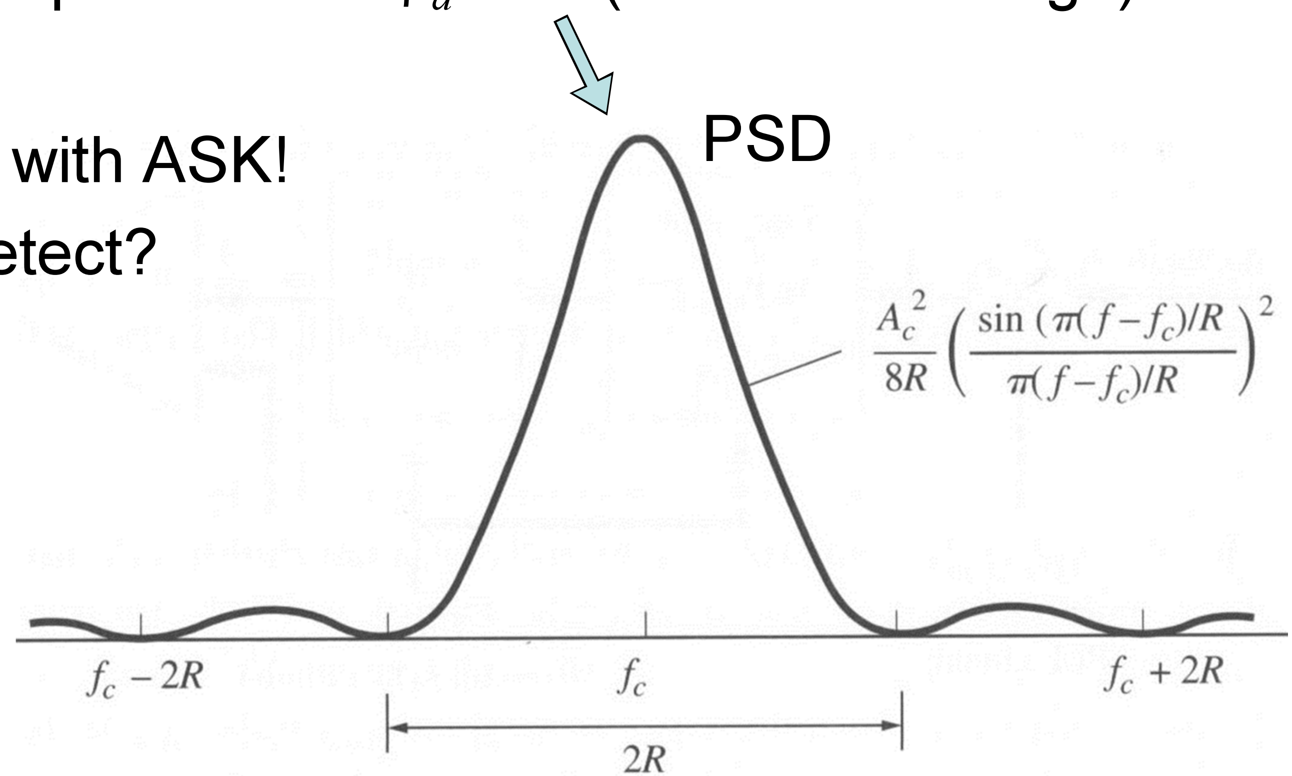
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

- BPSK signal representation: $x(t) = A_c \cos\left(\omega_c t + \Delta\varphi \cdot m(t)\right)$

  where $m(t) = \pm 1$ is bipolar message.
- Another form of the BPSK signal -> $$x(t) = A_c \cos\Delta\varphi \cos\omega_c t - A_c \sin\Delta\varphi \cdot m(t) \sin\omega_c t$$

Binary Phase Shift Keying

- Digital modulation index: $\beta_d = \dfrac{2\Delta\varphi}{\pi}$
- Important special case $\beta_d = 1$ (random message)
- Compare with ASK!
- How to detect?



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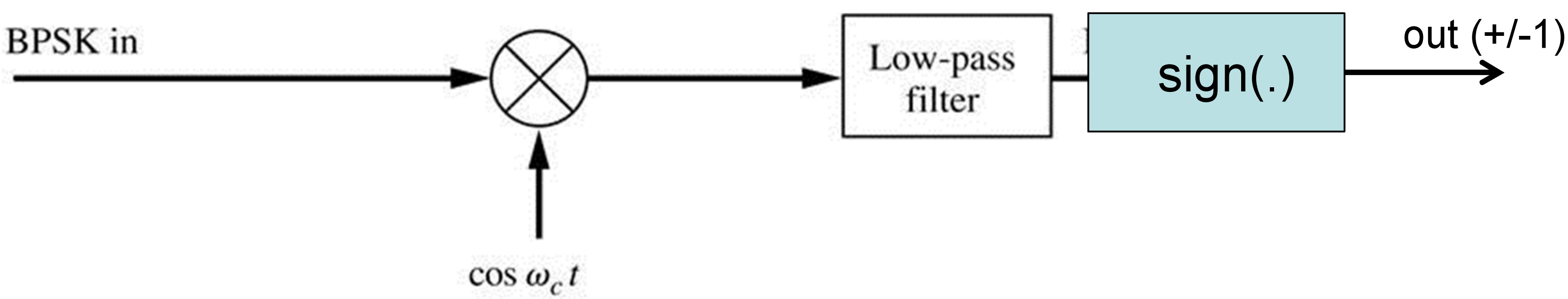
Detection of BPSK

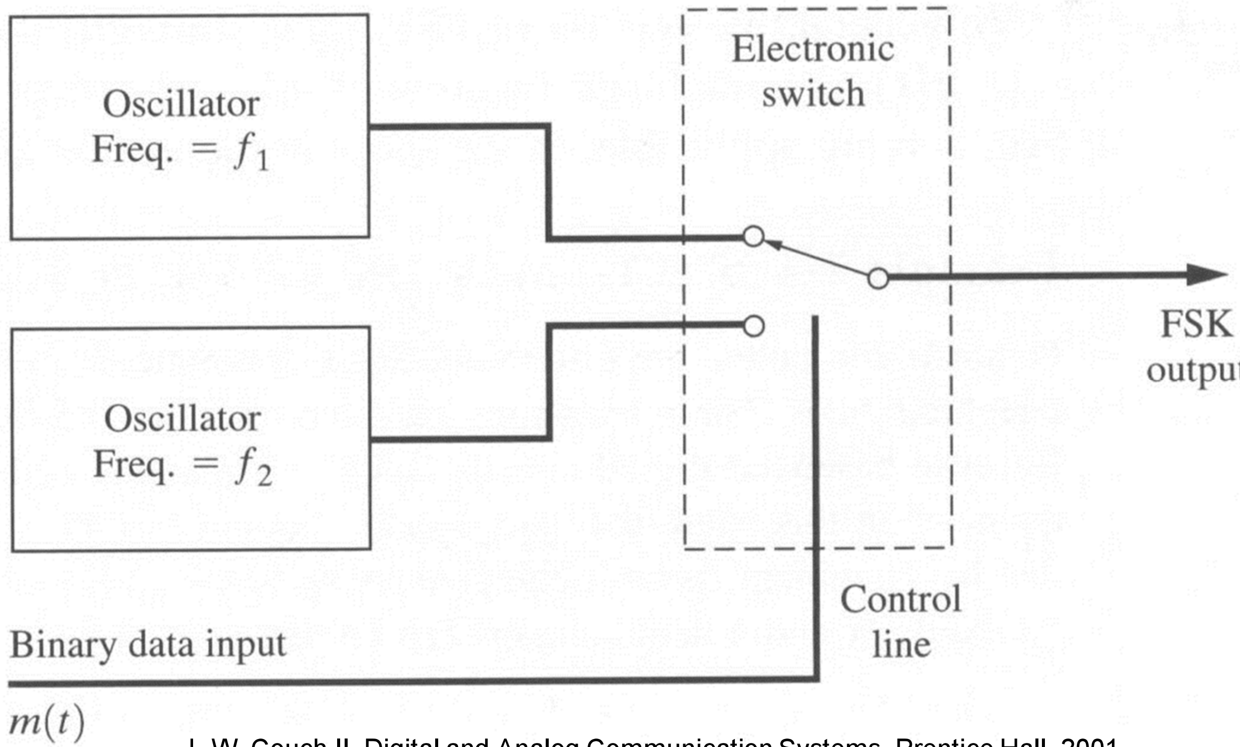
- Same as analog PM
- Add a quantizer (sign(*) function) to improve performance (noise immunity)



(a) Detection of BPSK (Coherent Detection)

Frequency Shift Keying

- Discontinuous FSK: $x(t)=\begin{cases} A_c\cos(\omega_1 t+\theta_1), & \text{mark (1)} \\ A_c\cos(\omega_2 t+\theta_2), & \text{space (0)} \end{cases}$



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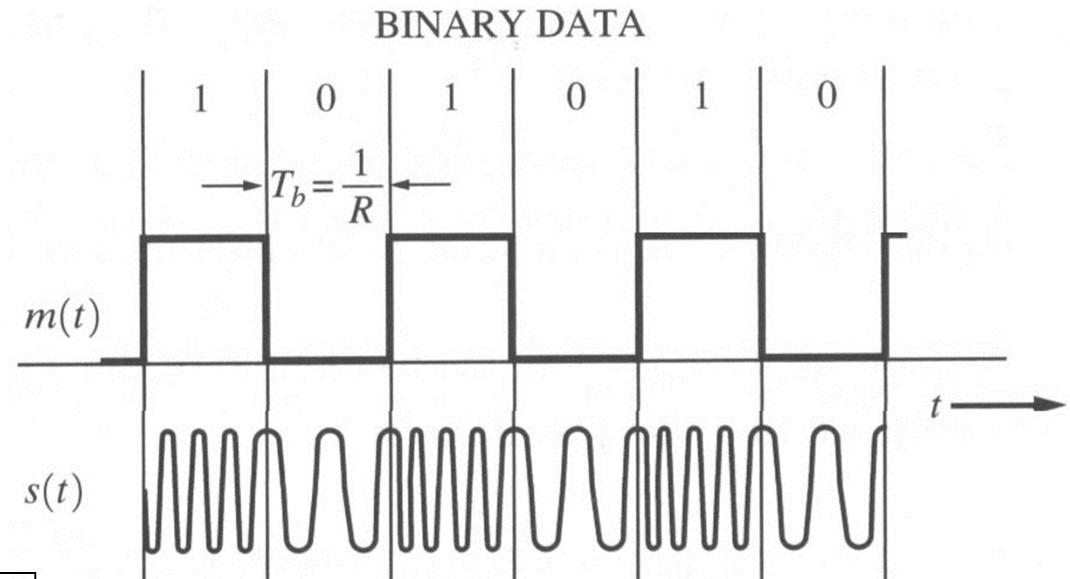
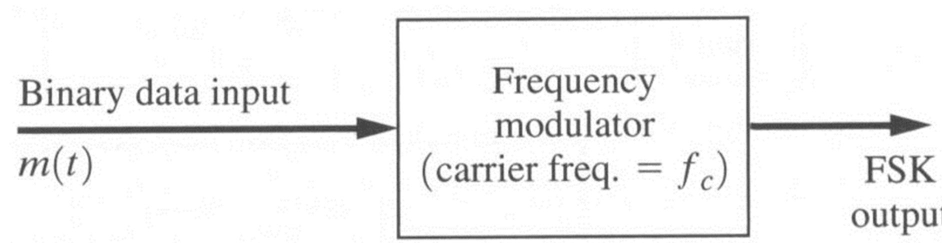
Not popular (spectral noise + PLL problems)

Frequency Shift Keying

- Continuous FSK:



$$x(t) = A_c \cos\left(\omega_c t + \Delta\Omega \int_0^t m(\tau) d\tau \right)$$



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Pulse-Amplitude Modulation (PAM)

- Baseband modulation (no carrier yet)
- Baseband signal represents digital data (e.g. binary)
- PAM: a conversion of an analog signal to a pulse-type signal in which the pulse amplitude carriers the analog information.
- This is the 1st step in converting an analog signal (waveform) to a digital signal.

$$\left\{b_1 \ldots b_n\right\} \to \left\{A_1 \ldots A_n\right\} \to x(t) = \sum_{k=1}^{n} A_k s(t-kT)$$

Pulse-Amplitude Modulation

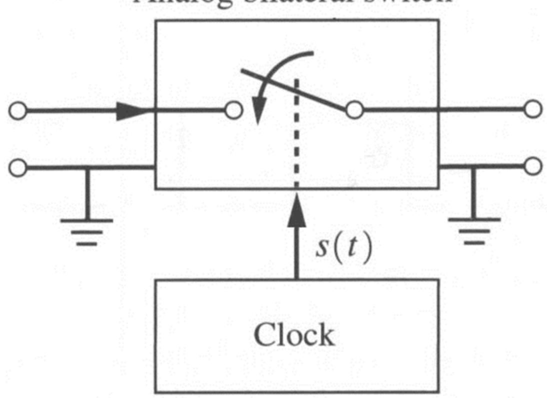
- Based on the sampling theorem: analog band-limited (to F_{max}) signal can be represented by its samples taken at $f_s \geq 2F_{\max}$
- PAM provides pulse-like waveform that contains the same information as the original analog signal. Pulse rate [pulses/s] is the same as f_s .
- Pulse shape can be any. Discuss rectangular pulse waveform first.
- Two types of sampling: natural sampling (gating) and instantaneous sampling (flat-top or sample-and-hold).

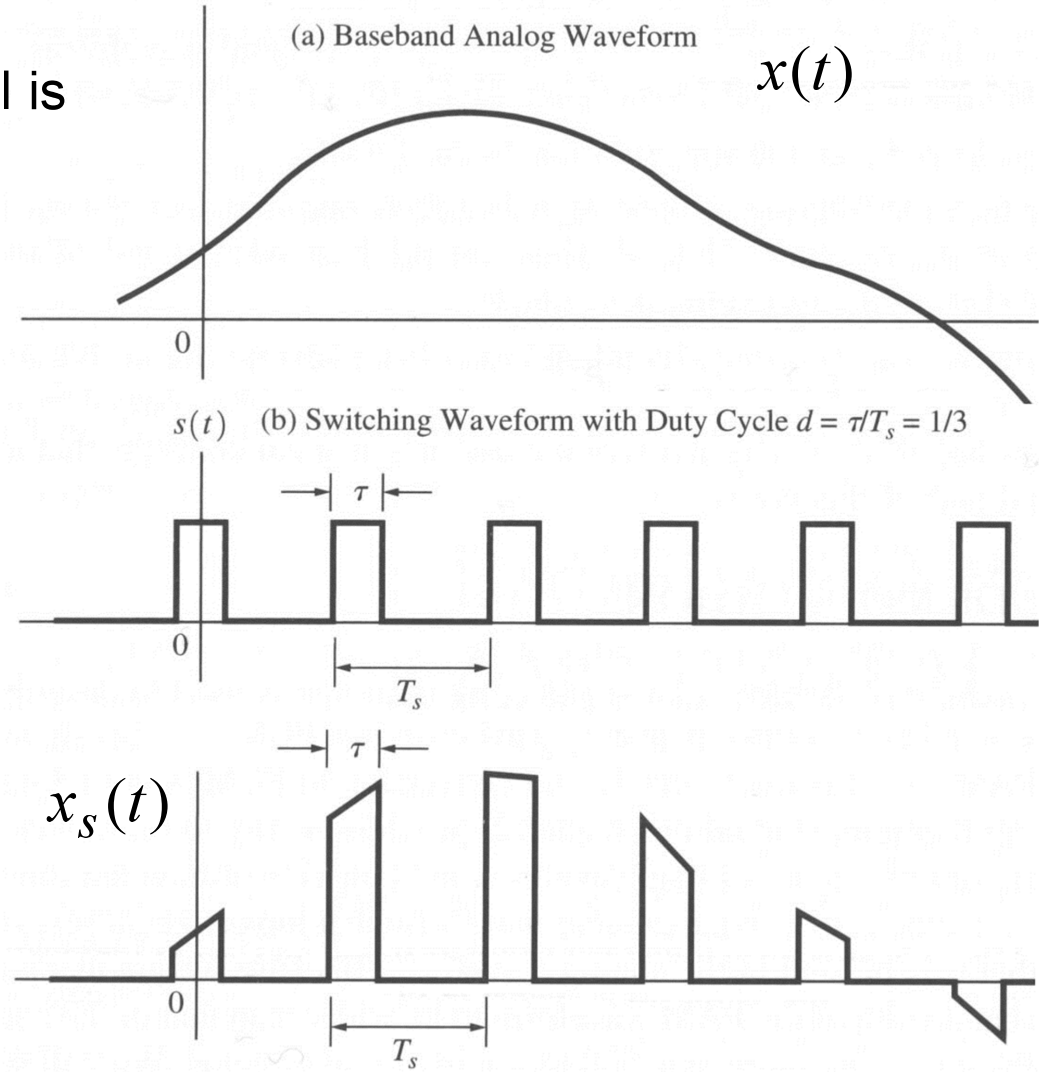
Natural Sampling (Gating)

- The sampled (PAM) signal is

$$x_s(t) = s(t)x(t),$$
$$s(t) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_s}{\tau}\right)$$

where $f_s = 1/T_s \geq 2F_{\max}$





(c) Resulting PAM Signal (natural sampling, $d = \tau/T_s = 1/3$)

Natural Sampling (Gating): Spectrum

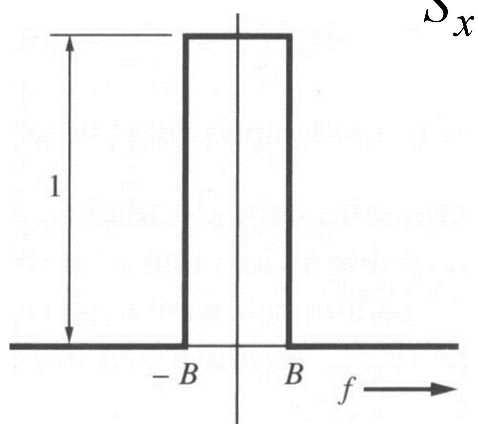
- Spectrum (FT) of the sampled (PAM) signal is

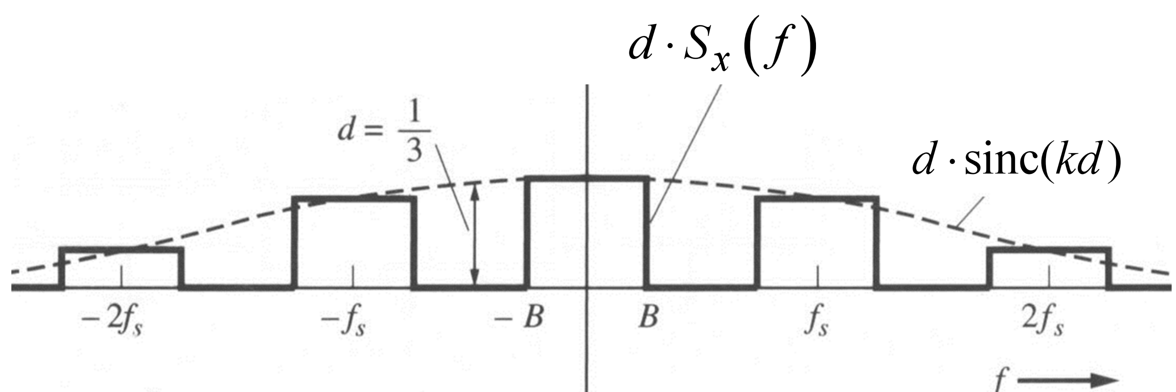
$$S_{x_s}(f) = FT\left[x_s(t)\right] = d\sum_{k=-\infty}^{\infty} \text{sinc}(kd) S_x\left(f - kf_s\right),$$

where $d = \tau / T_s$ is the duty cycle of s(t).

- **Example**:

- original signal spectrum $S_x(f)$ — sampled signal spectrum $S_{x_s}(f)$





- similar to ideal (delta-function) sampling?

Natural Sampling: Proof

- Start with $x_s(t) = s(t)x(t) \leftrightarrow S_{x_s}(f) = S_x(f) * S_s(f)$
- Find Fourier series of s(t):

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_s t}, \; c_n = d \cdot \mathrm{sinc}(nd)$$

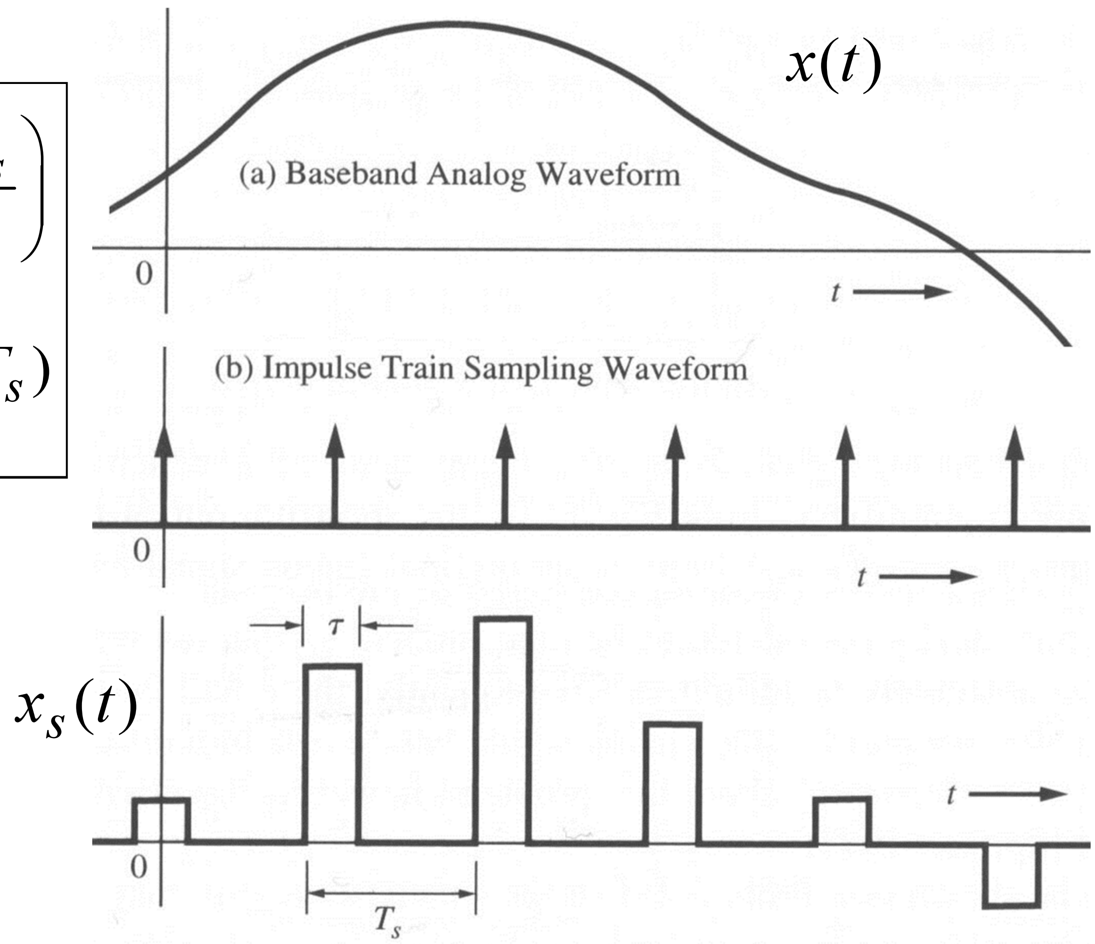
- FT of s(t) is $S_s(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_s)$
- Finally, $S_{x_s}(f) = S_x(f) * S_s(f) = \sum_{n=-\infty}^{\infty} c_n S_x(f - nf_s)$
- This concludes the proof.
- How to recover (demodulate) the original signal?

Instantaneous Sampling

- Also known as flat-top PAM or sample-and-hold.
- The sampled signal is

$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\Pi\left(\frac{t-kT_s}{\tau}\right)$$

$$= \Pi\left(\frac{t}{\tau}\right) * \sum_{k=-\infty}^{\infty} x(kT_s)\delta(t-kT_s)$$



(c) Resulting PAM Signal (flat-top sampling, $d = \tau/T_s = 1/3$)

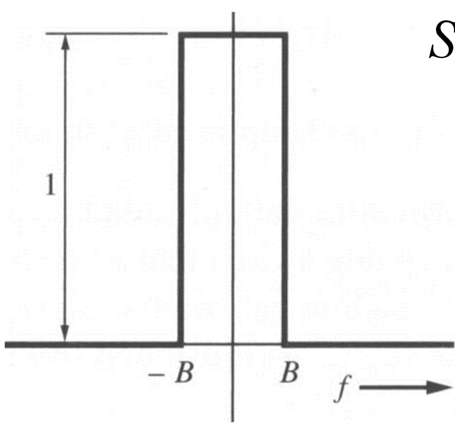
Instantaneous Sampling: Spectrum

- The sampled signal spectrum (FT) is

$$S_{x_s}(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} S_x\left(f - kf_s\right), \; H(f) = \tau \cdot \mathrm{sinc}(\tau f)$$

- **Example:**

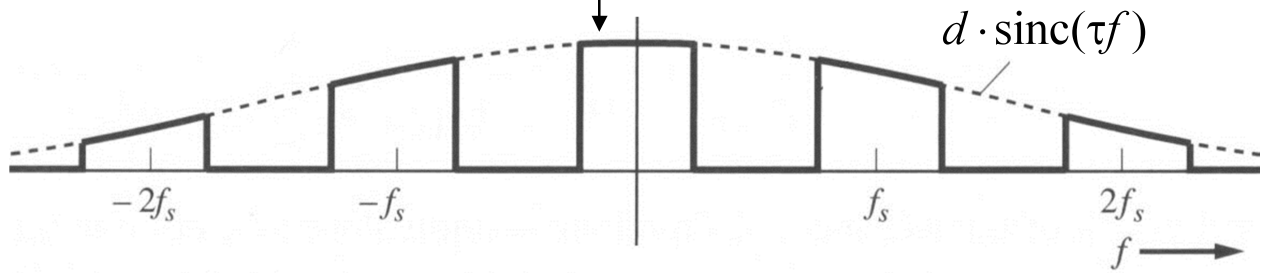
- original signal spectrum $S_x\left(f\right)$



sampled signal spectrum $S_{x_s}\left(f\right)$

$d \cdot \mathrm{sinc}(\tau f) \cdot S_x\left(f\right)$

$d \cdot \mathrm{sinc}(\tau f)$



- Proof – homework. How to recover (demodulate) x(t) ?

Baseband PAM: Generic Case

- Basic pulse shape is not necessarily rectangular. The information is represented by the pulse amplitude A_m.
- M-ary PAM signal waveform:

$$x_m(t) = A_m s(t),\ m = 1,2,...M,\ 0 \le t \le T$$

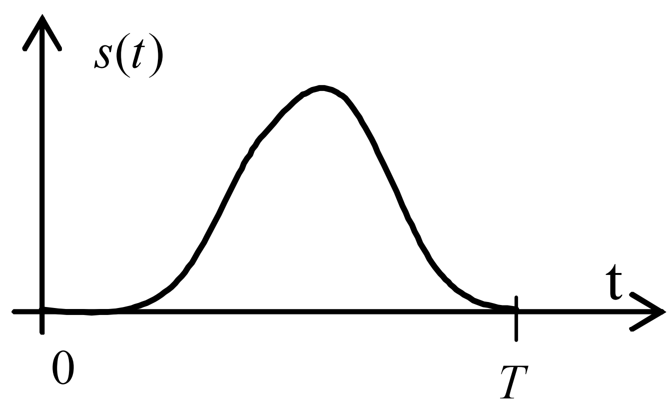
- s(t) – signal waveform, T – symbol interval, M – the number of symbols.
- The information transmitted by one symbol:

$$n_b = \log_2 M \ \text{[bits]}$$

- Transmitted signal sequence

$$x_T(t) = \sum_k A_k s(t - kT)$$

generic signal waveform



PAM: Energy, Spectrum

- Baseband signal energy: $E_m = \int_0^T x_m^2(t)dt = A_m^2 E_s, \; E_s = \int_0^T s^2(t)dt$
- Depends on m.
- Bandpass PAM signal:

$$x_m(t) = A_m s(t)\cos\omega_c t, \; m = 1, 2, \ldots M, \; 0 \le t \le T$$

- Its spectrum (FT): $S_{x_m}(f) = \frac{A_m}{2}\left(S_s(f - f_c) + S_s(f + f_c)\right)$
- It is DSB-SC signal! The bandwidth is twice of that of baseband signal.
- Bandpass signal energy: $E_m = \int_0^T x_m^2(t)dt = \frac{A_m^2}{2} E_s$
- Similar modulation formats: PPM, PWM.

Summary

- Basic digital modulation formats. Unipolar and bipolar NRZ baseband signals. ASK (OOK), PSK and FSK. Spectra and bandwidth.
- PAM. Instantaneous and flat-top sampling. Spectra of sampled signals. Recovery (demodulation) of the original signal. Generic form of a PAM signal.

- **Homework**: Reading: Couch, 3.1, 3.2, 5.9. Study carefully all the examples, make sure you understand and can solve them with the book closed.