ELG7177: MIMO Comunications, Lecture 2

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A Brief Review of Communications

- Communication system: transmit information from point A to point B
  - analog/digital
  - wireline/wireless
  - single user/multiuser
- Communication network: multiple As and Bs (or multi-user)
- Extensive applications
  - Internet
  - WiFi
  - cell phones
  - TV/radio broadcast
  - GPS
- Active R&D: 5G

Block Diagram of a Communication System

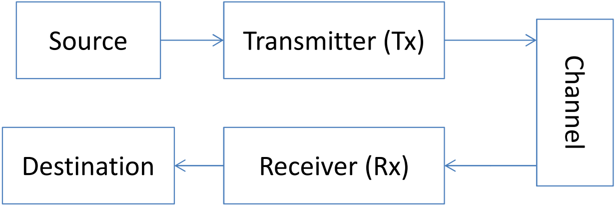

Figure: A high-level view of a communication system

A Digital Communication System

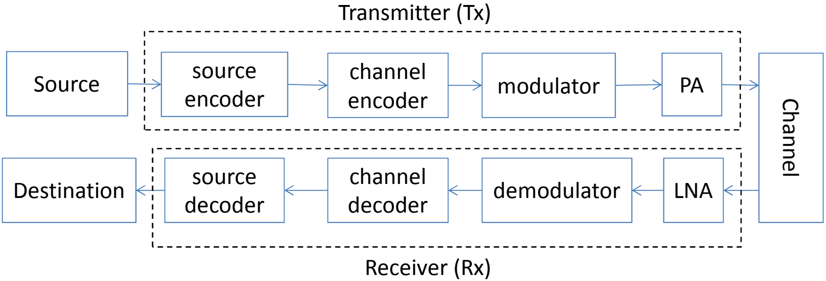

Figure: A high-level view of a digital communication system

A Wireless Communication System[1][2]

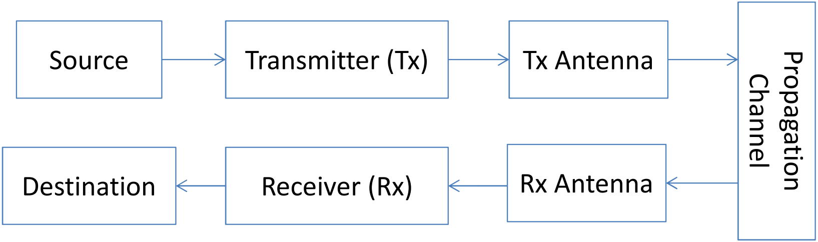

Figure: Block diagram of a wireless communication system

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[1] T.S. Rappaport, Wireless Communications: Principles and Practice, Prentice Hall, New Jersey, 2002. (2nd Edition)

[2] D. Tse, P. Viswanath, Fundamentals of Wireless Communications, Cambridge University Press, 2005

A Wireless Communication System

Major challenges, due to the wireless propagation channel

- out of designer's control
- low SNR (large path loss, 100s dB)
- multipath propagation → fading
    - frequency selectivity (delay spread)
    - time selectivity/variability (Doppler spread)
    - inaccurate/unavaiable channel state information
- interference
- limited/expensive bandwidth

How to combat?

A Wireless Communication System

How to combat major challenges?

- frequency/time domain processing: at their limits
    - modulation
    - coding
    - filtering
- space-domain processing: ”last frontier”
- via multiple (”smart”) antennas
- current active R&D: 5G

Modern Wireless Communication Systems[3][4]

- Key objectives of 5G
    - 1000× rate
    - wide availability
    - low latency
    - multiple services
- How? Key technologies:
    - massive MIMO
    - mmWaves
    - NOMA
    - HetNet

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[3] J G. Andrews et al., What Will 5G Be?, IEEE JSAC, v. 32, n.6, pp. 1065–1083, June 2014.

[4] M. Shafi et al, 5G: A Tutorial Overview of Standards, Trials, Challenges, Deployment, and Practice, IEEE JSAC, Part I, v. 35, N.6, pp. 1201–1221, June 2017.

Digital Communications[5][6][7]

- modulation
    - BPSK
    - QPSK
    - QAM
- signalling
    - sinc
    - raised-cosine
    - etc.
- optimum receiver: ML
    - matched filter
    - sampler
    - decision device

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[5] J.R. Barry, E.A. Lee, D.G. Messerschmitt, Digital Communications (3rd Ed.), Kluwer, Boston, 2004.

[6] J.M. Wozencraft, I.M. Jacobs, Principles of Communication Engineering, Wiley: New York, 1965.

[7] A. Lapidoth, A Foundation in Digital Communication, Cambridge University Press, 2017.

Digital Communications: Key Performance Metrics

- transmission rate, [bit/s]
- error rate/probability, BER/SER
- fading: outage probability

Digital Communications: fundamental limits

- from information theory[8]
- single user: channel capacity: [bit/s] or [bit/ch. use]
- fading: outage capacity
- benchmark for actual system performance
- optimal system design (Tx, Rx)
- much less is known about networks

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[8] T.M. Cover, J.A. Thomas, Elements of Information Theory, John Wiley & Sons, 2006.

Digital Communications: channel model

- AWGN channel (discrete-time)

$$y_k = x_k + \xi_k \tag{1}$$

  - x_k = Tx signal
  - y_k = Rx signal
  - ξ_k = noise (i.i.d. Gaussian)

Fundamental Limit: Channel Capacity

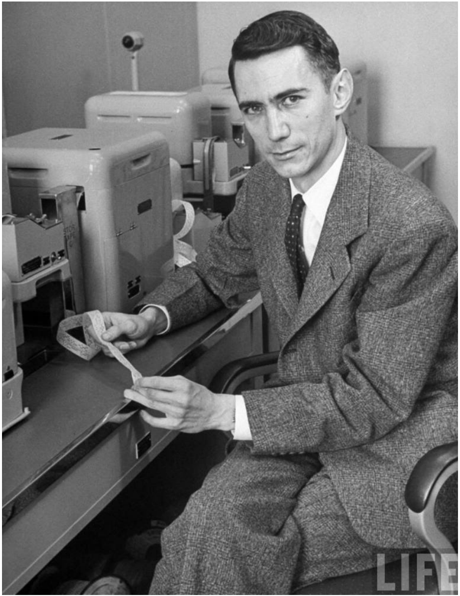
- largest transmission rate s.t. power & reliability constraints

$$R < C = \Delta f \log(1+\gamma) \text{ [bit/s]} \tag{2}$$

  - Δf = channel bandwidth
  - $\gamma = P_x/P_\xi$ = SNR
  - power constraint: $\sigma_x^2 \leq P_x$
  - relaibility constraint: arbitrary-low error probability
- equivalently, spectral efficiency:

$$C = \log(1+\gamma) \text{ [bit/s/Hz]} \tag{3}$$

Fundamental Limit: Channel Capacity



Claude Shannon, Farther of Information Theory:

Apr. 30, 1916 (Petoskey, Michigan, US) - Feb. 24, 2001 (Medford, Massachusetts, US).

Channel Capacity & M-ary Modulation

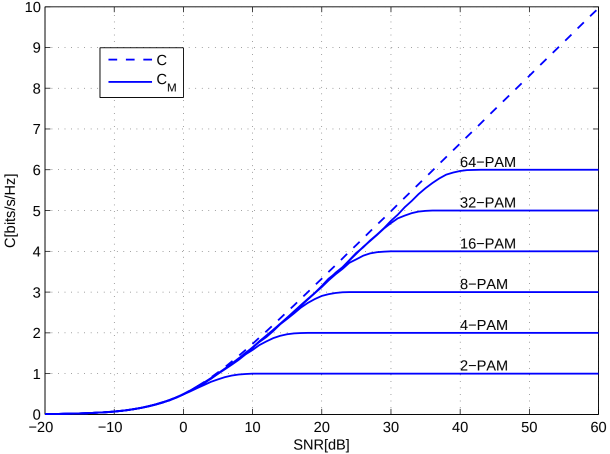


Figure: The constellation capacity of M-PAM[9]

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[9] Y. Han, Optimization of Modulation Constrained Digital Transmission, MS Thesis, University of Ottawa, 2017.

Fundamental Limit: Channel Capacity

- $R = C$ is not possible, but R can be close to C
- in practice,

$$R = \log(1 + \gamma/\Gamma) \text{ [bit/s/Hz]} \tag{4}$$

where $\Gamma > 1$ is the SNR gap to capacity

- $\Gamma \to 1$ for good (capacity-approaching) system
- depends on modulation and coding[10][11]

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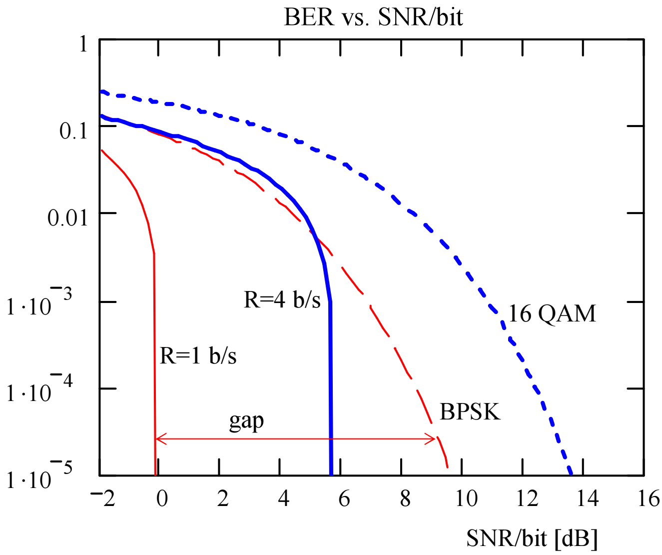
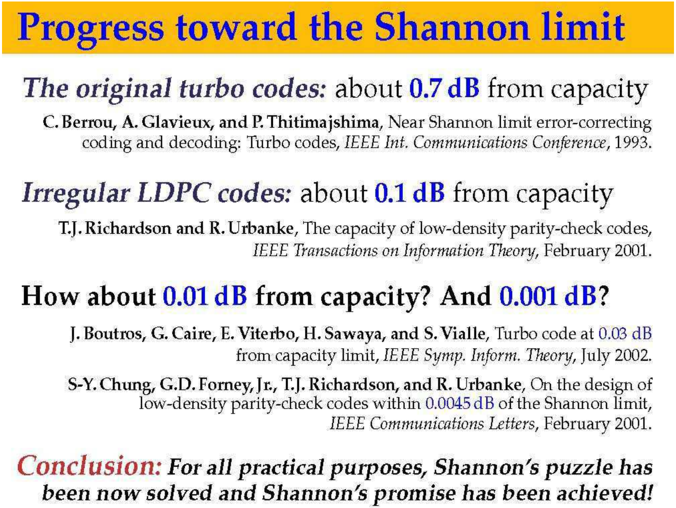
[10] G.D. Forney, G. Ungerboeck, Modulation and Coding for Linear Gaussian Channels, IEEE Trans. Info. Theory, v. 44, no. 6, pp. 2384–2415, Oct. 1998.

[11] J. Cioffi, EE 379A - Digital Communication: Signal Processing, Stanford University, Winter 2008.

SNR-Gap-to-Capacity in Practice



Progress Towards the Capacity[12]



Progress toward the Shannon limit

The original turbo codes: about **0.7 dB** from capacity

C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon limit error-correcting coding and decoding: Turbo codes, *IEEE Int. Communications Conference*, 1993.

Irregular LDPC codes: about **0.1 dB** from capacity

T.J. Richardson and R. Urbanke, The capacity of low-density parity-check codes, *IEEE Transactions on Information Theory*, February 2001.

How about 0.01 dB from capacity? And 0.001 dB?

J. Boutros, G. Caire, E. Viterbo, H. Sawaya, and S. Vialle, Turbo code at 0.03 dB from capacity limit, *IEEE Symp. Inform. Theory*, July 2002.

S-Y. Chung, G.D. Forney, Jr., T.J. Richardson, and R. Urbanke, On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit, *IEEE Communications Letters*, February 2001.

Conclusion: ***For all practical purposes, Shannon's puzzle has been now solved and Shannon's promise has been achieved!***

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[12] A.Vardy, What's New and Exciting in Algebraic and Combinatorial Coding Theory? Plenary Talk at ISIT-06.

Channel Capacity: two fundamental resources

From the capacity expression,

$$C = \Delta f \log(1 + \gamma) \text{ [bit/s]} \tag{5}$$

C can be increased by increasing

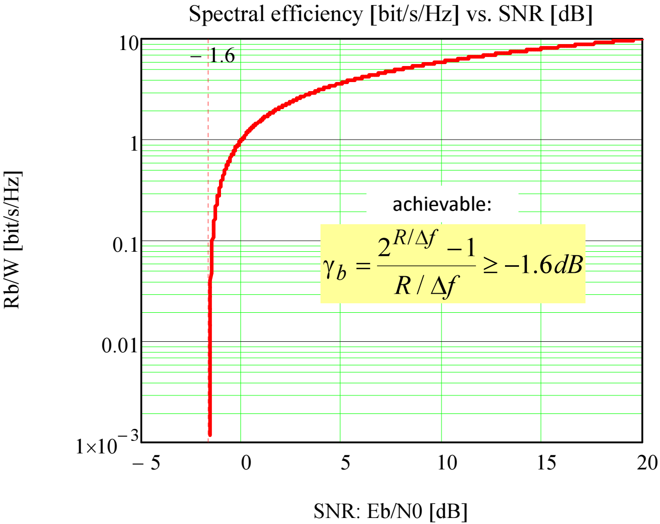
1. bandwidth Δf (expensive in wireless)
2. power P_x, via the SNR $\gamma = P_x / P_\xi$
3. anything else?

Spectral/Power Efficiency: Fundamental Tradeoff

- power efficiency: $\gamma_b = \text{SNR(energy)/bit}$
- spectral efficiency: $R/\Delta f$ [bit/s/Hz]
- the tradeoff:

$$\gamma_b \geq \frac{2^{R/\Delta f} - 1}{R/\Delta f} \geq \ln 2 = \text{-1.6 dB} \tag{6}$$

Spectral/Power Efficiency: Fundamental Tradeoff



achievable:

$$\gamma_b = \frac{2^{R/\Delta f} - 1}{R/\Delta f} \geq -1.6 dB$$

Fundamental Tradeoff in Practice[13]

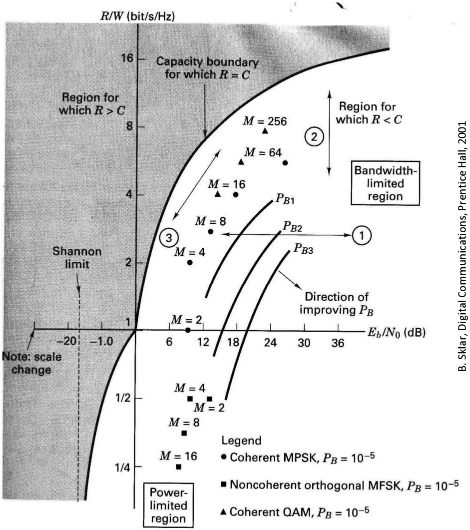


Figure 9.6 Bandwidth-efficiency plane.

B. Sklar, Digital Communications, Prentice Hall, 2001

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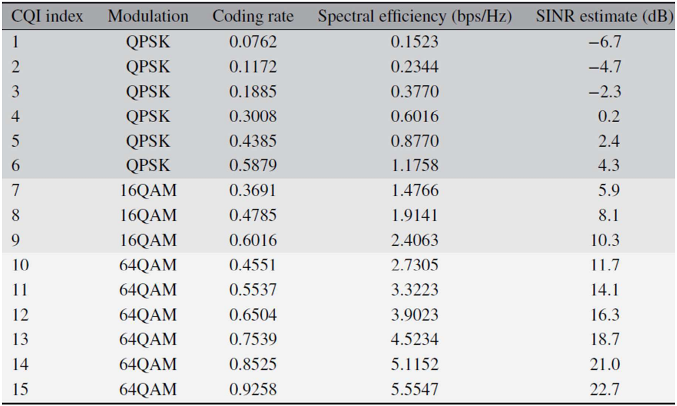
[13] B. Sklar, Digital Communications, Prentice Hall, 2001.

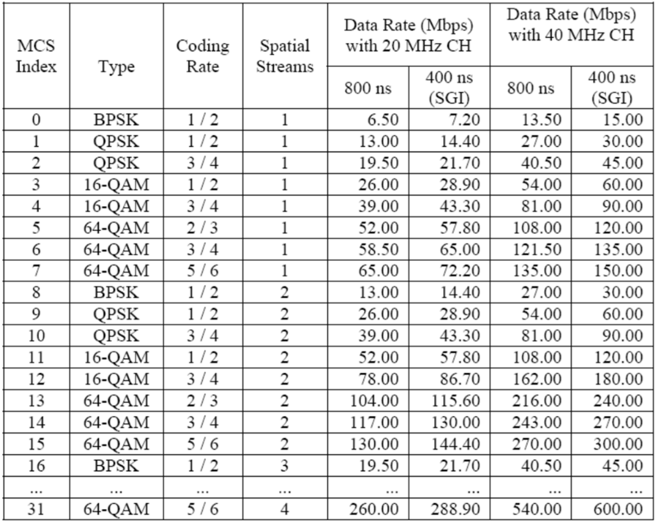
Practical Example: Spectral Efficiency of LTE/4G



CQI index	Modulation	Coding rate	Spectral efficiency (bps/Hz)	SINR estimate (dB)
1	QPSK	0.0762	0.1523	−6.7
2	QPSK	0.1172	0.2344	−4.7
3	QPSK	0.1885	0.3770	−2.3
4	QPSK	0.3008	0.6016	0.2
5	QPSK	0.4385	0.8770	2.4
6	QPSK	0.5879	1.1758	4.3
7	16QAM	0.3691	1.4766	5.9
8	16QAM	0.4785	1.9141	8.1
9	16QAM	0.6016	2.4063	10.3
10	64QAM	0.4551	2.7305	11.7
11	64QAM	0.5537	3.3223	14.1
12	64QAM	0.6504	3.9023	16.3
13	64QAM	0.7539	4.5234	18.7
14	64QAM	0.8525	5.1152	21.0
15	64QAM	0.9258	5.5547	22.7

Figure: Spectral efficiency of LTE/4G cell phones

Practical Example: IEEE 802.11n (WiFi)



MCS Index	Type	Coding Rate	Spatial Streams	Data Rate (Mbps) with 20 MHz CH		Data Rate (Mbps) with 40 MHz CH	
				800 ns	400 ns (SGI)	800 ns	400 ns (SGI)
0	BPSK	1 / 2	1	6.50	7.20	13.50	15.00
1	QPSK	1 / 2	1	13.00	14.40	27.00	30.00
2	QPSK	3 / 4	1	19.50	21.70	40.50	45.00
3	16-QAM	1 / 2	1	26.00	28.90	54.00	60.00
4	16-QAM	3 / 4	1	39.00	43.30	81.00	90.00
5	64-QAM	2 / 3	1	52.00	57.80	108.00	120.00
6	64-QAM	3 / 4	1	58.50	65.00	121.50	135.00
7	64-QAM	5 / 6	1	65.00	72.20	135.00	150.00
8	BPSK	1 / 2	2	13.00	14.40	27.00	30.00
9	QPSK	1 / 2	2	26.00	28.90	54.00	60.00
10	QPSK	3 / 4	2	39.00	43.30	81.00	90.00
11	16-QAM	1 / 2	2	52.00	57.80	108.00	120.00
12	16-QAM	3 / 4	2	78.00	86.70	162.00	180.00
13	64-QAM	2 / 3	2	104.00	115.60	216.00	240.00
14	64-QAM	3 / 4	2	117.00	130.00	243.00	270.00
15	64-QAM	5 / 6	2	130.00	144.40	270.00	300.00
16	BPSK	1 / 2	3	19.50	21.70	40.50	45.00
...	...	...	...	...	...	...	...
31	64-QAM	5 / 6	4	260.00	288.90	540.00	600.00

Figure: Data rates of WiFi routers

The SNR

The SNR is

$$\mathrm{SNR} = \gamma = \frac{P_x}{P_\xi} \tag{7}$$

P_x = signal power at the receiver (Rx),
$P_\xi = kTF\Delta f$ = Rx noise power,
$k = 1.38 \cdot 10^{-23}$ [J/K] = Boltzman constant,
Δf = bandwidth [Hz],
F = Rx noise figure (typically a few dB)
T = Rx temperature [deg. K]

P_x: from the link budget

The Rx signal power P_x is

$$P_x = P_t \frac{G_t G_r}{L_p} \tag{8}$$

P_t = Tx signal power,
G_t, G_r = Tx and Rx antenna gains,
L_p = propagation channel path loss (large, 50...150 dB or even more).

Fade margin and other losses can be added too (to make it worse (:

SNR: impact of antennas

- via antenna gain G
- isotropic antenna: $G = 1$
- nearly isotropic: G is close to 1
- highly-directional antenna: large $G \gg 1$
- antenna array of N elements (antennas): $G = N$ in many cases

G_t can be accounted for via effective isotropic radiated power (EIRP):

$$P_E = P_t \cdot G_t \tag{9}$$

Impact of Antennas on the Capacity

- With the old-fashioned use of directional antennas,

$$C = \Delta f \log(1 + G_t G_r \gamma_{iso}) \tag{10}$$

γ_{iso} = Rx SNR with isotropic antennas (when $G_t = G_r = 1$).

- but it increases with G_t, G_r only logarithmically (very slow)
- Can we do better???

Impact of Antennas: Historical Perspective

- What is the best way to use antenna arrays?
- SISO
- MISO/SIMO
- MIMO

SISO: single antennas at both ends

- single-antenna systems: $N = 1 = G_t = G_r$

$$C = \log(1+\gamma) \text{ [bit/s/Hz]} \tag{11}$$

where $\gamma = \gamma_{iso}$

- SE is not large (unless the SNR is very large)
- fading degrades performance
- simple design

Multiple Antennas (Array) at One End: MISO/SIMO

- Antenna array at one end (beamforming)

$$C = \log(1 + N\gamma) \tag{12}$$

- SE is larger, but not much (only logarithmic in N)
- fading can be reduced
- more complex design (N antennas + circuitry)

Multiple Antennas at Both Ends: Old Fashion

- Try old-fashioned use of antenna arrays at both ends
- Tx + Rx beamforming: $G_t = G_r = N > 1$,

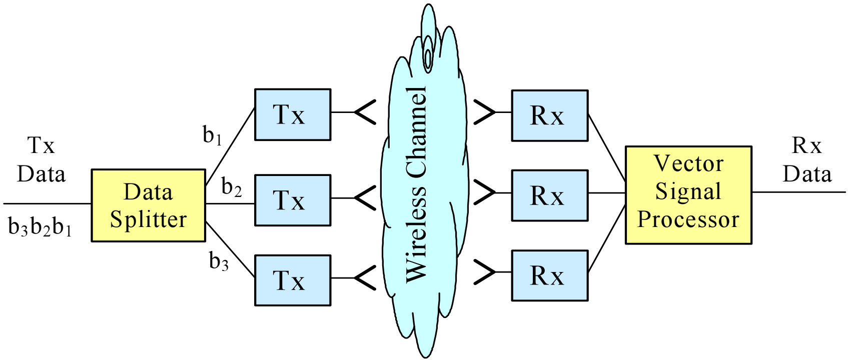
$$C = \Delta f \log(1 + N^2 \gamma) \tag{13}$$

- larger SE, but still only logarithmic in N -> very slow increase
- fading can be reduced
- more complex design ($2 \cdot N$ antennas $+ 2 \cdot N$ circuitry)

Multiple Antennas at Both Ends: Old Fashion

Can we do better?

True MIMO: launch multiple bit streams!

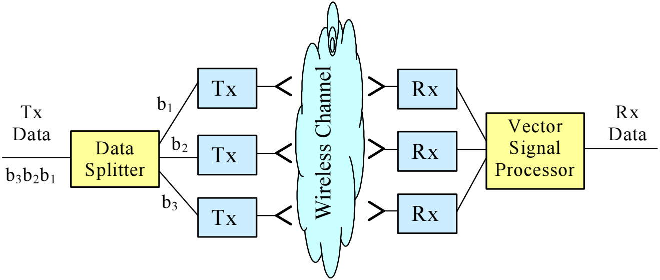


- Multi-stream transmission (not beamforming):

$$C = \log |\mathbf{I} + \gamma \mathbf{H}\mathbf{H}^{+}| \tag{14}$$

where **H** is the channel matrix.

True MIMO: launch multiple bit streams!

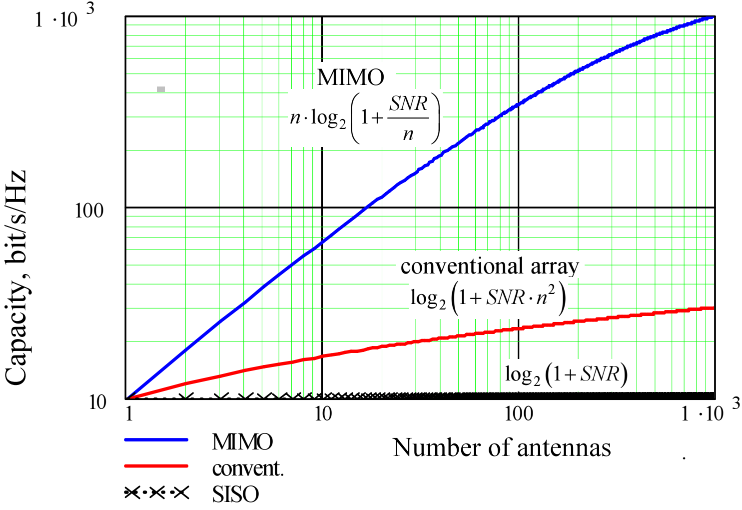


- Under favorable propagation,

$$C = N \log(1 + \gamma/N) \tag{15}$$

  i.e. almost linear in N -> much faster increase !
- Much larger SE with large N!
- large N -> massive MIMO = key technology for 5G.

True MIMO: launch multiple bit streams!



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Summary

- brief review of communications
- wireless & digital communications
- key performance metrics
- fundamental limits
- impact of antennas