

CSI5165: Revolving Door Algorithms: Examples

Lucia Moura

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Ranking Algorithm Example 1

$$\text{RANK}(136) = r = ?$$

Look at $\downarrow \binom{6}{3} = +20$, then $\uparrow \binom{3}{2} = -3$, then $\downarrow \binom{1}{1} = +1$ then -1

123

156

134

256

234

356

124

456

145

146

245

246

345

346

135

 $\underline{1}\bar{3}6 \quad -\binom{3}{2} = -3 \downarrow +\binom{1}{1} = 1 - 1 \quad r = 17$

235

236

125

 $12\underline{6} \quad +\binom{6}{3} = +20 \uparrow$

Ranking Algorithm Example 2

$$\text{RANK}(245) = r = ?$$

Look at $\downarrow \binom{5}{3} = +10$, then $\uparrow \binom{4}{2} = -6$, then $\downarrow \binom{2}{1} = +2$ then -1

123		156
134		256
234		356
124		456
<u>1</u> 45	$-\binom{4}{2} = -6 \downarrow$	146
<u>2</u> 45	$+\binom{2}{1} = +2 - 1$	246
345		346
135		136
235		236
1 <u>2</u> 5	$+\binom{5}{3} = +10 \uparrow$	126

Unranking Algorithm: Example 1

- $n = 6$, $k = 3$, $r = 12$, $T = [?, ?, ?]$

Unranking Algorithm: Example 1

- $n = 6, k = 3, r = 12, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, [12] \dots, \binom{5}{3} = 10, \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$

Unranking Algorithm: Example 1

- $n = 6, k = 3, r = 12, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, [12] \dots, \binom{5}{3} = 10, \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 5, k = 2, r = 20 - 12 - 1 = 7, T = [?, ?, 6]$

Unranking Algorithm: Example 1

- $n = 6, k = 3, r = 12, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots [12] \dots, \binom{5}{3} = 10, \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 5, k = 2, r = 20 - 12 - 1 = 7, T = [?, ?, 6]$
- $\binom{5}{2} = 10, \dots [7] \dots, \binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, \binom{2}{2} = 1$

Unranking Algorithm: Example 1

- $n = 6, k = 3, r = 12, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots [12] \dots, \binom{5}{3} = 10, \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 5, k = 2, r = 20 - 12 - 1 = 7, T = [?, ?, 6]$
- $\binom{5}{2} = 10, \dots [7] \dots, \binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, \binom{2}{2} = 1$
- $n = 4, k = 1, r = 10 - 7 - 1 = 2, T = [?, 5, 6]$

Unranking Algorithm: Example 1

- $n = 6, k = 3, r = 12, T = [?, ?, ?]$
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- $n = 5, k = 2, r = 20 - 12 - 1 = 7, T = [?, ?, 6]$
- $\binom{5}{2} = 10, \dots [7] \dots, \binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, \binom{2}{2} = 1$
- $n = 4, k = 1, r = 10 - 7 - 1 = 2, T = [?, 5, 6]$
- $\binom{4}{1} = 4, \dots, \binom{3}{1} = 3, \dots [2], \dots, \binom{2}{1} = 2, \dots, \binom{1}{1} = 1$

Unranking Algorithm: Example 1

- $n = 6, k = 3, r = 12, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots [12] \dots, \binom{5}{3} = 10, \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 5, k = 2, r = 20 - 12 - 1 = 7, T = [?, ?, 6]$
- $\binom{5}{2} = 10, \dots [7] \dots, \binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, \binom{2}{2} = 1$
- $n = 4, k = 1, r = 10 - 7 - 1 = 2, T = [?, 5, 6]$
- $\binom{4}{1} = 4, \dots, \binom{3}{1} = 3, \dots [2], \dots, \binom{2}{1} = 2, \dots, \binom{1}{1} = 1$
- $T = [3, 5, 6]$

Unranking Algorithm: Example 2

- $n = 6$, $k = 3$, $r = 7$, $T = [?, ?, ?]$

Unranking Algorithm: Example 2

- $n = 6, k = 3, r = 7, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, \binom{5}{3} = 10, \dots, [7] \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$

Unranking Algorithm: Example 2

- $n = 6, k = 3, r = 7, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, \binom{5}{3} = 10, \dots, [7] \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 4, k = 2, r = 10 - 7 - 1 = 2, T = [?, ?, 5]$

Unranking Algorithm: Example 2

- $n = 6, k = 3, r = 7, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, \binom{5}{3} = 10, \dots, [7] \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 4, k = 2, r = 10 - 7 - 1 = 2, T = [?, ?, 5]$
- $\binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, [2] \dots, \binom{2}{2} = 1$

Unranking Algorithm: Example 2

- $n = 6, k = 3, r = 7, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, \binom{5}{3} = 10, \dots, [7] \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 4, k = 2, r = 10 - 7 - 1 = 2, T = [?, ?, 5]$
- $\binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, [2] \dots, \binom{2}{2} = 1$
- $n = 2, k = 1, r = 3 - 2 - 1 = 0, T = [?, 3, 5]$

Unranking Algorithm: Example 2

- $n = 6, k = 3, r = 7, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, \binom{5}{3} = 10, \dots [7] \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 4, k = 2, r = 10 - 7 - 1 = 2, T = [?, ?, 5]$
- $\binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots [2] \dots, \binom{2}{2} = 1$
- $n = 2, k = 1, r = 3 - 2 - 1 = 0, T = [?, 3, 5]$
- $\binom{2}{1} = 2, \dots, \binom{1}{1} = 1, \dots [0]$

Unranking Algorithm: Example 2

- $n = 6, k = 3, r = 7, T = [?, ?, ?]$
- $\binom{6}{3} = 20, \dots, \binom{5}{3} = 10, \dots, [7] \dots, \binom{4}{3} = 4, \dots, \binom{3}{3} = 1$
- $n = 4, k = 2, r = 10 - 7 - 1 = 2, T = [?, ?, 5]$
- $\binom{4}{2} = 6, \dots, \binom{3}{2} = 3, \dots, [2] \dots, \binom{2}{2} = 1$
- $n = 2, k = 1, r = 3 - 2 - 1 = 0, T = [?, 3, 5]$
- $\binom{2}{1} = 2, \dots, \binom{1}{1} = 1, \dots, [0]$
- $T = [1, 3, 5]$

Successor Algorithm: Case A2

- { 1 2 3 7 10 12 }

Successor Algorithm: Case A2

- { 1 2 3 7 10 12 }
- { 1 2 3 7 10 12 }
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Successor Algorithm: Case A2

- { 1 2 3 7 10 12 }

- { 1 2 3 7 10 12 }

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- { ? ? ? 8 10 12 }

Successor Algorithm: Case A2

- { 1 2 3 7 10 12 }
- { 1 2 3 7 10 12 }

			↑	↓		
- { ? ? ? 8 10 12 }
- { 1 2 7 8 10 12 }

Successor Algorithm: Case A1

- { 1 2 3 7 8 12 }

Successor Algorithm: Case A1

• { 1 2 3 7 8 12 }

• { 1 2 3 7 8 12 }

↓ ↑ ↓

Successor Algorithm: Case A1

- { 1 2 3 7 8 12 }

- { 1 2 3 7 8 12 }

↓ ↑ ↓
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- { ? ? ? ? 7 12 }

Successor Algorithm: Case A1

- { 1 2 3 7 8 12 }

- { 1 2 3 7 8 12 }

↓ ↑ ↓

- { ? ? ? ? 7 12 }

- { 1 2 3 4 7 12 }

Successor Algorithm: Case B1

- { 1 2 3 4 7 12 }

Successor Algorithm: Case B1

- { 1 2 3 4 7 12 }

- { 1 2 3 4 7 12 }

↑ ↓ ↑ ↓

Successor Algorithm: Case B1

- { 1 2 3 4 7 12 }

- { 1 2 3 4 7 12 }

↑ ↓ ↑ ↓

- { ? ? ? 5 7 12 }

↑ ↓ ↑ ↓

Successor Algorithm: Case B1

- { 1 2 3 4 7 12 }

- { 1 2 3 4 7 12 }

↑ ↓ ↑ ↓

- { ? ? ? 5 7 12 }

↑ ↓ ↑ ↓

- { 1 2 4 5 7 12 }

Successor Algorithm: Case B2

- { 6 9 10 12 16 19 }

Successor Algorithm: Case B2

- { 6 9 10 12 16 19 }
- { 6 9 10 12 16 19 }
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Successor Algorithm: Case B2

- { 6 9 10 12 16 19 }
- { **6** 9 10 12 16 19 }
 ↑ ↓ ↑ ↓ ↑ ↓
- { **5** 9 10 12 16 19 }