ELG3175: Introduction to Communication Systems, Winter 2024, © S. Loyka

Assignment #7

Due: Mar. 27 (Wed.)**, 4pm, MRN 021** (the lecture). Hard copies only. Late/electronic/email submissions will **not** be accepted..

1) A signal $x(t) = 2\cos(2\pi ft)$, where $f = 1kHz$, is sampled at time instants $t = nT_s, n = 0, \pm 1, \pm 2$ where the sampling rate $f_s = 4kHz$ and the duty cycle $d = 0.5$. Assume that the natural sampling is used. (a) Sketch the sampled signal for $0 \le t \le 8T_s$. (b) An ideal low-pass filter with gain $1/d$ is used to recover $x(t)$ from the sampled signal. Find the filter output (i.e., the recovered signal) and compare it with the original signal. (c) Repeat (a) and (b) when the flat-top sampling is used with the same duty cycle. Is the filter output the same as in (b)? Why?

2) In a CD player, the sampling rate is 44 KHz and the samples are quantized using a 16-bit/sample quantizer. Determine the resulting number of bits for a piece of music with duration of 60 minutes. Additionally, (a) assuming that the signal is a sinusoid with the amplitude equal to the maximum allowed level of the quantizer, find SQNR in dB, (b) if SQNR of 60 dB is acceptable, how many bits are required to represent the quantizer output? (c) repeat (a) and (b) when the sinusoid amplitude is half of the maximum allowed level of the quantizer (hint: you need to change the derivation of SQNR in Lecture 11 a bit in this case, as discussed in the class).

3) Let $w(t)$ be an analog signal (waveform) and $w_s(t)$ be its naturally-sampled signal. Show that $w(t)$ can be recovered from $w_s(t)$ using a demodulator as shown in Fig. 3.4 of the textbook. Find the proportionality constant C for given n, ω_s, B.

4) Consider an audio signal with spectral components limited to the band of [30 Hz, 3 kHz] (assume it is a sinusoid of frequency anywhere within this band). A PCM signal is generated with a sampling rate of 9000 samples/s. The required SQNR is 40 dB. (1) What is the minimum number of uniform quantizing levels needed, and what is the minimum number of bits per sample needed? (2) Find the minimum bandwidth required for such a system. (3) Repeat (1) and (2) when a μ-law compander is used with $\mu = 255$ and compare the results. Why the companding does not provide advantage? *Note:* For a μ-law compander PCM system, $SQNR \approx 3N^2/[\ln(1+\mu)]^2$, i.e. does not depend (approximately) on the peak factor!

5) Design a delta-modulator for a sinusoidal signal whose frequency is located anywhere in the voice band, [20 Hz, 20 kHz], and A= 1 V. Find a sampling frequency such that SQNR is at least 50 dB and no slope overload distortion is present. Find also the step size. Assume that the reconstruction filter cut-off frequency is 40 kHz. Repeat the same when (i) the maximum frequency is 200 kHz (the reconstruction filter cut-off frequency is 400 kHz), and (ii) A=10 V. Compare the results. What is the effect of signal maximum frequency? Amplitude?

6) A data source produces binary data at a rate of 10 kbits/s, which is transmitted using binary PAM with a raised-cosine pulse, roll-off factor is 0.1. Find the required channel bandwidth. How would it change if the roll-off factor would be 0.9? Repeat these if a 16-level PAM system is used.

7) 8 message signals of maximum frequency = 2 kHz each are transmitted over the same channel (time-division multiplexing) using a PCM - binary PAM (raised cosine) system. The quantizing error must not exceed 1% of the peak signal amplitude and the sampling guard band is 25%. Find the minimum transmission bandwidth if roll-off factor = 0.2.

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All spectra should be sketched as they would appear on a spectrum analyzer.
Please include in your solutions all the intermediate results and their numerical values (if applicable). **Detailed solutions with explanations are required**, not just the final answers/equations; **all symbols used must be defined**, including units used, if applicable (e.g. f = frequency [Hz]). Missing explanations, symbol definitions/units will be penalized. Your answers should demonstrate the full extent of your knowledge and the latter will determine your marks.

Plagiarism (i.e. "cut-and-paste" from a student to a student, other forms of "borrowing" the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that

particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean’s office for further investigation.

Please read appropriate chapters of the textbook first, study all the examples, attempt to do them with the closed book. Remember the learning efficiency pyramid!